CHAPTER 5
FREE CONVECTION

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Laboratorium Efisiensi dan Konservasi Energi

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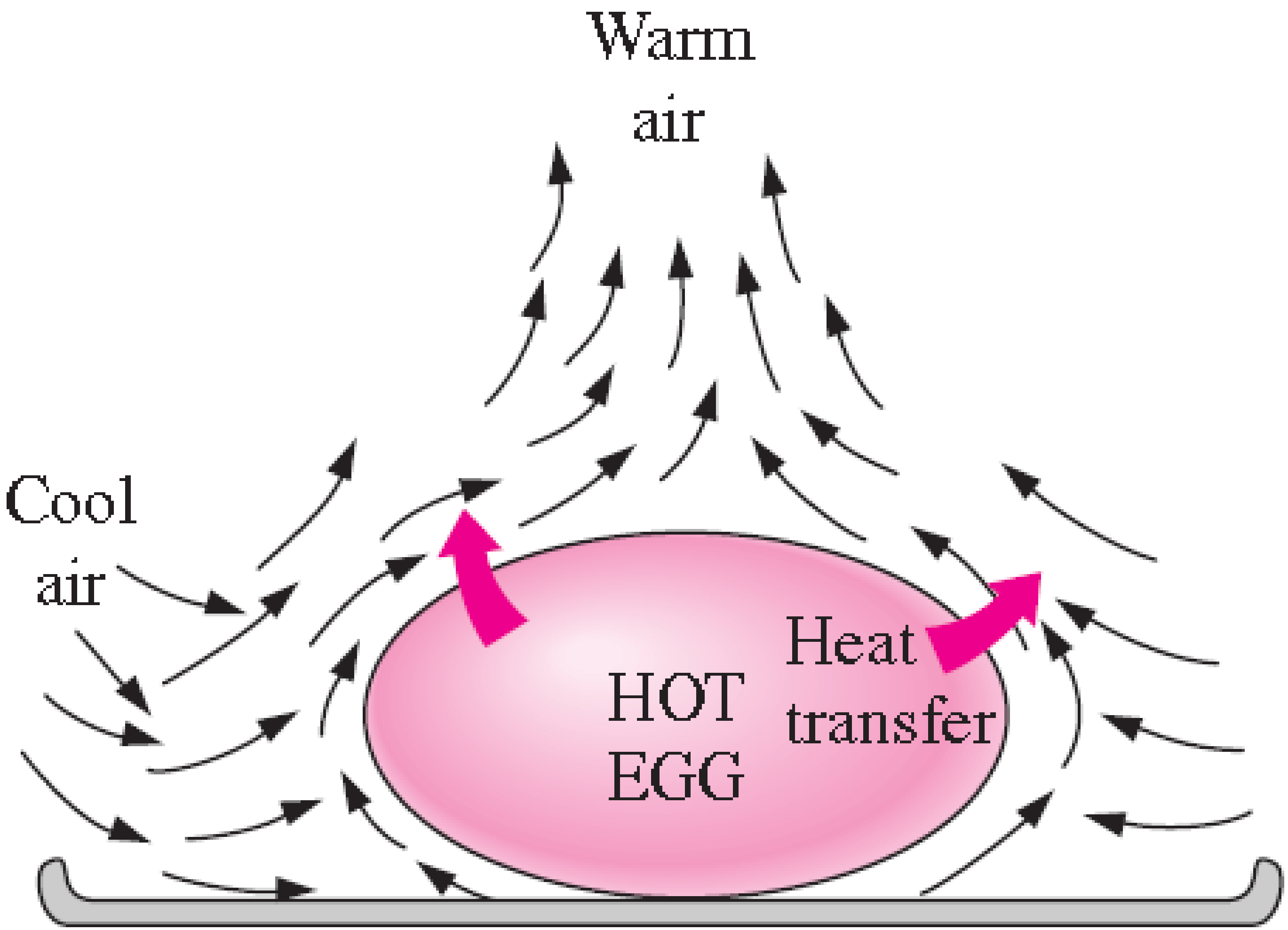
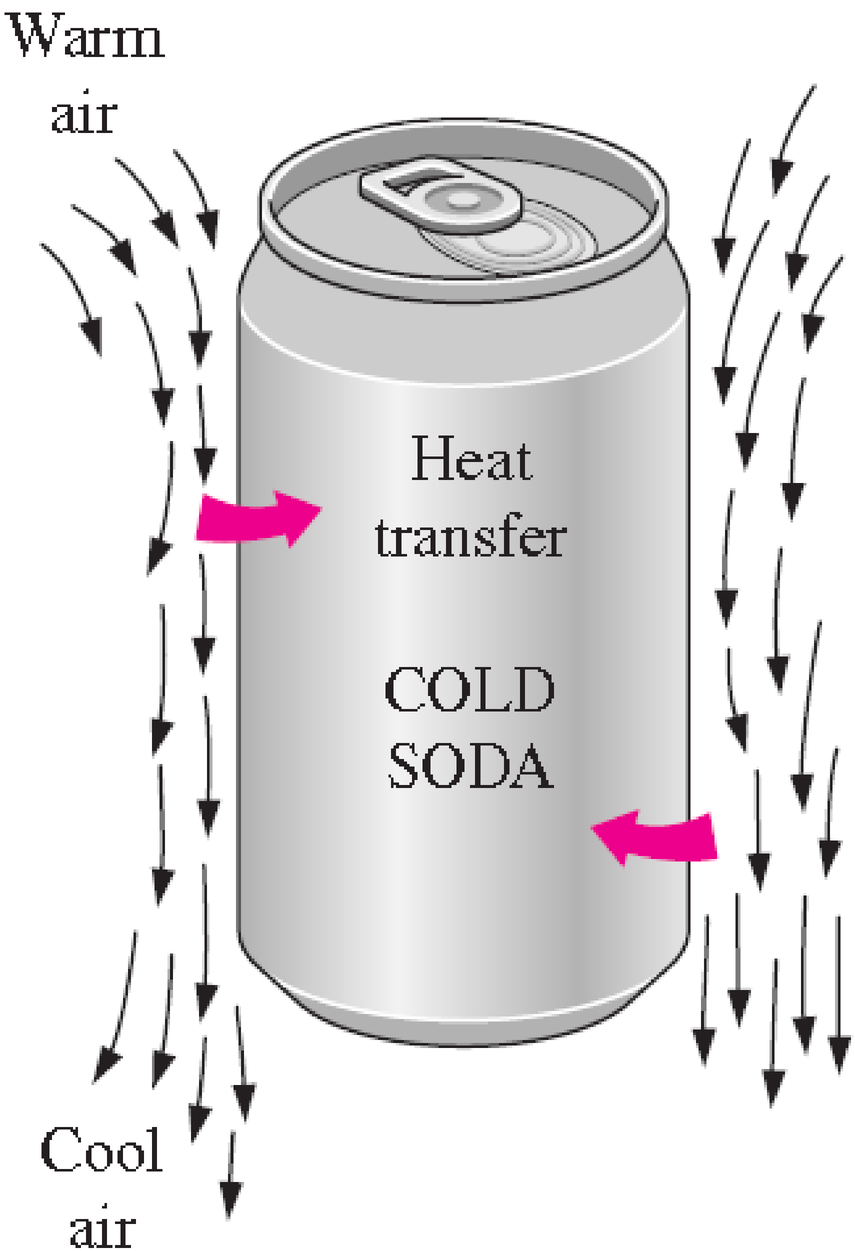
FREE CONVECTION



The cooling of a boiled egg in a cooler environment by natural convection.



The warming up of a cold drink in a warmer environment by natural convection

Natural Convection

Where we've been ……

- Up to now, have considered ***forced*** convection, that is an external driving force causes the flow.

Where we're going:

- Consider the case where fluid movement is by buoyancy effects caused by temperature differential




Events due to natural convection

- Weather events such as a thunderstorm
- Glider planes
- Radiator heaters
- Hot air balloon

- Heat flow through and on outside of a double pane window
- Oceanic and atmospheric motions
- Coffee cup example ….

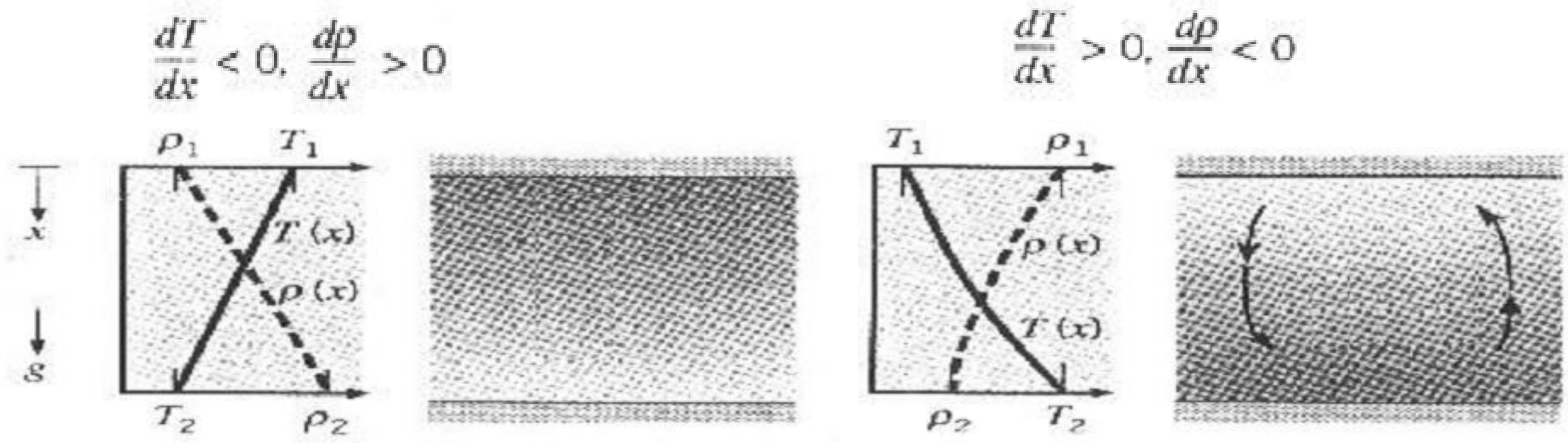
Small velocity

Natural Convection

- New terms
  - ***Volumetric thermal expansion coefficient***
  - ***Grashof number***
  - ***Rayleigh number***
- Buoyancy is the driving force
  - Stable versus unstable conditions
- Nusselt number relationship for laminar free convection on hot or cold surface
- Boundary layer impacts: laminar $\Rightarrow$ turbulent

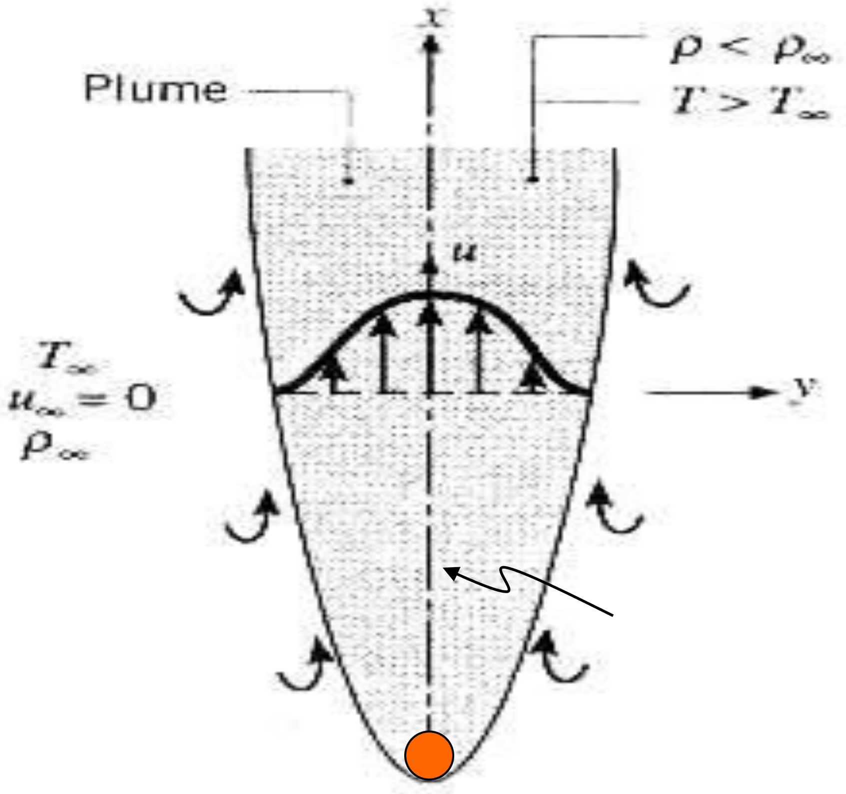
Buoyancy is the driving force in Natural Convection

- ***Buoyancy*** is due to combination of
  - Differences in fluid density
  - Body force proportional to density
  - Body forces namely, gravity, also Coriolis force in atmosphere and oceans
- Convection flow is driven by buoyancy in unstable conditions
- Fluid motion may be
  (no constraining surface) or along a surface



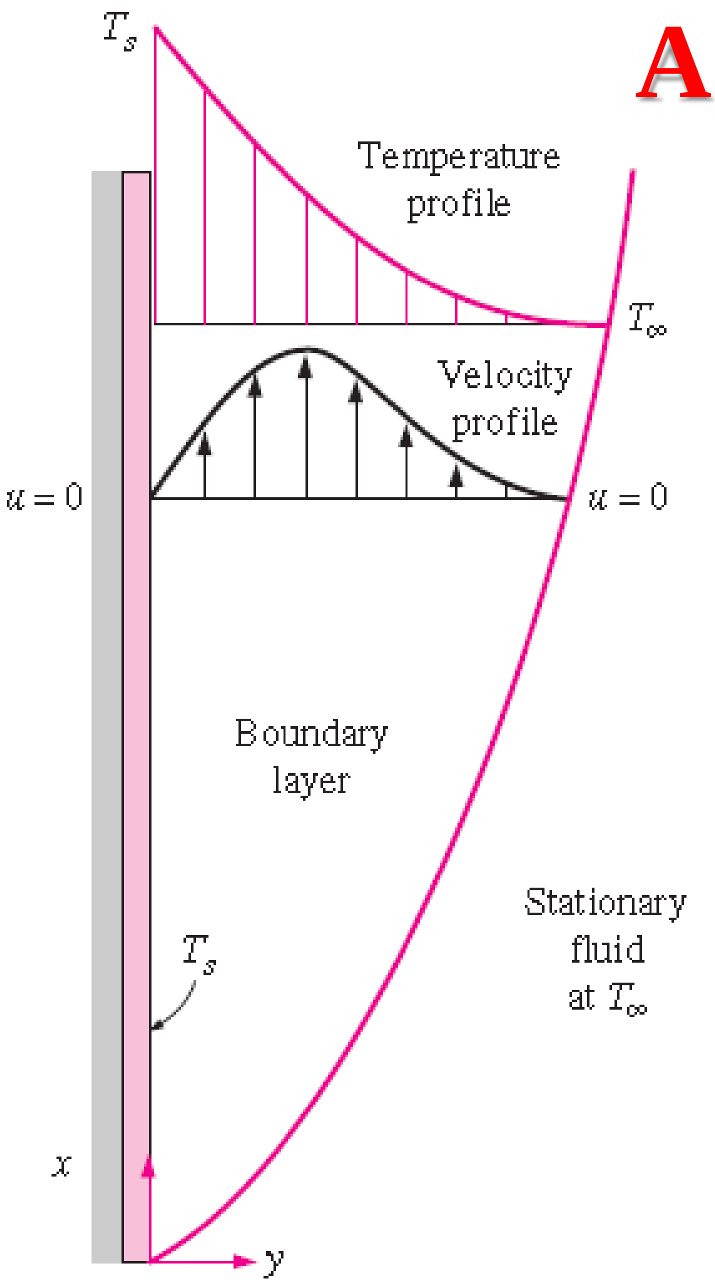
Buoyancy is the driving force

- Free boundary layer flow



Heated wire or hot pipe

A heated vertical plate



Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at T_s inserted in a fluid at temperature T_∞.

Natural Convection Boundary Layer : Governing Equations

- The difference between the two flows (forced flow and free flow) is that, in free convection, a major role is played by buoyancy forces.

$$X = -\rho g$$ ← Very important

•Consider the x-momentum equation.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - g + \nu\frac{\partial^2 u}{\partial y^2}$$

•As we know, $\partial p / \partial y = 0$, hence the x-pressure gradient in the boundary layer must equal that in the quiescent region outside the boundary layer.

Pascal Law :

$$\frac{\partial P}{\partial x} = -\rho_\infty g$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\left(-\rho_\infty g\right) - \mathrm{g} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \mathrm{g}\left(\frac{\Delta\rho}{\rho}\right) + \nu\frac{\partial^2 u}{\partial y^2}$$

Buoyancy force $\Delta\rho = \rho_\infty - \rho$

Governing Equations

- Define β, the volumetric thermal expansion coefficient.

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

For all liquids and gases

$$\beta \approx -\frac{1}{\rho}\frac{\Delta \rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T}$$

$$\rho_\infty - \rho \approx \rho\beta(T - T_\infty)$$

Density gradient is due to the temperature gradient

$$\text{For an ideal gas: } P = \frac{RT}{\rho} \Rightarrow \rho = \frac{P}{RT}$$

$$Thus: \beta = \frac{1}{T}$$

Governing Equations

- Buoyancy effects replace pressure gradient in the momentum equation.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= g\beta(T-T_\infty)+\nu\frac{\partial^2 u}{\partial y^2}$$

•The buoyancy effects are confined to the momentum equation, so the mass and energy equations are the same.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

Strongly coupled and must be solved simultaneously

Dimensionless Similarity Parameter

$$x^* \equiv \frac{x}{L} \quad and \quad y^* \equiv \frac{y}{L}$$

$$u^* \equiv \frac{u}{u_0} \quad and \quad v^* \equiv \frac{v}{u_0} \qquad T^* = \frac{\mathrm{T} - \mathrm{T}_\infty}{T_s - \mathrm{T}_\infty}$$

where L is a characteristic length, and

u_0 is an arbitrary reference velocity

- ***The x-momentum and energy equations are***

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{g\beta(T_s - T_\infty)L}{u_0^2}\mathrm{T}^* + \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L \Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$$

Dimensionless Similarity Parameter

- Define new dimensionless parameter,

$$Gr_L = \frac{g\beta(T_s - T_\infty)L}{u_0^2}\left(\frac{u_0 L}{\nu}\right)^2 = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

•***Grashof number*** in natural convection is analogous to the Reynolds number in forced convection.

•***Grashof number*** indicates the ratio of the buoyancy force to the viscous force.

•Higher Gr number means increased natural convection flow

$$\frac{Gr_L}{\mathrm{Re}_L^2} << 1 \quad \text{forced}$$

$$\frac{Gr_L}{\mathrm{Re}_L^2} >> 1 \quad \text{natural}$$

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$ for vertical flat plates

$$\mathrm{Gr}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}$$ for pipes

$$\mathrm{Gr}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}$$ for bluff bodies

The transition to turbulent flow occurs in the range for natural convection from vertical flat plates. At higher Grashof numbers, the boundary layer is turbulent; at lower Grashof numbers, the boundary layer is laminar.

where the L and D subscripts indicates the length scale basis for the Grashof Number.

g = acceleration due to Earth's gravity

β = volumetric thermal expansion coefficient (equal to approximately 1/T, for ideal fluids, where T is absolute temperature)

T_s = surface temperature

T_∞ = bulk temperature

L = length

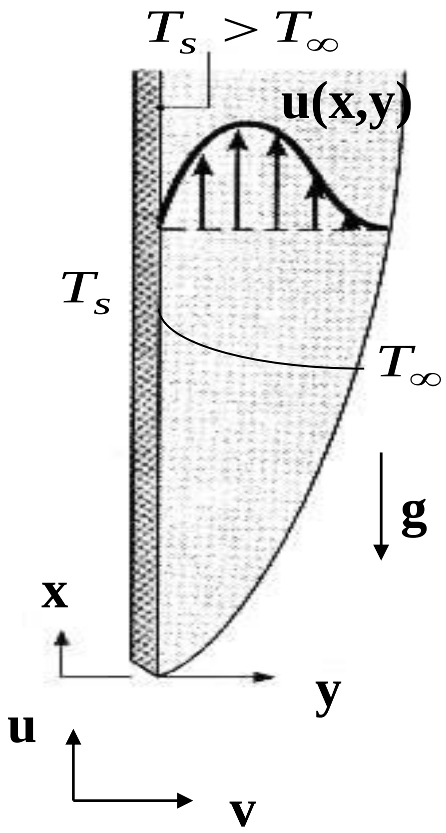
D = diameter

ν = kinematic viscosity

Franz Grashof

Born	11 July 1826 Düsseldorf, Germany
Died	26 October 1893 (aged 67) Karlsruhe, Germany
Nationality	German
Fields	Engineering

Laminar Free Convection on Vertical Surface



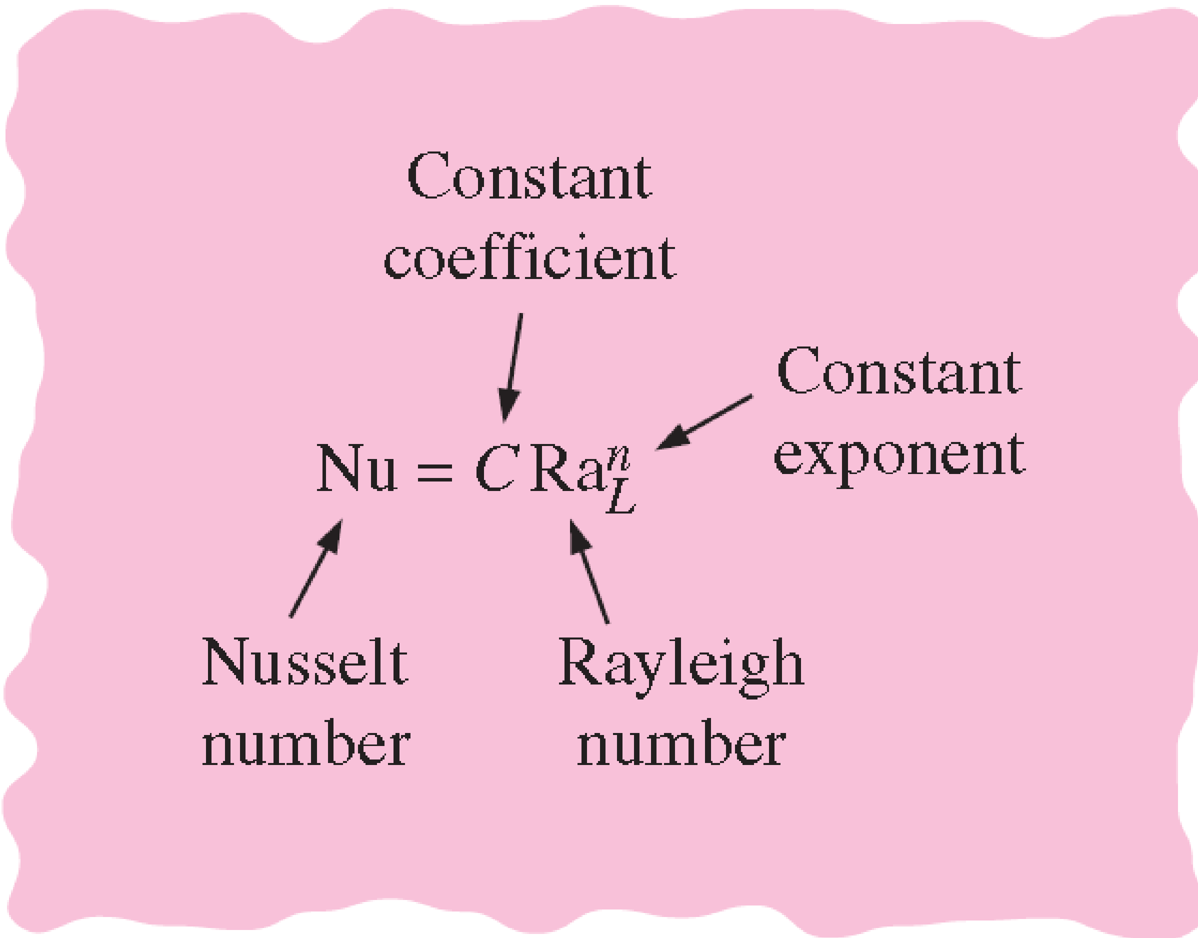
- As $y \to \infty$: $u = 0$, $T = T_\infty$
- As $y \to 0$: $u = 0$, $T = T_s$

- With little or no external driving flow, Re $\cong$ 0 and forced convection effects can be safely neglects

$$\frac{Gr_L}{\mathrm{Re}_L^2} >> 1$$

$$Nu_L = f(Gr_L, \mathrm{Pr})$$

The simple empirical correlations for the average *Nusselt number* Nu in natural convection are of the form :



$$\mathrm{Nu} = \frac{hL_c}{k} = C(\mathrm{Gr}_L\,\mathrm{Pr})^n = C\,\mathrm{Ra}_L^n$$

where Ra_L is the **Rayleigh number,** which is the product of the Grashof and Prandtl numbers:

$$\mathrm{Ra}_L = \mathrm{Gr}_L\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\mathrm{Pr}$$

- The values of the constants *C* and *n* depend on the *geometry* of the surface and the *flow regime*, which is characterized by the range of the Rayleigh number.
- The value of *n* is usually ¼ for laminar flow and 1/3 for turbulent flow, while the value of the constant *C* is normally less than 1.
- All fluid properties are to be evaluated at the film temperature $T_f = (T_s + T\infty)/2$.

Empirical solution for the ***local Nusselt number*** in laminar free convection

$$Nu_x = \frac{hx}{k} = \left(\frac{Gr_L}{4}\right)^{1/4} \cdot f(\mathrm{Pr})$$

Where

$$f(\mathrm{Pr}) = \frac{0.75\sqrt{\mathrm{Pr}}}{\left(0.609 + 1.221\sqrt{\mathrm{Pr}} + 1.238\,\mathrm{Pr}\right)^{1/4}}$$

Average Nusselt # =

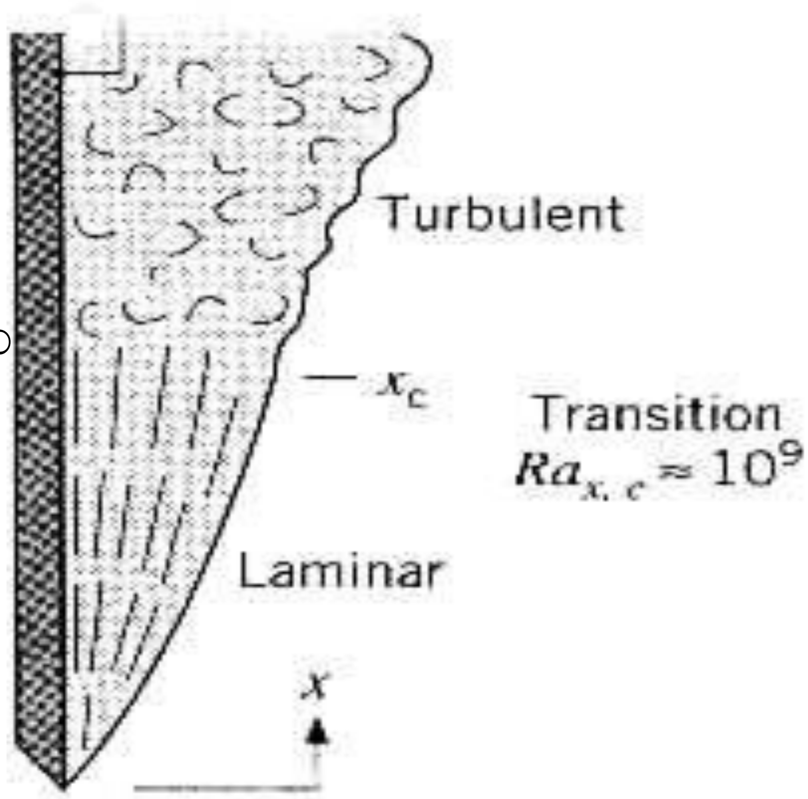
$$\overline{Nu_L} = \frac{\bar{h}\,L}{k} = \frac{4}{3}\left(\frac{Gr_L}{4}\right)^{1/4} \cdot f(\mathrm{Pr})$$

Effects of Turbulence

- Just like in forced convection flow, hydrodynamic instabilities may result in the flow.
- For example, illustrated for a heated vertical surface:
- Define the ***Rayleigh number*** for relativemagnitude of buoyancy and viscous forces

$$Ra_{x,c} = Gr_{x,c} \Pr \qquad T_s > T_\infty$$

$$= \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha}$$



Empirical Correlations

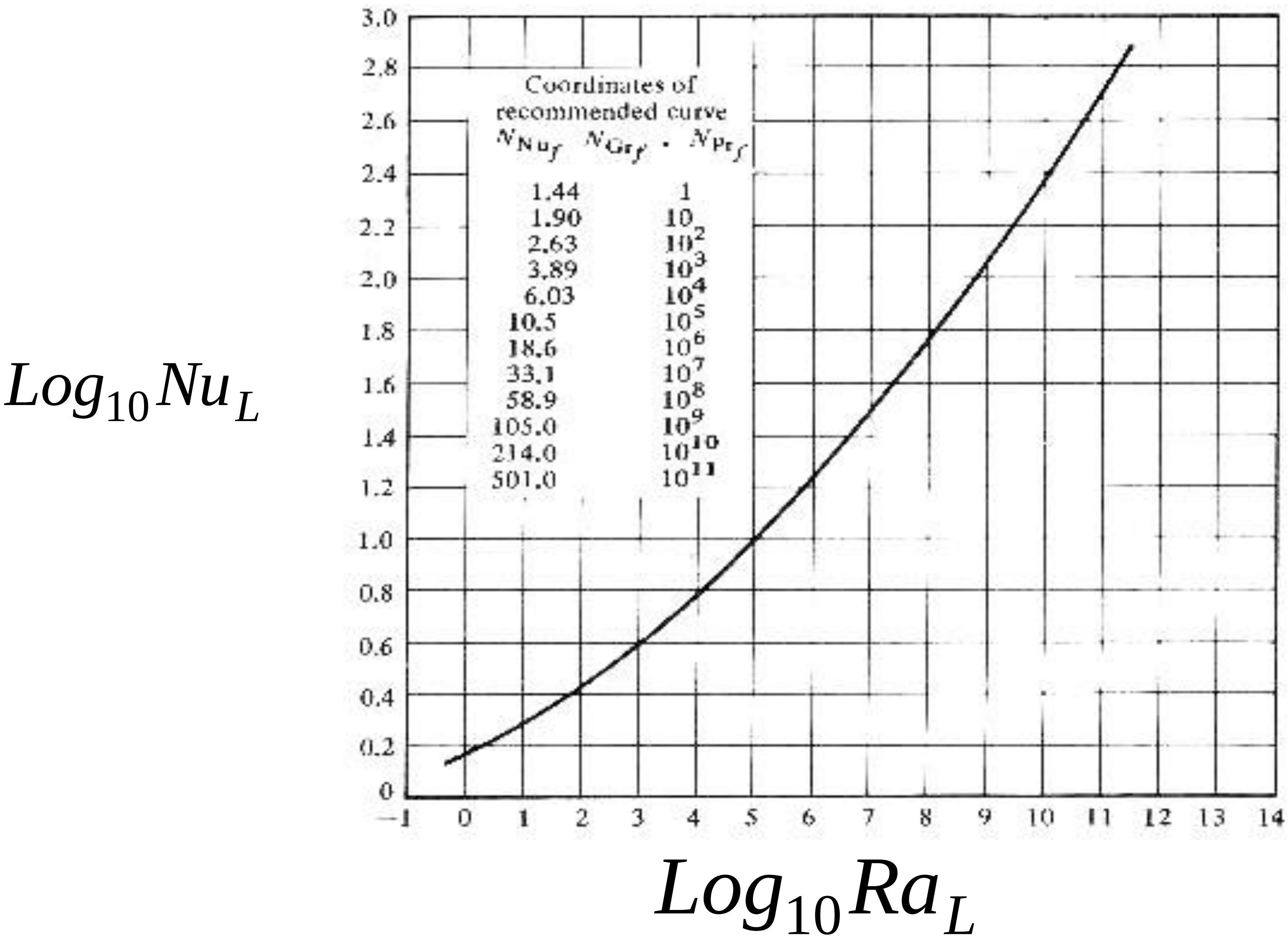
Typical correlations for heat transfer coefficient developed from experimental data are expressed as:

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = CRa_L^n$$

$$Ra_L = Gr_L \cdot \mathrm{Pr} = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

$$\begin{cases} n = 1/4 & \text{For Turbulent} \\ n = 1/3 & \text{For Laminar} \end{cases}$$

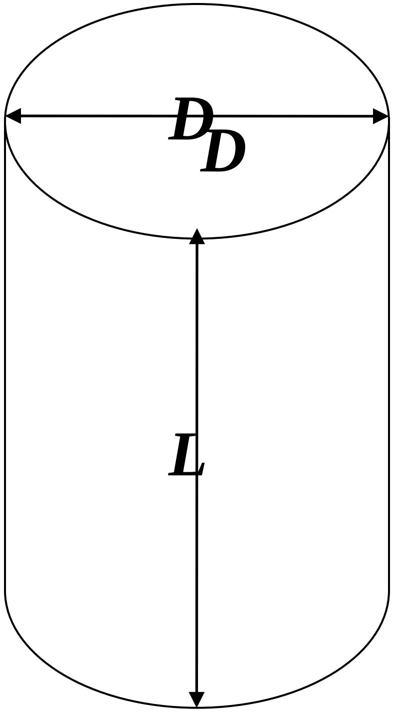
Vertical Plate at constant *Ts*



•Alternative applicable to entire Rayleigh number range (for constant *Ts*)

$$\overline{Nu_L} = \left\{ 0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + (0.492/\mathrm{Pr})^{9/16}\right]^{8/27}} \right\}^2$$

Vertical Cylinders

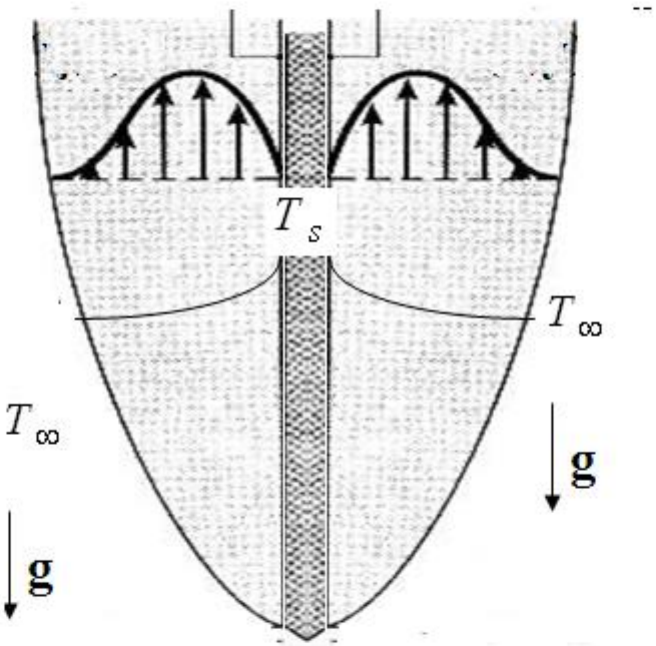


•Use same correlations for vertical flat plate if:

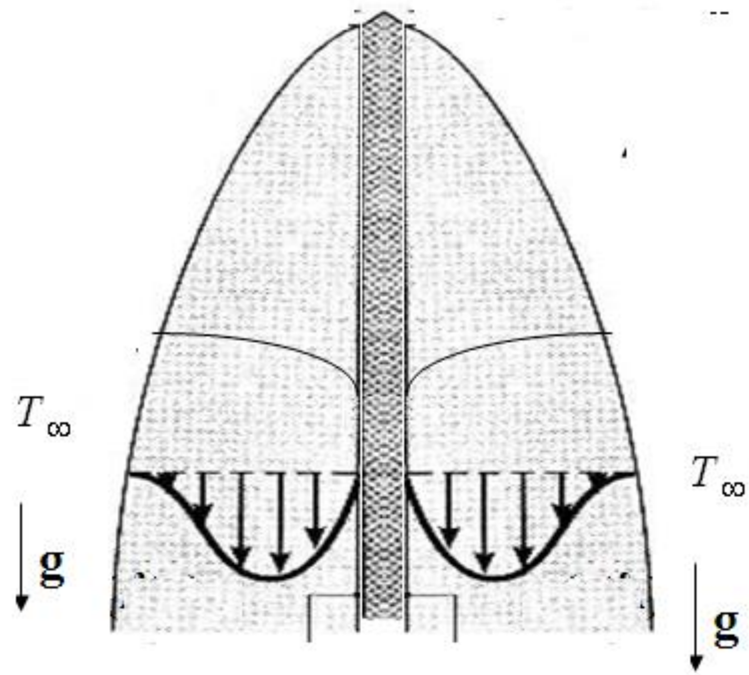
$$\frac{D}{L} \gtrsim \frac{35}{Gr_L^{1/4}}$$

Free Convection : Vertical Plate

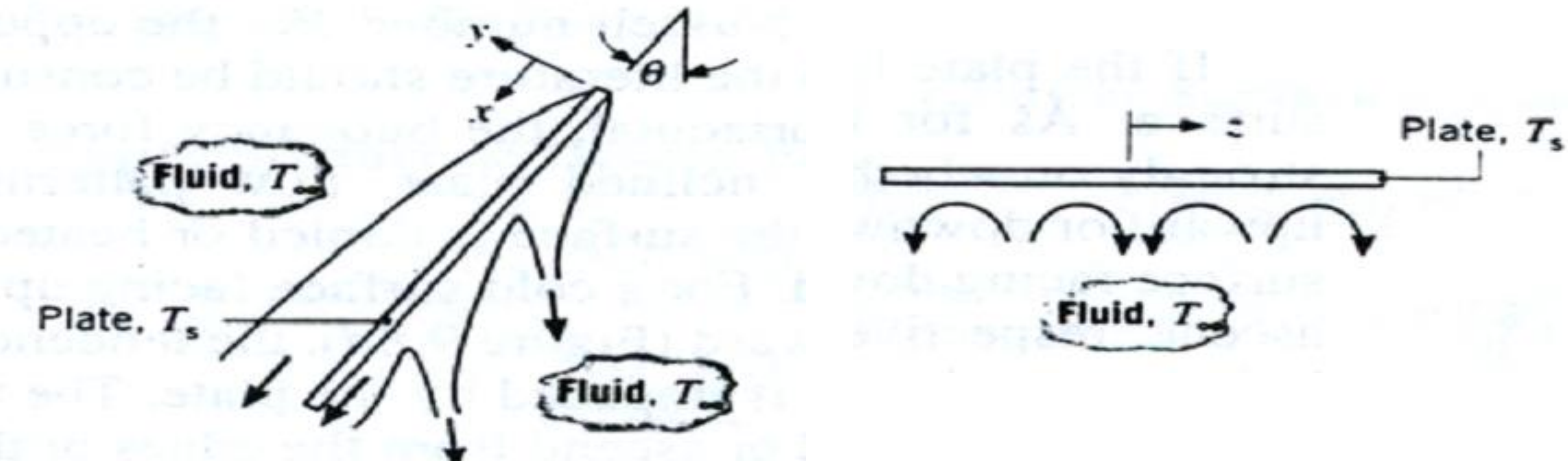
Hot plate or Cold fluid



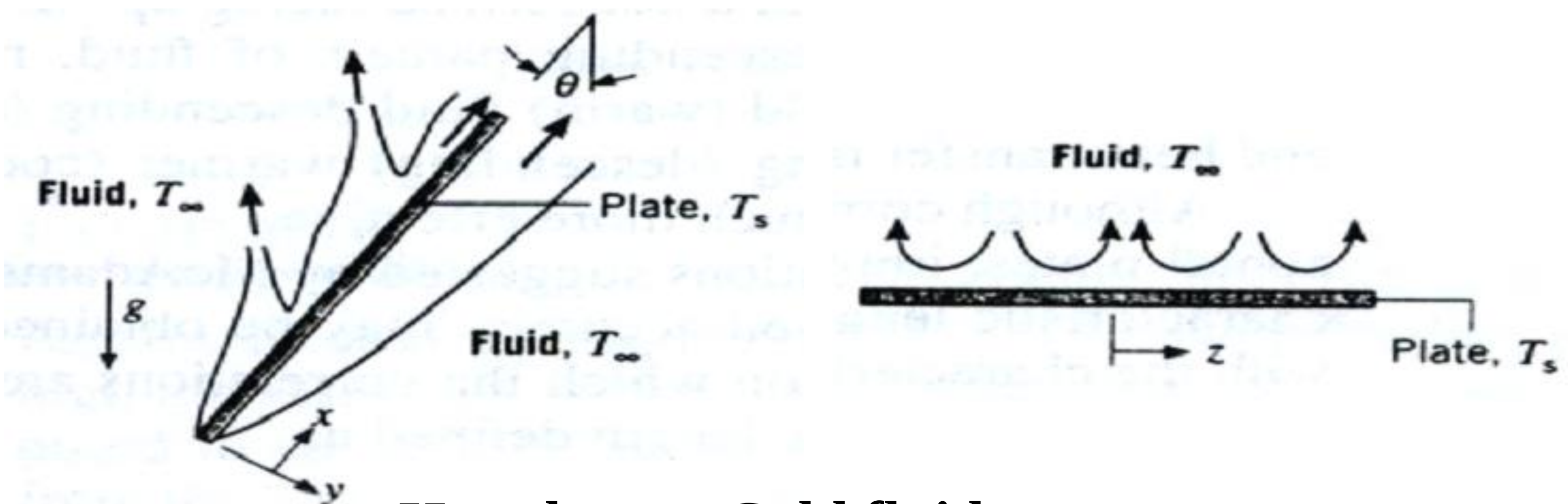
Cold plate or Hot fluid



Free Convection from Inclined Plate



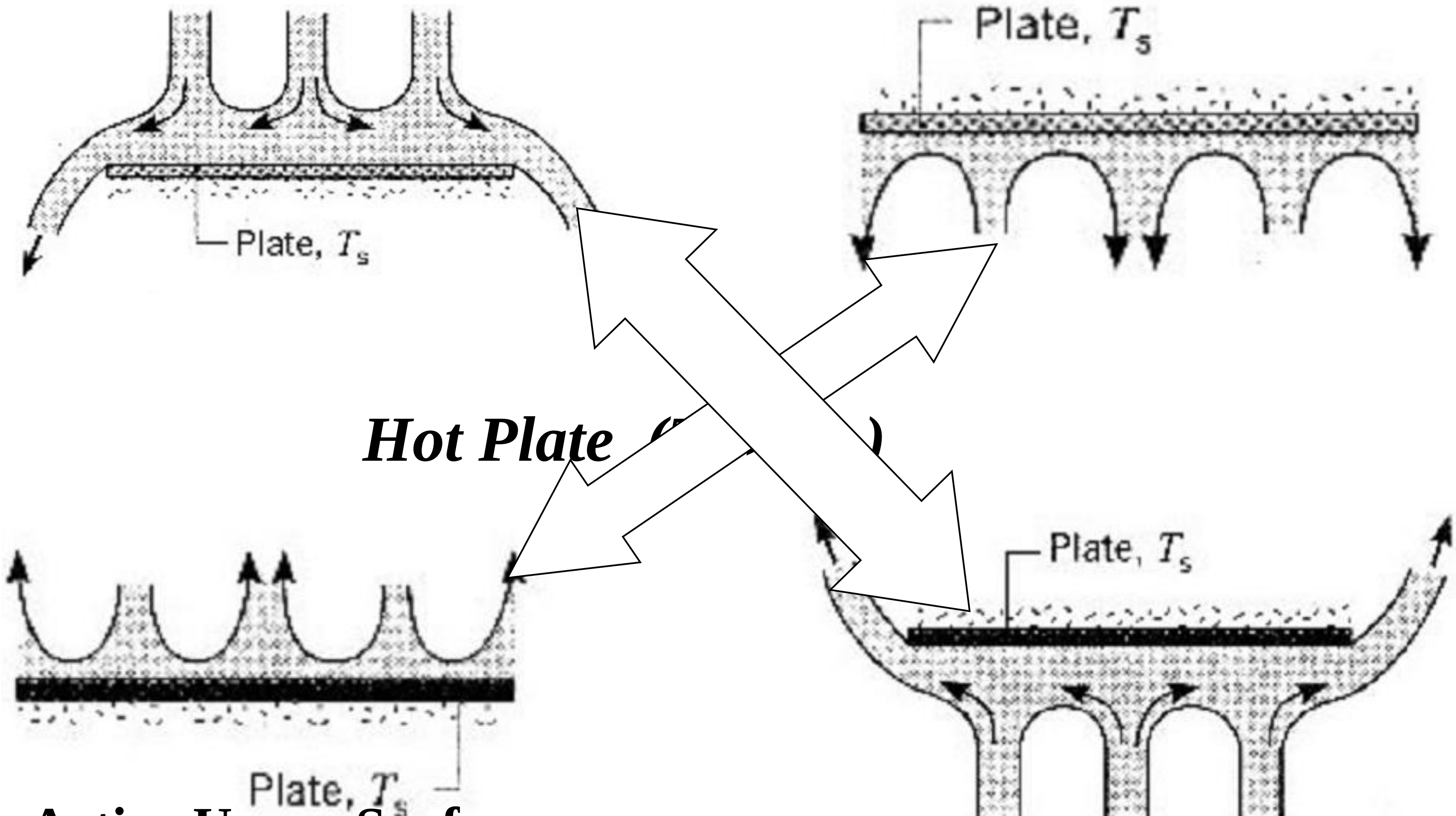
Cold plate or Hot fluid



Hot plate or Cold fluid

Horizontal Plate

Cold Plate ($T_s < T_\infty$)



Hot Plate

Active Upper Surface

Active Lower Surface

Empirical Correlations : **Horizontal Plate**

- Define the characteristic length, *L* as

$$L \equiv \frac{A_s}{P}$$

- **Upper surface of heated plate, or Lower surface of cooled plate :**

$$\overline{Nu_L} = 0.54\, Ra_L^{1/4} \qquad \left(10^4 \le Ra_L \le 10^7\right)$$

$$\overline{Nu_L} = 0.15\, Ra_L^{1/3} \qquad \left(10^7 \le Ra_L \le 10^{11}\right)$$

- **Lower surface of heated plate, or Upper surface of cooled plate :**

$$\overline{Nu_L} = 0.27\, Ra_L^{1/4} \qquad \left(10^5 \le Ra_L \le 10^{10}\right)$$

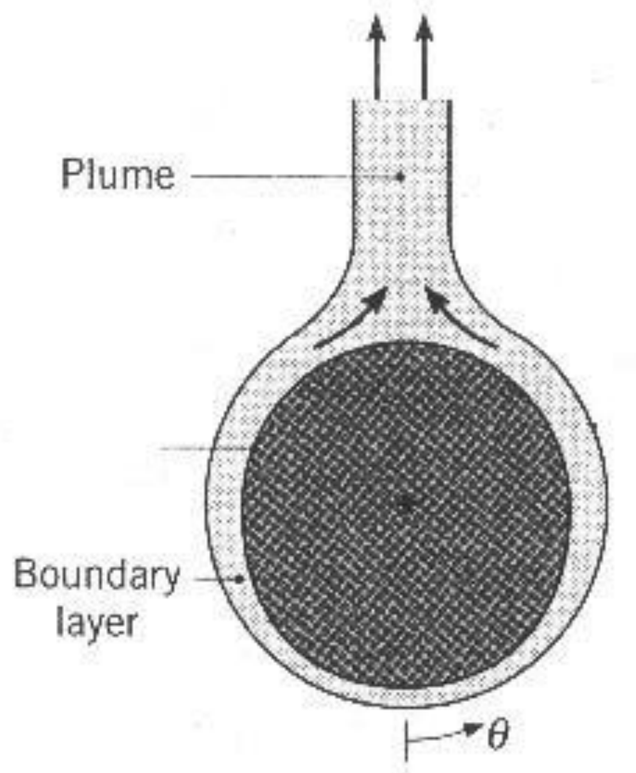
Note: Use fluid properties at the ***film temperature***

$$T_f = \frac{T_s + T_\infty}{2}$$

Empirical Correlations : **Long Horizontal Cylinder**

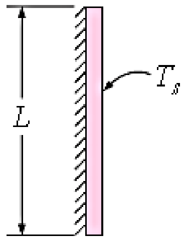
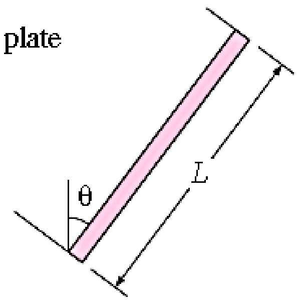
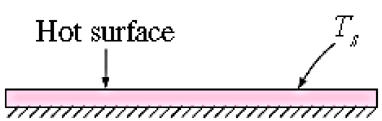
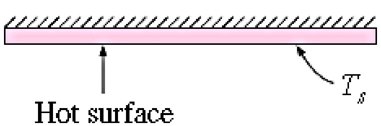
- Very common geometry (pipes, wires)
- For isothermal cylinder surface, use general form equation for computing Nusselt #

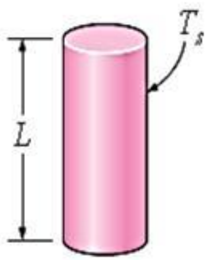
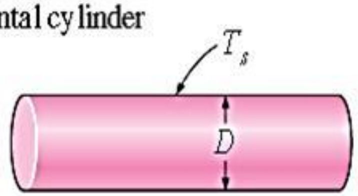
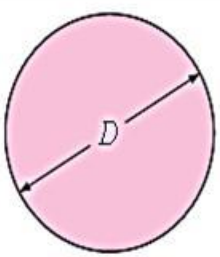
$$\overline{Nu_D} = \frac{\bar{h}D}{k} = CRa_D^n$$



Constants for general Nusselt number Equation

Ra_D	C	n
10^{-10} - 10^{-2}	0.675	0.058
10^{-2} - 10^{+2}	1.02	0.148
10^{2} - 10^{4}	0.850	0.188
10^{4} - 10^{7}	0.480	0.250
10^{7} - 10^{12}	0.125	0.333

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ (9-19) $Nu = 0.1Ra_L^{1/3}$ (9-20) $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $Ra < 10^9$
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ (9-22) $Nu = 0.15Ra_L^{1/3}$ (9-23) $Nu = 0.27Ra_L^{1/4}$ (9-24)

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$ (9-25)
Sphere 	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	$Nu = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)

The End

Terima kasih

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