INTERNAL FORCED CONVECTION

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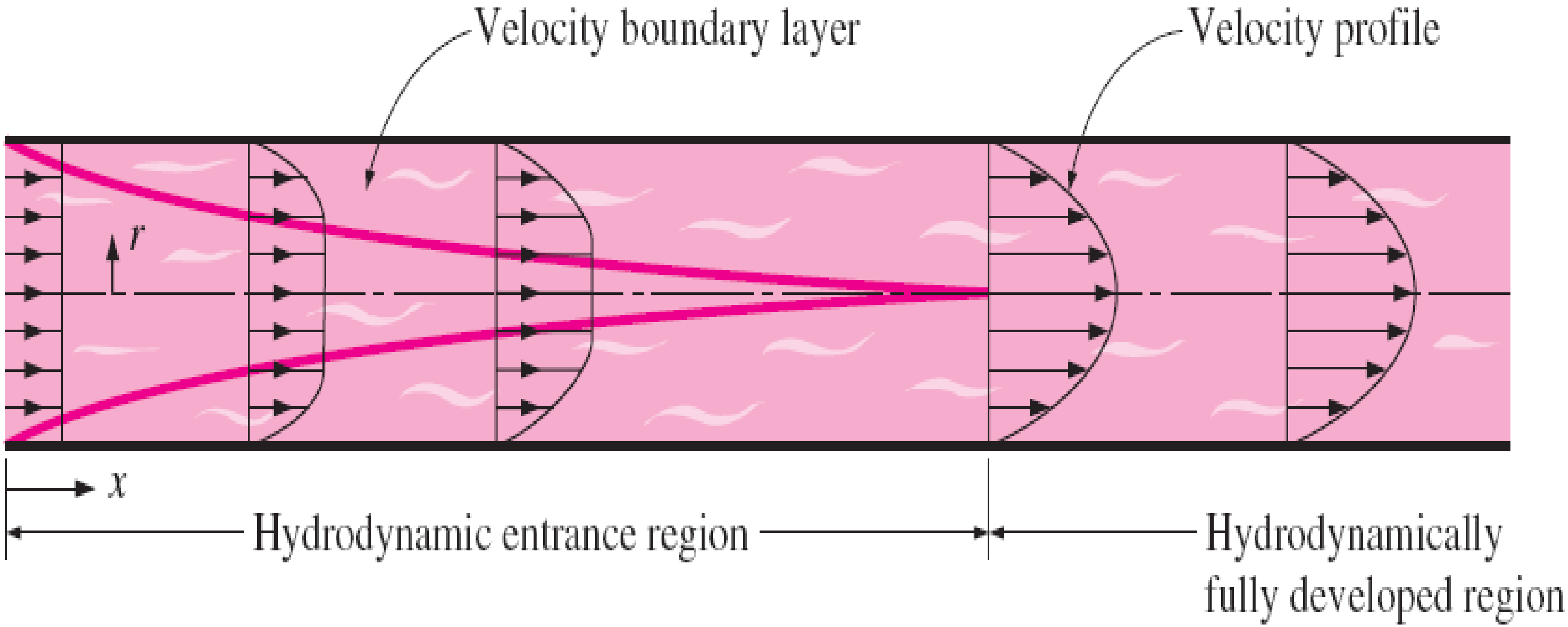
Internal Flow

The development of the boundary layer for *laminar* flow in a circular tube is represented in Fig. 17.11. Because of viscous effects, the uniform velocity profile at the entrance will gradually change to a parabolic distribution as the boundary layer begins to fill the tube in the entrance region.



Velocity boundary layer
Velocity profile
r
x
Hydrodynamic entrance region
Hydrodynamically
fully developed region

Beyond the *hydrodynamic entrance length*, the velocity profile no longer changes, and we speak of the flow as *hydrodynamically fully developed*. The extent of the entrance region, as well as the shape of the velocity profile, depends upon Reynolds number, which for internal flow has the form

$$\mathrm{Re}_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu} = \frac{4\dot{m}}{\pi D \mu}$$

where u_m *is the mean (average) velocity; D, the tube diameter, is the characteristic length;* and $\dot{m}$ is the mass flow rate. In a fully developed flow, the *critical Reynolds number corresponding* to the onset of turbulence is

$$Re_D \approx 2300$$

although much larger Reynolds numbers ($Re_D \approx 10{,}000$) are needed to achieve fully turbulent conditions. For *laminar flow* ($Re_D = 2300$), the *hydrodynamic entry length* has the form

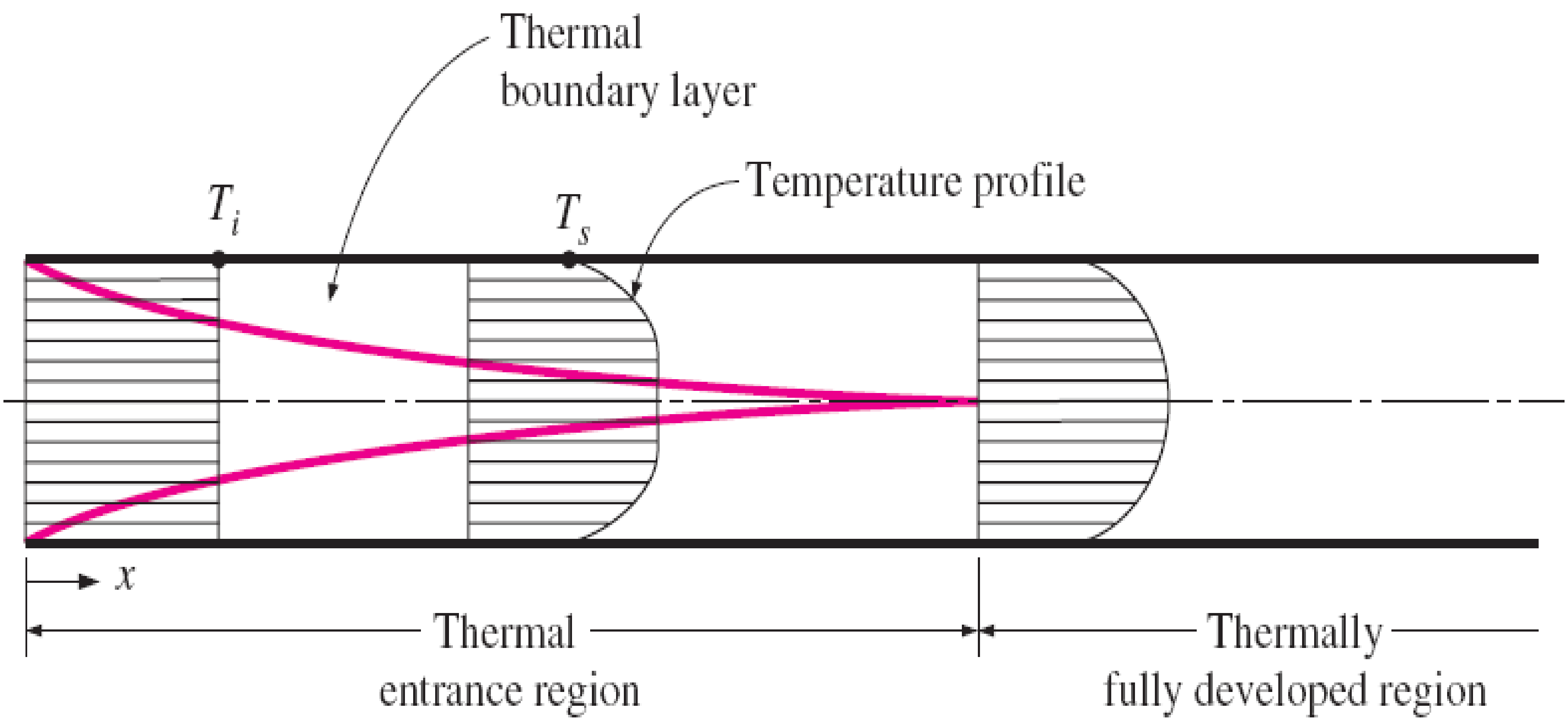
$$(X_L/D)_{lam} \leq 0.05\, Re_D$$

while for *turbulent flow,* the entry length is approximately independent of Reynolds number and that, as a first approximation

$$10 \le \left(\frac{x_{fd,h}}{D}\right)_{\text{turb}} \le 60 \qquad 17.40$$

For the purposes of this text, we shall assume fully developed turbulent flow for (x/D) > 10.

If fluid enters the tube at $x = 0$ with a uniform temperature $T(\mathbf{r,0})$ that is less than the *constant* tube surface temperature, $\mathbf{T_s}$, convection heat transfer occurs, and a thermal boundary layer begins to develop.



Thermal
boundary layer
Temperature profile
T_i
T_s
x
Thermal
entrance region
Thermally
fully developed region

In the ***thermal entrance region,*** the temperature of the central portion of the flow outside the *thermal boundary layer, δt,* remains unchanged, but in the boundary layer, the temperature increases sharply to that of the tube surface.

At the ***thermal entrance length,*** $x_{fd,t}$, the thermal boundary layer has filled the tube, the fluid at the centerline begins to experience heating, and the *thermally fully developed flow* condition has been reached. For *laminar flow,* the thermal entry length may be expressed as

$$\left(\frac{x_{fd,t}}{D}\right)_{\text{lam}} \le 0.05\,Re_D\,Pr \qquad [Re_D < 2300]$$

From this relation and by comparison of the hydrodynamic and thermal boundary layers of Fig. 17.11*a* and 17.11*b*, it is evident that we have represented a fluid with a $Pr < 1$ (gas), as the hydrodynamic boundary layer **has developed more slowly** than the thermal boundary layer ($x_{fd,h} > x_{fd,t}$). For liquids having $Pr > 1$, the inverse situation would occur.

$$Pr = \frac{\upsilon}{\alpha}$$

For *turbulent flow,* conditions are nearly independent of Prandtl number, and to a first approximation the *thermal entrance length* is

$$\left(\frac{x_{fd,t}}{D}\right)_{\text{turb}} = 10 \qquad [Re_D \geq 10{,}000]$$

The Mean Temperature.

The temperature and velocity profiles at a *particular* location in the flow direction *x* each depend on radius, *r*. The *mean* temperature of the fluid, also referred to as the average or bulk temperature, shown on the figure as $T_m(x)$, is defined in terms of the energy transported by the fluid as it moves past location *x*.

For incompressible flow, with constant specific heat c_p, the *mean temperature* is found from

$$T_m = \frac{\int_{A_c} uT\,dA_c}{u_m A_c}$$

where u_m is the mean velocity. For a circular tube, dA_c = 2πrdr, and it follows that :

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} uTrdr \qquad 17.44$$

The ***mean temperature*** is the fluid reference temperature used for determining the convection heat rate with Newton’s law of cooling and the overall energy balance.

Newton’s Law of Cooling. To determine the convective heat flux at the tube surface, Newton’s law of cooling, also referred to as the ***convection rate equation,*** is expressed as

$$q_s'' = q_{\text{conv}}'' = h(T_s - T_m)$$

where *h* is the *local* convection coefficient. Depending upon the method of surface heating (cooling), T_s can be a constant or can vary, but the mean temperature will always change in the flow direction. Still, the *convection coefficient is a constant for the fully developed conditions* we examine next.

Fully Developed Conditions. The temperature profile can be conveniently represented as the dimensionless ratio (T_s - T)/(T_s - T_m). While the temperature profile $T(r)$ continues to change with x, the *relative shape* of the profile given by this *temperature ratio* is independent of x for fully developed conditions. The requirement for such a condition is mathematically stated as

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right]_{fd,t} = 0 \qquad 17.46$$

where T_s is the tube surface temperature, T is the local fluid temperature, and T_m is the mean temperature. Since the temperature ratio is independent of x, the derivative of this ratio with respect to r must also be independent of x.

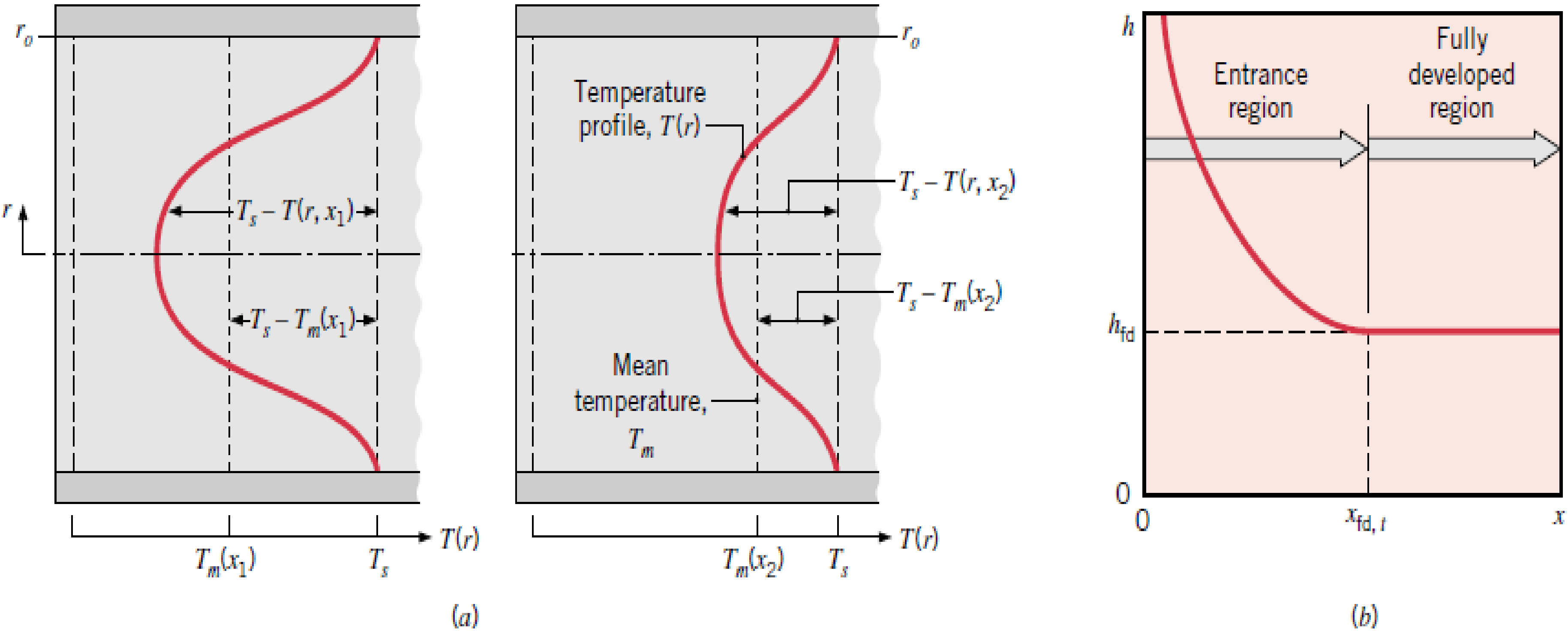


Figure 17.12 Thermally fully developed flow characteristics for constant surface temperature heating. (*a*) Relative shape of the temperature profile remains unchanged in the flow direction ($x_2 > x_1$). (*b*) Convection coefficient is constant for $x > x_{fd,t}$.

Evaluating this derivative at the tube surface (note that *Ts* and *Tm* are constants in sofar as differentiation with respect to *r* is concerned), we obtain

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\bigg|_{r=r_o} = \frac{-\partial T/\partial r|_{r=r_o}}{T_s - T_m} \neq f(x)$$

Substituting for $\partial T/\partial r$ from Fourier's law, is of the form

$$q_s'' = k\frac{\partial T}{\partial r}\bigg|_{r=r_o}$$

and for q''_s from Newton's law of cooling, we obtain

$$\frac{h}{k} \neq f(x) = \text{constant}$$

Hence, *in the thermally fully developed flow of a fluid with constant properties, the local convection coefficient is a constant, independent of x*. The last equation is not satisfied in the entrance region where h varies with x.

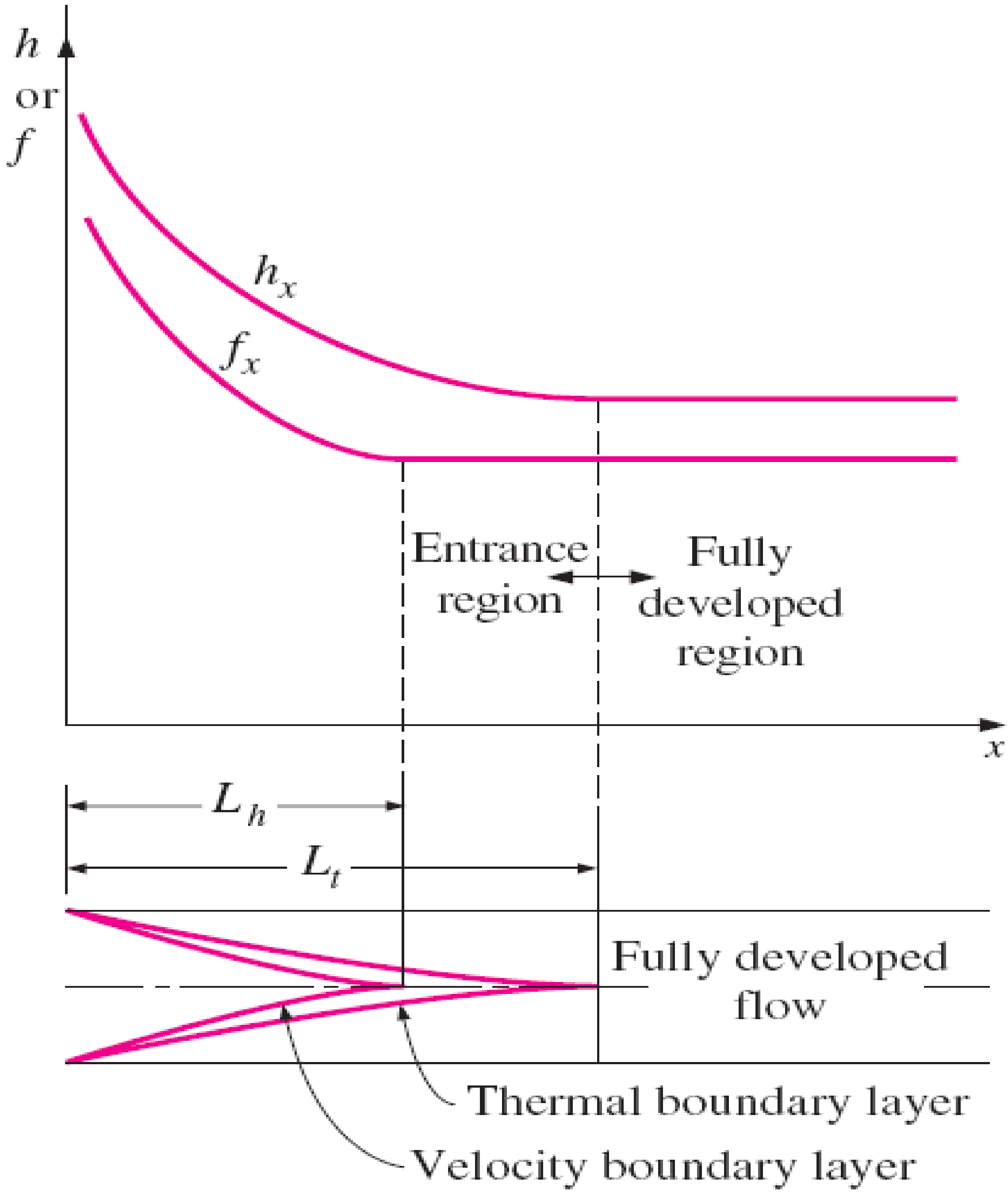
Because the thermal boundary layer thickness is zero at the tube entrance, the coefficient is extremely large near $x = 0$, and decreases markedly as the boundary layer develops, until the constant value associated with the fully developed conditions is reached.

Hydrodynamically fully developed:

$$\frac{\partial \mathcal{V}(r,x)}{\partial x} = 0 \quad \longrightarrow \quad \mathcal{V} = \mathcal{V}(r)$$

Thermally fully developed:

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right] = 0$$


h
or
f
h_x
f_x
Entrance
region
Fully
developed
region
x
L_h
L_t
Fully developed
flow
Thermal boundary layer
Velocity boundary layer

Energy Balances and Methods of Heating

Because the flow in a tube is completely enclosed, an energy balance may be applied to determine the convection heat transfer rate, q_{conv}, in terms of the difference in temperatures at the tube inlet and outlet.

From an energy balance applied to a differential control volume in the tube, we will determine how the mean temperature $T_m(x)$ varies in the flow direction with position along the tube for two *surface thermal conditions* (methods of heating/cooling).

Overall Tube Energy Balance.

Fluid moves at a constant flow rate $\dot{m}$ and convection heat transfer occurs along the wall surface. Assuming that fluid kinetic and potential energy changes are negligible, there is no shaft work, and regarding c_p as constant, the energy rate balance reduces to give

$$q_{\text{conv}} = \dot{m} c_p (T_{m,o} - T_{m,i})$$

where T_m denotes the mean fluid temperature and the subscripts *i* and *o* denote inlet and outlet conditions, respectively. It is important to recognize that this *overall energy balance is a general expression that applies irrespective of the nature of the surface thermal or tube flow conditions*.

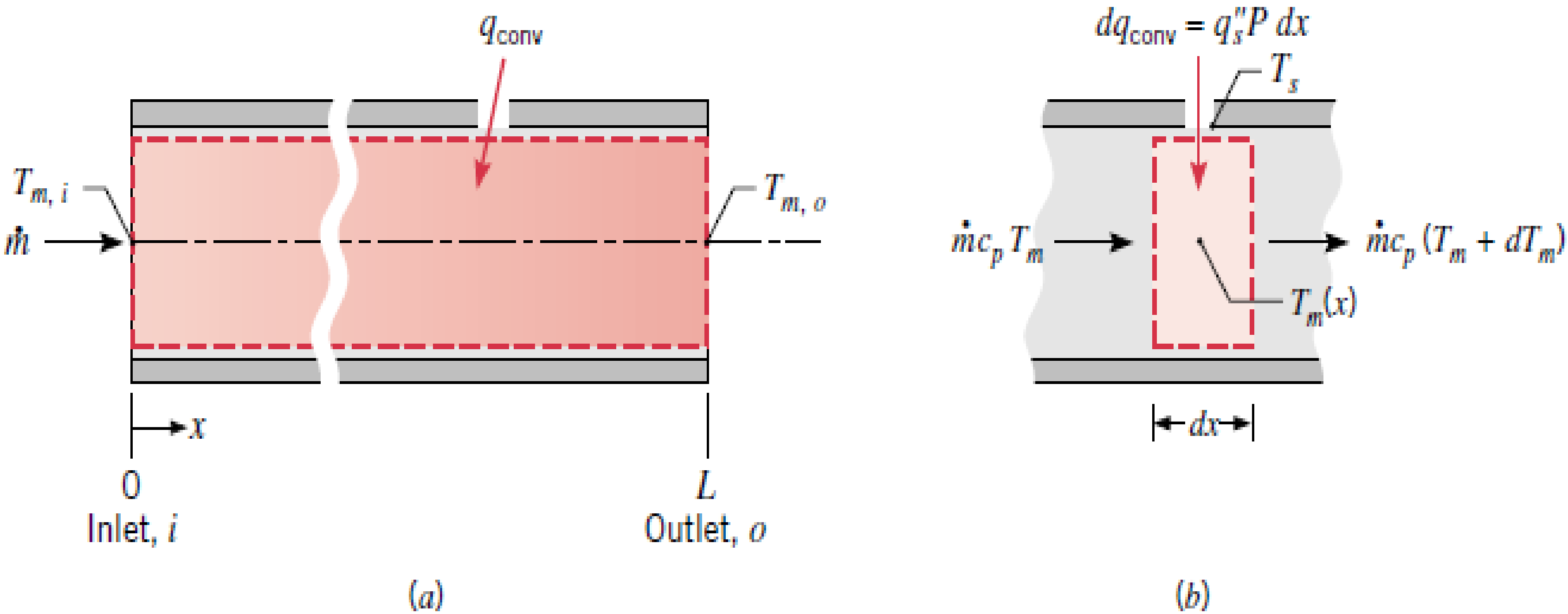


Figure 17.14 Energy balances for steady flow in a tube. (*a*) Overall tube balance for the convection heat rate, Eq. 17.48. (*b*) Balance on a differential control volume for determining $T_m(x)$, Eq. 17.50.

Energy Balance on a Differential Control Volume. We can apply the same analysis to a differential control volume within the tube as shown in Fig. 17.14*b* by writing Eq. 17.48 in differential form

$$dq_{\text{conv}} = \dot{m} c_p dT_m \qquad 17.49$$

We can express the rate of convection heat transfer to the differential element in terms of the surface heat flux as

$$dq_{\text{conv}} = q_s'' P dx \qquad 17.50$$

where *P* is the surface perimeter. Combining Eqs. 17.49 and 17.50, it follows that

$$q_s'' P dx = \dot{m} c_p dT_m$$

By rearranging this result, we obtain an expression for the axial variation of T_m in terms of the *surface heat flux*

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} \qquad [\text{surface heat flux, } q_s'']$$

17.51

or, using Newton's law of cooling, Eq. 17.45, with $q''_s = h(T_s - T_m)$, in terms of the tube wall *surface temperature*

$$\frac{dT_m}{dx} = \frac{P}{\dot{m}c_p} h(T_s - T_m) \qquad [\text{surface temperature, } T_s]$$

17.52

Thermal Condition: Constant Surface Heat Flux, q''_s

For *constant surface heat flux* thermal condition (Fig. 17.15), we first note that it is a simple matter to determine the total heat transfer rate, q_{conv}. Since q''_s is independent of *x*, it follows that

$$q_{conv} = q''_s (P \cdot L) \qquad 17.53$$

This expression can be used with the overall energy balance, Eq. 17.48, to determine the fluid temperature change, $T_{m,o} - T_{m,i}$.

For constant q''_s it also follows that the right-hand side of Eq. 17.51 is a constant independent of x. Hence

$$\frac{dT_m}{dx} = \frac{q''_s P}{\dot{m} c_p} = \text{constant}$$

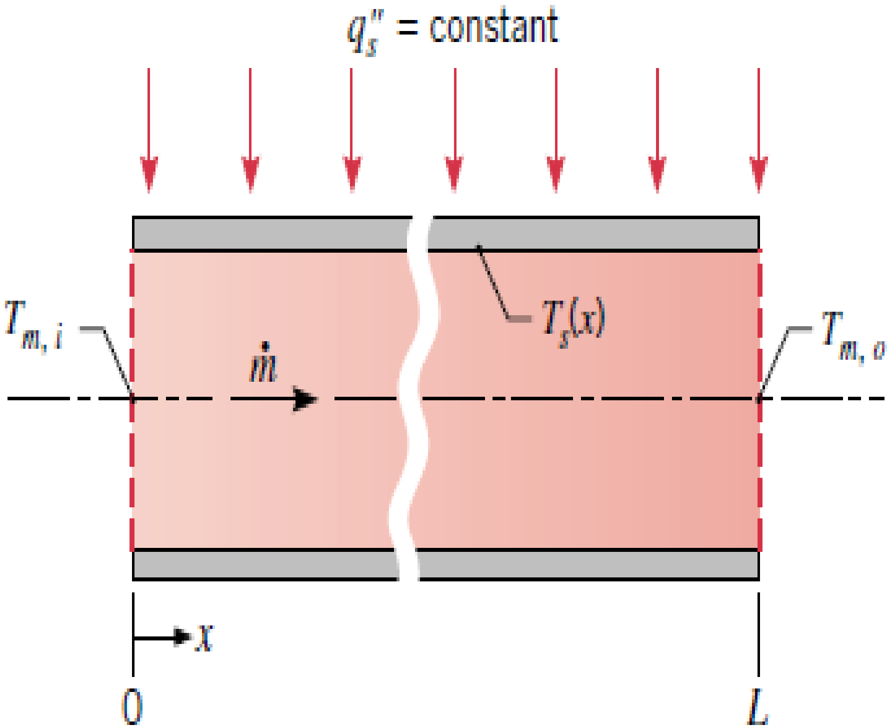


Figure 17.15 Internal flow through a circular tube with the surface thermal condition corresponding to *constant surface heat flux*, q_s''.

Integrating from $x = 0$ to some axial position x, we obtain the *mean temperature distribution, $T_m(x)$*

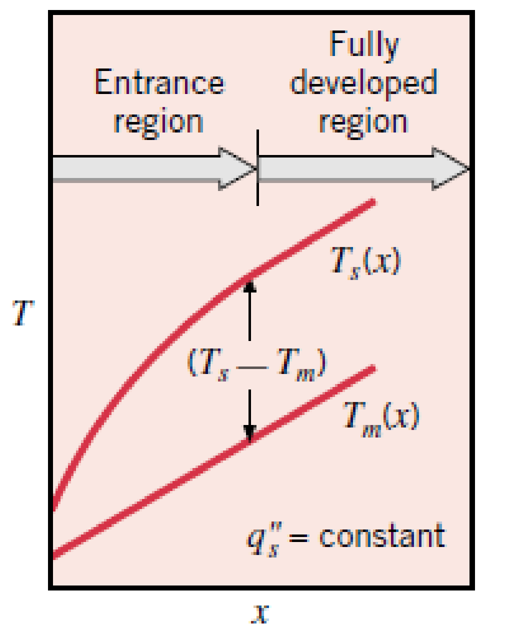


Figure 17.16

$$T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m} c_p} x \qquad [q_s'' = \text{constant}]$$

17.54

Thermal Condition: Constant Surface Temperature, T_s

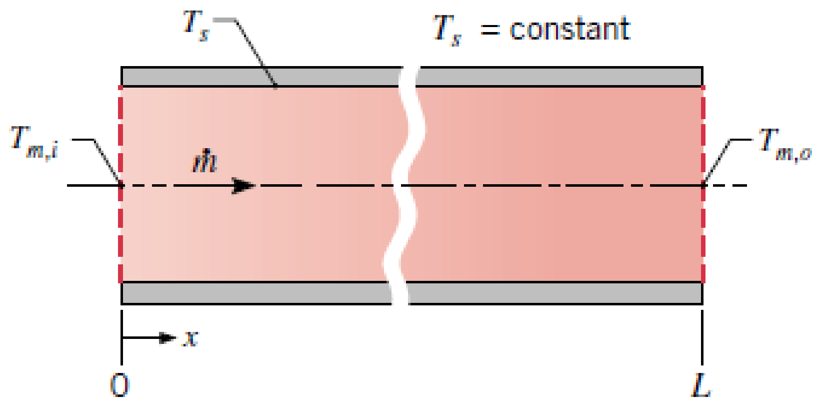


Figure 17.17 Internal flow through a circular tube with the surface thermal condition corresponding to *constant surface temperature*, T_s.

Results for the total heat transfer rate and the axial distribution of the mean temperature are entirely different for the *constant surface temperature* condition (Fig. 17.17). Defining ΔT as (T_s - T_m), Eq. 17.52 may be expressed as

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h \, \Delta T$$

With $P/\dot{m}c_p$ constant, separate variables and integrate from the tube inlet to the outlet

$$\int_{\Delta T_i}^{\Delta T_o} \frac{d(\Delta T)}{\Delta T} = -\frac{P}{\dot{m}c_p}\int_0^L h\,dx$$

or

$$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\left(\frac{1}{L}\int_0^L h\,dx\right)$$

From the definition of the average convection heat transfer coefficient, Eq. 17.8, it follows that

$$\ln \frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m} c_p} \bar{h}_L \qquad [T_s = \text{constant}]$$ 17.55a

where $\bar{h}_L$, or simply $\bar{h}$, is the average value of *h* for the entire tube. Alternatively, taking the exponent of both sides of the equation

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m} c_p} \bar{h}\right) \qquad [T_s = \text{constant}]$$

17.55b

If we had integrated from $x = 0$ to some axial position, we obtain the *mean temperature distribution*, $T_m(x)$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}\right) \qquad [T_s = \text{constant}]$$

17.56

where $\bar{h}$ is now the average value of *h* from the tube inlet to *x*. This result tells us that the temperature difference $(T_s - T_m)$ *decreases exponentially* with distance along the tube axis. The axial surface and mean temperature distributions are therefore as shown in Fig. 17.18.

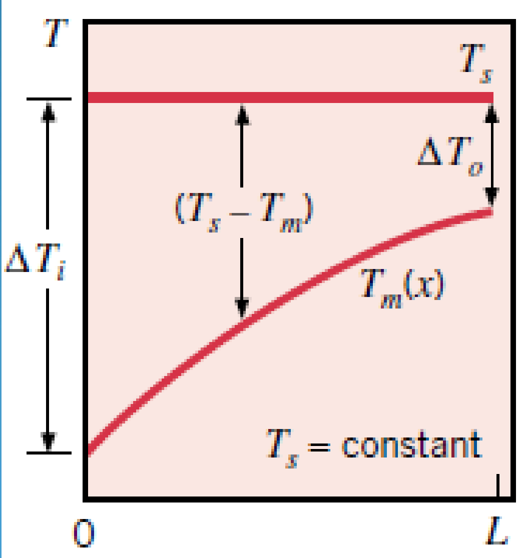


Figure 17.18

Determination of an expression for the total heat transfer rate q_{conv} is complicated by the exponential nature of the temperature decrease. Expressing Eq. 17.48 in the form

$$q_{\text{conv}} = \dot{m}c_p[(T_s - T_{m,i}) - (T_s - T_{m,o})] = \dot{m}c_p(\Delta T_i - \Delta T_o)$$

and substituting for $\dot{m}c_p$ from Eq. 17.55a, we obtain the *convection rate equation*

$$q_{\text{conv}} = \bar{h}A_s\Delta T_{\text{lm}} \qquad [T_s = \text{constant}] \qquad 17.57$$

where A_s is the tube surface area (A_s = P.L) and ΔT_{lm} is the ***log mean temperature difference*** (***LMTD***)

$$\Delta T_{\text{lm}} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)} \qquad 17.58$$

Convection Correlations for Tubes: Fully Developed Region

To use many of the foregoing results for internal flow, the convection coefficients must be known. In this section we present correlations for estimating the coefficients for *fully developed laminar* and *turbulent* flows in *circular* and *noncircular tubes*.

Laminar Flow

The problem of laminar flow ($Re_D < 2300$) in tubes has been treated theoretically, and the results can be used to determine the convection coefficients. For flow in a *circular* tube characterized by *uniform surface heat flux* and *laminar, fully developed conditions,* the *Nusselt number is a constant,* independent of Re_D, Pr, and axial location

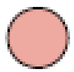
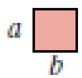
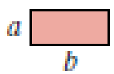
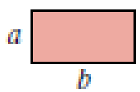
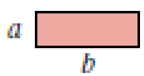
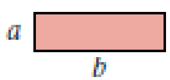
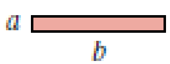
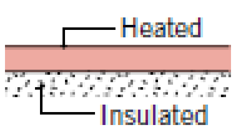
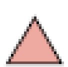
$$Nu_D = \frac{hD}{k} = 4.36 \qquad [q_s'' = \text{constant}] \tag{17.61}$$

When the thermal surface condition is characterized by a *constant surface temperature,* the results are of similar form, but with a smaller value for the Nusselt number

$$\mathrm{Nu}_D = \frac{hD}{k} = 3.66 \qquad [T_s = \text{constant}]$$

17.62

Table 17.4 Nusselt Numbers for Fully Developed Laminar Flow in Noncircular Tubes for Constant T_s and q_s'' Surface Thermal Conditions[a]

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ Constant q_s''	Constant T_s
	—	4.36	3.66
	1.0	3.61	2.98
	1.43	3.73	3.08
	2.0	4.12	3.39
	3.0	4.79	3.96
	4.0	5.33	4.44
	8.0	6.49	5.60
	∞	8.23	7.54
	∞	5.39	4.86
	—	3.11	2.47

[a]The characteristic length is the hydraulic diameter, D_h, Eq. 17.63.

Guide for Selection of Internal Flow Correlations

Table 17.5 Summary of Forced Convection Heat Transfer Correlations for Internal Flow in Smooth Circular Tubes[c]

Flow/Surface Thermal Conditions	Correlation[a,b]		Restrictions on Applicability
Laminar, fully developed, $(x_{fd}/D) > 0.05\,Re_D Pr$			
Constant q''_s	$Nu_D = 4.36$	(17.61)	$Pr \geq 0.6$, $Re_D \leq 2300$
Constant T_s	$Nu_D = 3.66$	(17.62)	$Pr \geq 0.6$, $Re_D \leq 2300$
Turbulent, fully developed, $(x_{fd}/D) > 10$			
Constant q''_s or T_s (*Dittus-Boelter*)	$Nu_D = 0.023\,Re_D^{4/5}\,Pr^n$	(17.64)	$0.6 \leq Pr \leq 160$, $Re_D \geq 10{,}000$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
Constant q''_s or T_s (*Sieder-Tate*)	$Nu_D = 0.027\,Re_D^{4/5}\,Pr^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$	(17.65)	$0.7 \leq Pr \leq 16{,}700$, $Re_D \geq 10{,}000$

REFERENCES

1. **Y. A. Cengel**. *Heat Transfer: A Practical Approach*, Mc Graw-Hill Education, New York, 2007.
2. **F. Kreith**. *Principles of Heat Transfer*. Harper International Edition, New York, 1985
3. **J. P. Holman**. *Heat Transfer*, Mc Graw-Hill Book Company, New York, 1996.
4. **S. Kakac & Y. Yener**. *Convective Heat Transfer*. CRC Press, Boca Raton, 1995.
5. **Sinaga, Nazaruddin, A. Suwono, Sularso, and P. Sutikno**. *Kaji Numerik dan Eksperimental Pembentukan Horseshoe Vortex pada Pipa Bersirip Anular*, Prosiding, Seminar Nasional Teknik Mesin II, Universitas Andalas, Padang, Desember 2003
6. **Sinaga, Nazaruddin, A. Suwono dan Sularso**. *Pengamatan Visual Pembentukan Horshoe Vortex pada Susunan Gormetri Pipa Bersirip Anular*, Prosiding, Seminar Nasional Teknik Mesin II, Universitas Andalas, Padang, Desember 2003.
7. **Sinaga, Nazaruddin**. *Pengaruh Parameter Geometri dan Konfigurasi Berkas Pipa Bersirip Anular Terhadap Posisi Separasi di Permukaan Sirip*, Jurnal Ilmiah Poros, Jurusan Teknik Mesin FT Universitas Tarumanegara, Vol. 9 No. 1, Januari, 2006.
8. **Cahyono, Sukmaji Indro, Gwang-Hwan Choe, and Nazaruddin Sinaga**. *Numerical Analysis Dynamometer (Water Brake) Using Computational Fluid Dynamic Software*. Proceedings of the Korean Solar Energy Society Conference, 2009.
9. **Sinaga, Nazaruddin**. *Pengaruh Model Turbulensi Dan Pressure-Velocity Copling Terhadap Hasil Simulasi Aliran Melalui Katup Isap Ruang Bakar Motor Bakar*, Jurnal Rotasi, Volume 12, Nomor 2, ISSN:1411-027X, April 2010.
10. **Nazaruddin Sinaga, Abdul Zahri.** *Simulasi Numerik Perhitungan Tegangan Geser Dan Momen Pada Fuel Flowmeter Jenis Positive Displacement Dengan Variasi Debit Aliran Pada Berbagai Sudut Putar Rotor*, Jurnal Teknik Mesin S-1, Vol. 2, No. 4, Tahun 2014.
11. **Nazaruddin Sinaga**. *Kaji Numerik Aliran Jet-Swirling Pada Saluran Annulus Menggunakan Metode Volume Hingga*, Jurnal Rotasi Vol. 19, No. 2, April 2017.

12. **Nazaruddin Sinaga**. *Analisis Aliran Pada Rotor Turbin Angin Sumbu Horisontal Menggunakan Pendekatan Komputasional*, Eksergi, Jurnal Teknik Energi POLINES, Vol. 13, No. 3, September 2017.
13. **Muchammad, M., Sinaga, N., Yunianto, B., Noorkarim, M.F., Tauviqirrahman, M.** *Optimization of Texture of The Multiple Textured Lubricated Contact with Slip*, International Conference on Computation in Science and Engineering, Journal of Physics: Conf. Series 1090-012022, 5 November 2018, IOP Publishing, Online ISSN: 1742-6596 Print ISSN: 1742-6588.
14. **Nazaruddin Sinaga, Mohammad Tauiviqirrahman, Arif Rahman Hakim, E. Yohana.** *Effect of Texture Depth on the Hydrodynamic Performance of Lubricated Contact Considering Cavitation*, Proceeding of International Conference on Advance of Mechanical Engineering Research and Application (ICOMERA 2018), Malang, October 2018.
15. **Syaiful, N. Sinaga, B. Yunianto, M.S.K.T. Suryo**. *Comparison of Thermal-Hydraulic Performances of Perforated Concave Delta Winglet Vortex Generators Mounted on Heated Plate: Experimental Study and Flow Visualization*, Proceeding of International Conference on Advance of Mechanical Engineering Research and Application (ICOMERA 2018), Malang, October 2018.
16. **Nazaruddin Sinaga, K. Hatta, N. E. Ahmad, M. Mel**. *Effect of Rushton Impeller Speed on Biogas Production in Anaerobic Digestion of Continuous Stirred Bioreactor*, Journal of Advanced Research in Biofuel and Bioenergy, Vol. 3 (1), December 2019, pp. 9-18.
17. **Nazaruddin Sinaga, Syaiful, B. Yunianto, M. Rifal**. Experimental and Computational Study on Heat Transfer of a 150 KW Air *Cooled Eddy Current Dynamometer*, Proc. The 2019 Conference on Fundamental and Applied Science for Advanced Technology (Confast 2019), Yogyakarta, Januari 21, 2019.
18. **Nazaruddin Sinaga**. *CFD Simulation of the Width and Angle of the Rotor Blade on the Air Flow Rate of a 350 kW Air-Cooled Eddy Current Dynamometer*, Proc. The 2019 Conference on Fundamental and Applied Science for Advanced Technology (Confast 2019), Yogyakarta, Januari 21, 2019.
19. **Anggie Restue, Saputra, Syaiful, and Nazaruddin Sinaga**. *2-D Modeling of Interaction between Free-Stream Turbulence and Trailing Edge Vortex*, Proc. The 2019 Conference on Fundamental and Applied Science for Advanced Technology (Confast 2019), Yogyakarta, January 21, 2019.
20. **E. Yohana, B. Farizki, N. Sinaga, M. E. Julianto, I. Hartati**. *Analisis Pengaruh Temperatur dan Laju Aliran Massa Cooling Water Terhadap*

Efektivitas Kondensor di PT. Geo Dipa Energi Unit Dieng, Journal of Rotasi, Vol. 21 No. 3, 155-159.

21. **B. Yunianto, F. B. Hasugia, B. F. T. Kiono, N. Sinaga**. *Performance Test of Indirect Evaporative Cooler by Primary Air Flow Rate Variations*, Prosiding SNTTM XVIII, 9-10 Oktober 2019, 1-7.