Forced Convection :

EXTERNAL FLOW

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In this chapter we will study the following topics :

- Governing parameters for forced convection
- The boundary layers in external flow
- Forced convection over a flat plate
- Flow across cylinders

7.1 Governing parameters for forced convection

The Nusselt number for forced convection is a function of Reynolds number, the Prandtl number and the shape and the orientation of the surface. The general correlation equation is

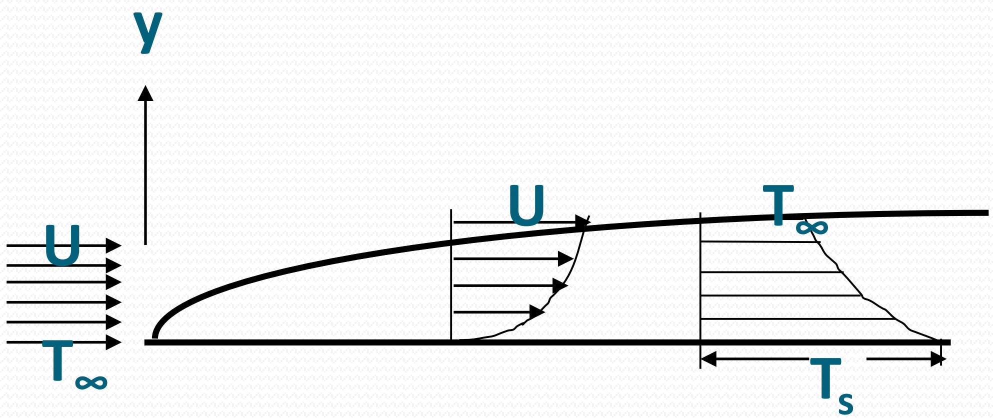
$$Nu_L = F(\mathrm{Re}_L, \mathrm{Pr}, S)$$

The exact forms of the Nussselt number equation depend on :

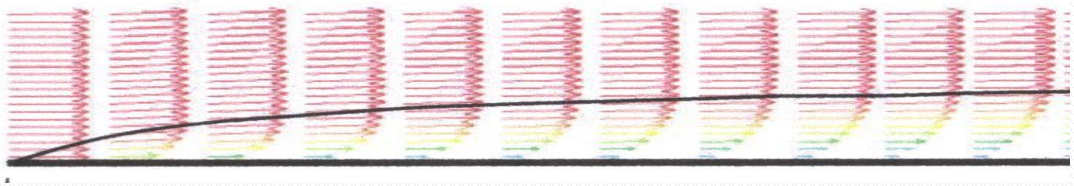
- **the type of flow :** laminar or turbulent
- **the shape of the surfaces** in contact with the pool of bulk moving fluid
- **the boundary conditions :** constant temperature or constant heat flux.

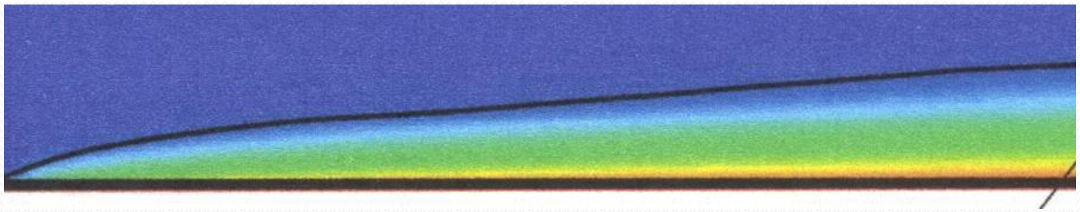
7-2 The Boundary Layers

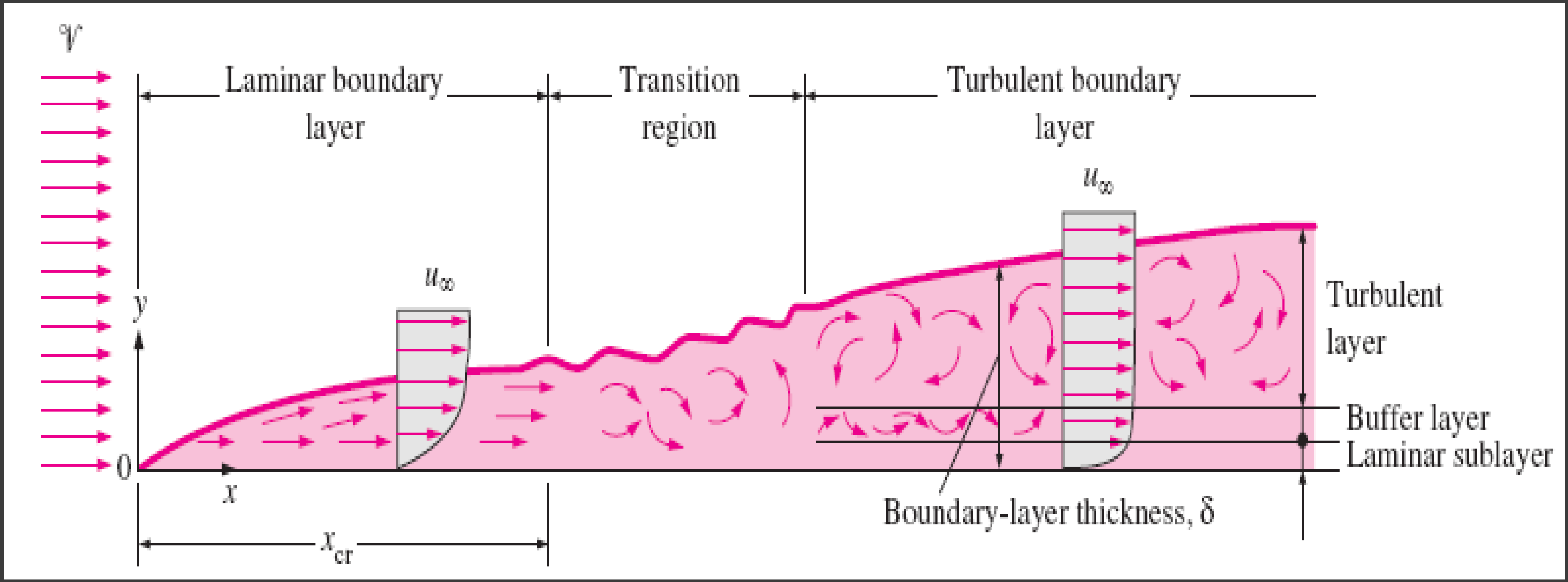
- Due to the bulk motion of the viscous fluid, there exist both hydraulic (velocity) and thermal boundary layers



$$\tau = \mu\left(\frac{du}{dy}\right)_{y=0}$$

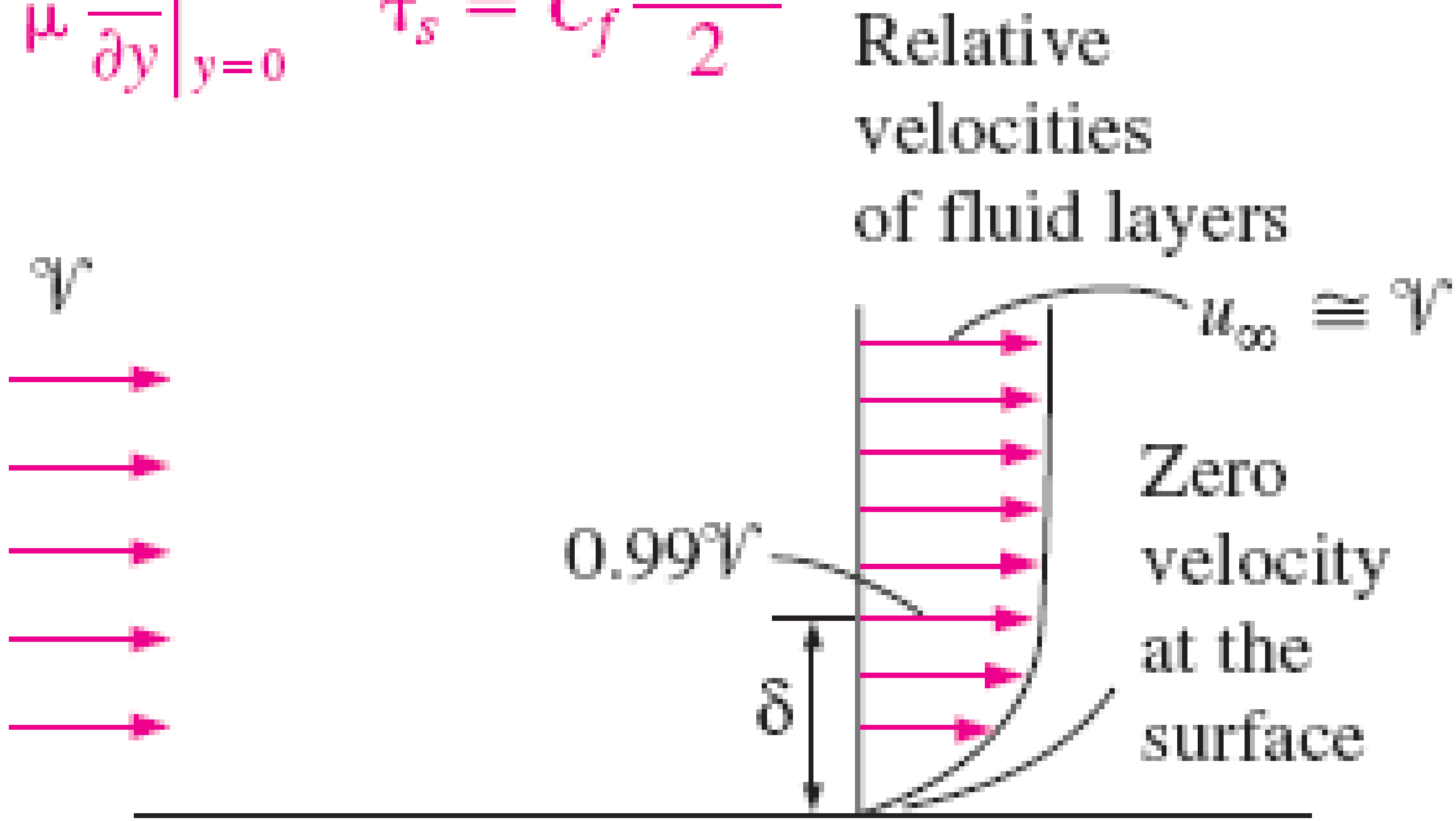




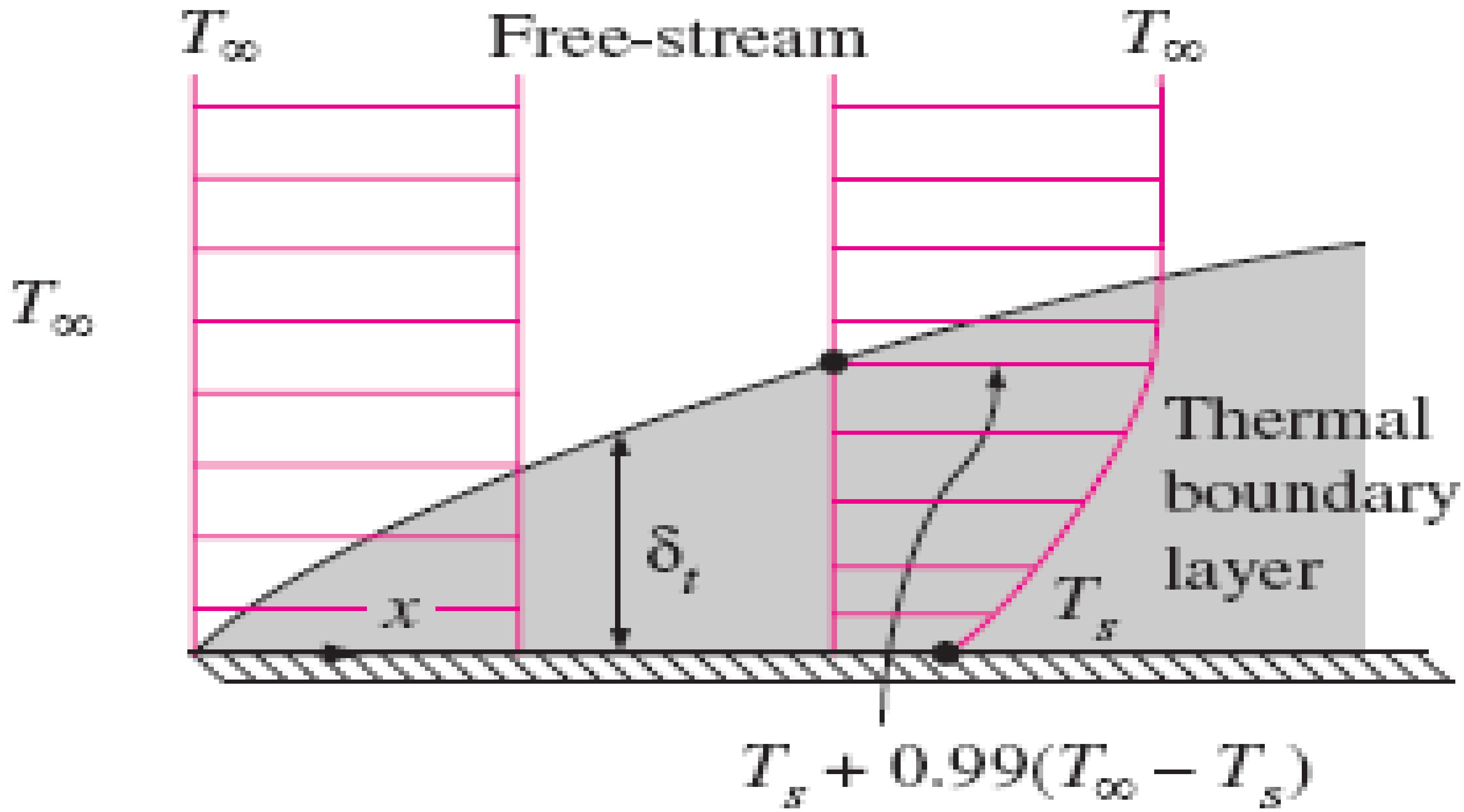


The development of the boundary layer for flow over a flat plate, and the different flow regimes.

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} \qquad \tau_s = C_f \frac{\rho \mathcal{V}^2}{2}$$



The development of a boundary layer on a surface is due to the no-slip condition.



Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface).

Prandtl and Reynolds Number

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless parameter Prandtl number,*

$$\mathrm{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{\mathcal{V} L_c}{\nu} = \frac{\rho \mathcal{V} L_c}{\mu}$$

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

- On the solid boundary, **no slip boundary condition** must exist for viscous fluid.

- The shear stress can also be written in terms of **frictional coefficient, c_f**

$$\tau = c_f \frac{\rho U^2}{2}$$

- The frictional force is

$$F_D = \tau A = c_f A \frac{\rho U^2}{2}$$

DERIVATION OF DIFFERENTIAL CONVECTION EQUATIONS*

- In this section we derive the governing equations of fluid flow in the boundary layers.
- To keep the analysis at a manageable level, we assume the flow to be **steady and two-dimensional,** and the **fluid to be Newtonian with constant properties** (density, viscosity, thermal conductivity, etc.).

- Consider the parallel flow of a fluid over a surface.
- We take the flow direction along the surface to be *x* and the direction normal to the surface to be *y*, and we choose a differential volume element of length *dx*, height *dy*, and unit depth in the *z*-direction (normal to the paper) for analysis (Fig. 6-20).

- The fluid flows over the surface with a uniform free-stream velocity u, but the velocity within boundary layer is two-dimensional:
- the x-component of the velocity is u, and the y-component is v.
- Note that $u = u(x, y)$ and $v = v(x, y)$ in steady two-dimensional flow.

- Next we apply three fundamental laws to this fluid element:
  - Conservation of mass,
  - conservation of momentum,
  - and conservation of energy
- to obtain the continuity, momentum, and energy equations for laminar flow in boundary layers.

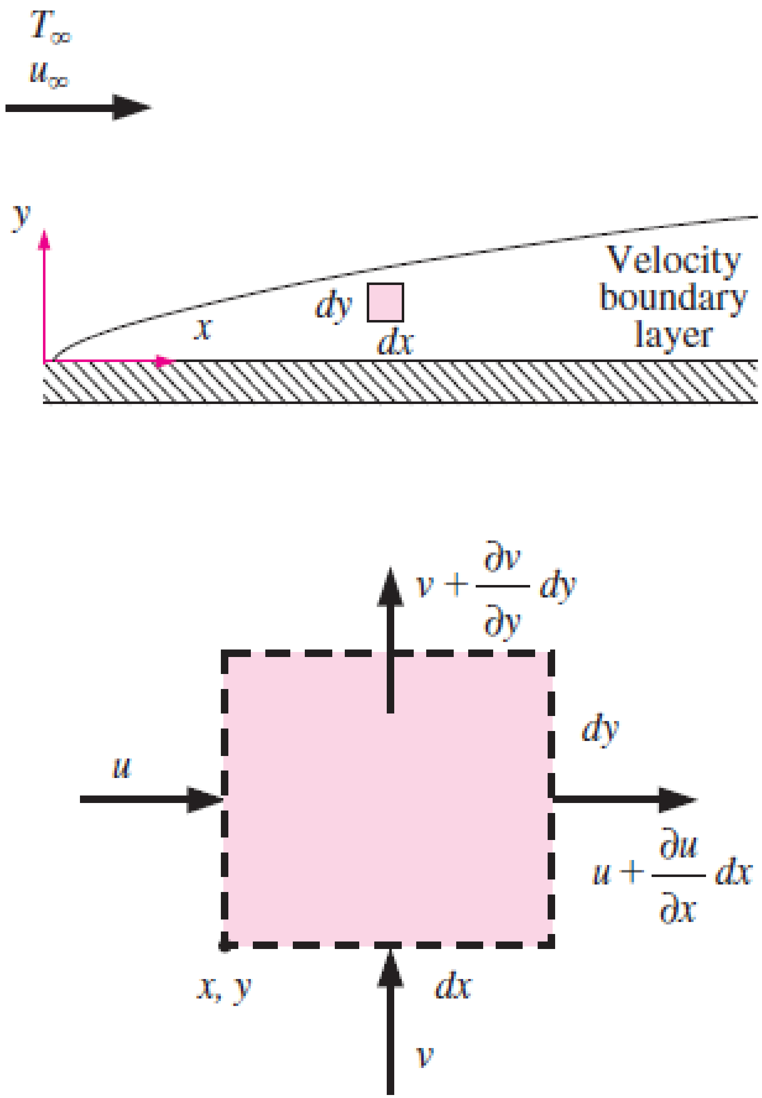


FIGURE 6–20

Differential control volume used in the derivation of mass balance in velocity boundary layer in two-dimensional flow over a surface.

Conservation of Mass Equation

- The conservation of mass principle is simply a statement that mass cannot be created or destroyed, and all the mass must be accounted for during an analysis.
- In steady flow, the amount of mass within the control volume remains constant, and thus the conservation of mass can be expressed as :

$$\begin{pmatrix} \text{Rate of mass flow} \\ \text{into the control volume} \end{pmatrix} = \begin{pmatrix} \text{Rate of mass flow} \\ \text{out of the control volume} \end{pmatrix}$$

- Noting that mass flow rate is equal to the product of density, mean velocity, and cross-sectional area normal to flow, the rate at which fluid enters the control volume from the left surface is $u(dy.1)$.
- The rate at which the fluid leaves the control volume from the right surface can be expressed as :

$$\rho\left(u + \frac{\partial u}{\partial x}dx\right)(dy \cdot 1)$$

- Repeating this for the *y* direction and substituting the results into Eq. 6-18, we obtain :

$$\rho u(dy \cdot 1) + \rho v(dx \cdot 1) = \rho\left(u + \frac{\partial u}{\partial x}dx\right)(dy \cdot 1) + \rho\left(v + \frac{\partial v}{\partial y}dy\right)(dx \cdot 1)$$

- Simplifying and dividing by *dx.dy.*1 gives :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

This is the *conservation of mass* relation, also known as the **continuity equation,** or **mass balance** for steady two-dimensional flow of a fluid with constant density.

Conservation of Momentum Equations

- The differential forms of the equations of motion in the velocity boundary layer are obtained by applying Newton's second law of motion to a differential control volume element in the boundary layer.

$$(\text{Mass})\begin{pmatrix}\text{Acceleration}\\ \text{in a specified direction}\end{pmatrix} = \begin{pmatrix}\text{Net force (body and surface)}\\ \text{acting in that direction}\end{pmatrix}$$

$$\delta m \cdot a_x = F_{\text{surface},\, x} + F_{\text{body},\, x}$$

- Noting that flow is steady and two-dimensional and thus u $u(x, y)$, the total differential of u is

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

- Then the acceleration of the fluid element in the x direction becomes

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

- The forces acting on a surface are due to pressure and viscous effects.
- In two-dimensional flow, the *viscous stress* at any point on an imaginary surface within the fluid can be resolved into two perpendicular components:
  - Normal to the surface called *normal stress* (which should not be confused with pressure)
  - along the surface called *shear stress*

- The normal stress is related to the velocity gradients $\partial u/\partial x$ and $\partial v/\partial y$, that are much smaller than $\partial u/\partial y$, to which shear stress is related.
- Neglecting the normal stresses for simplicity, the surface forces acting on the control volume in the *x*-direction will be as shown in Fig. 6-22.

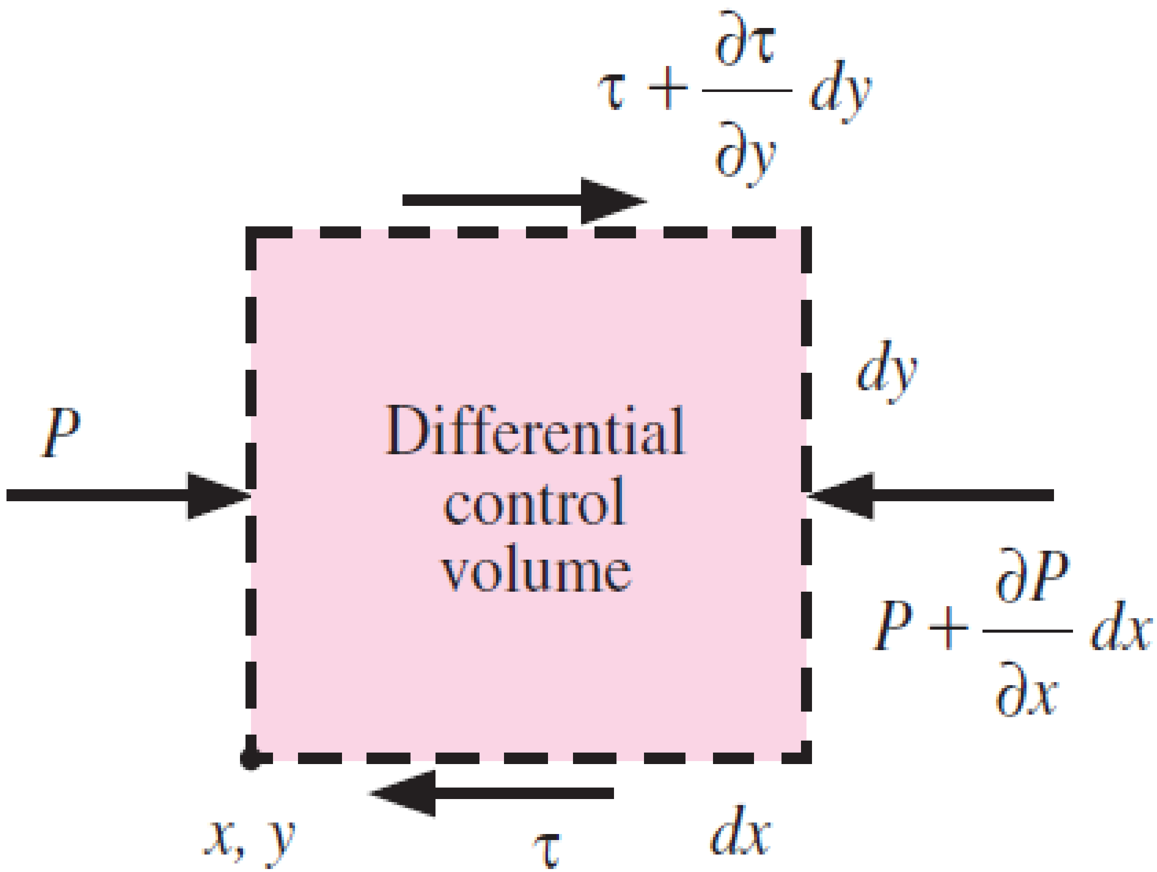


FIGURE 6–22
Differential control volume used in the derivation of *x*-momentum equation in velocity boundary layer in two dimensional flow over a surface.

- Then the net surface force acting in the *x*-direction becomes

$$F_{\text{surface},\,x} = \left(\frac{\partial \tau}{\partial y}\,dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}\,dx\right)(dy \cdot 1) = \left(\frac{\partial \tau}{\partial y} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1)$$

$$= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1)$$

since $\tau = \mu(\partial u/\partial y)$

- The momentum equation becomes:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x}$$

- This is the relation for the **conservation of momentum** in the *x*-direction, and is known as the ***x*-momentum equation.**

- If there is a body force acting in the *x*-direction, it can be added to the right side of the equation provided that it is expressed per unit volume of the fluid.

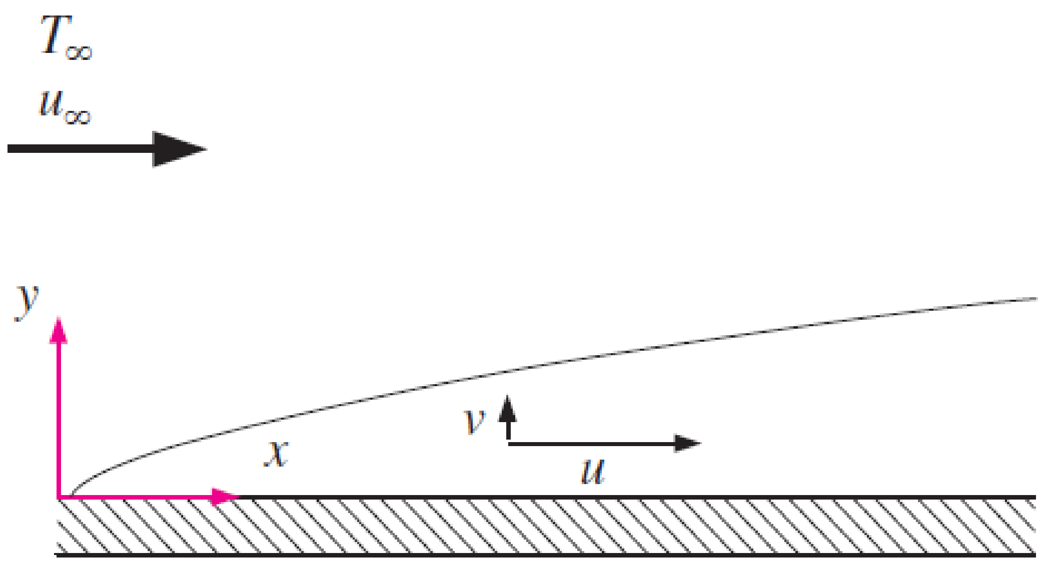
Navier-Stoke Equation

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x,$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\rho g_z.$$

Boundary layer approximations:



1) Velocity components:

$$v \ll u$$

2) Velocity grandients:

$$\frac{\partial v}{\partial x} \approx 0, \frac{\partial v}{\partial y} \approx 0$$

$$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$$

3) Temperature gradients:

$$\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$$

- When gravity effects and other body forces are negligible, and the boundary layer approximations are valid, applying Newton’s second law of motion on the volume element in the *y*-direction gives the *y-momentum equation* to be

$$\frac{\partial P}{\partial y} = 0$$

- That is, *the variation of pressure in the direction normal to the surface is negligible,* and thus $P = P(x)$ and $\partial P/\partial x = dP/dx$.
- It means that for a given *x,* the pressure in the boundary layer is equal to the pressure in the free stream.
- The pressure can be determined by a separate analysis of fluid flow in the free stream (is easier because of the absence of viscous effects)

- The velocity components in the free stream region of a flat plate are $u = u_\infty$ = constant and $v = 0$.
- Substituting these into the *x*-momentum equations (Eq. 6-28) gives $\partial P/\partial x = 0$.
- **Therefore, for flow over a flat plate, the pressure remains constant over the entire plate (both inside and outside the boundary layer).**

Conservation of Energy Equation

- The energy balance for any system undergoing any process is expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

- which states that the change in the energy content of a system during a process is equal to the difference between the energy input and the energy output.

Conservation of Energy Equation

- During a *steady-flow process,* the total energy content of a control volume remains constant (and thus $\Delta E_{\text{system}} = 0$), and the amount of energy entering a control volume in all forms must be equal to the amount of energy leaving it.
- Then the rate form of the general energy equation reduces for a steady-flow process to

$$\dot{E} \quad - \dot{E} \quad = 0$$

- Noting that energy can be transferred by heat, work, and mass only.
- The energy balance for a steady-flow control volume can be written explicitly as

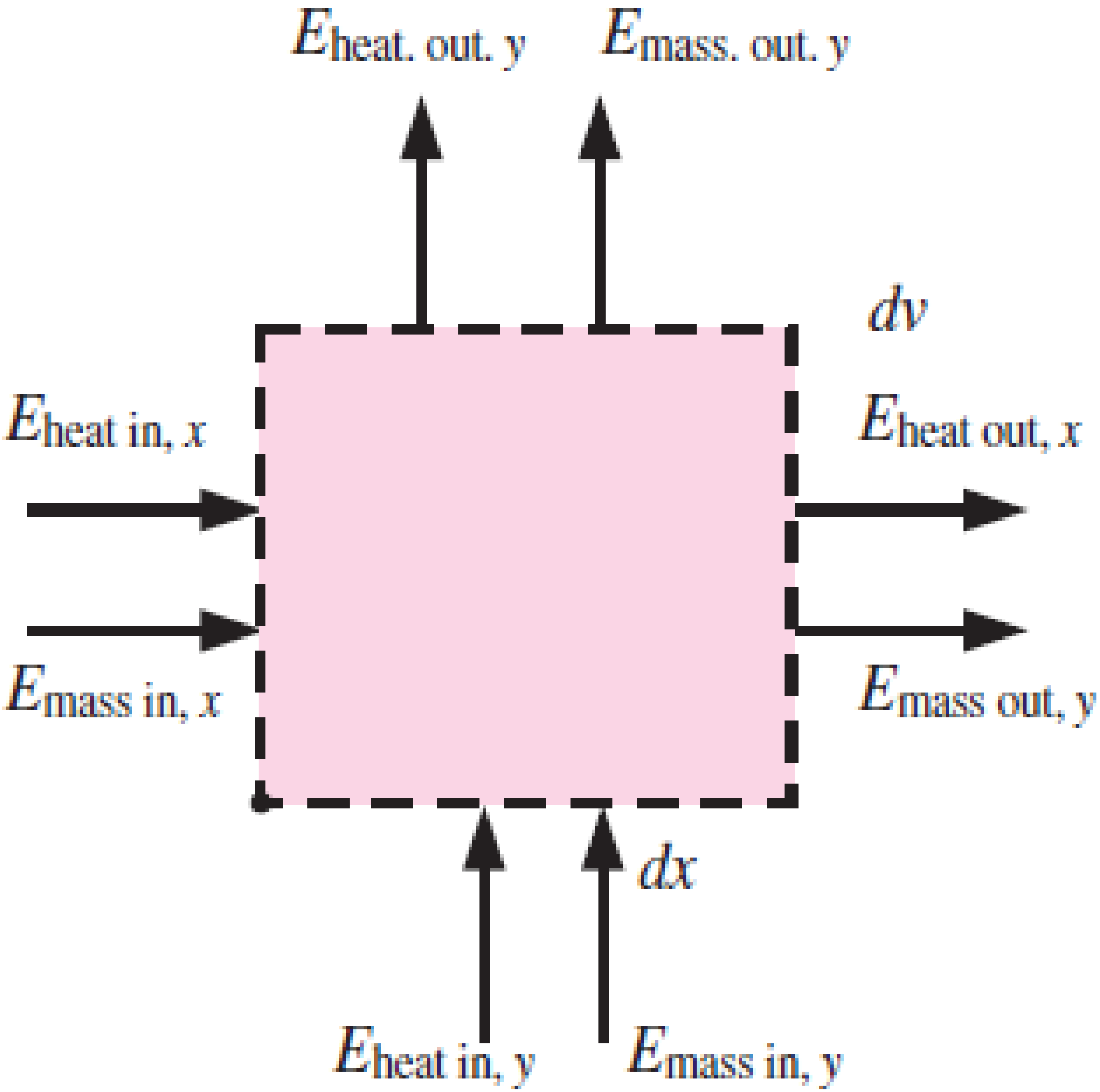
$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by work}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}} = 0$$

- The total energy of a flowing fluid stream per unit mass is $e_{stream} = h + \text{ke} + \text{pe}$, where h is the enthalpy, pe = gz is the potential energy, and ke = $V^2/2 = (u^2 + v^2)/2$ is the kinetic energy of the fluid per unit mass.

- The kinetic and potential energies are usually very small relative to enthalpy, and therefore it is common practice to neglect them.

$$e_{\mathrm{stream}} = h = C_p T.$$

- Tass flow rate of the fluid entering the control volume from the left is $u(dy.1)$



Eheat. out. y
Emass. out. y
dv
Eheat in, x
Eheat out, x
Emass in, x
Emass out, y
dx
Eheat in, y
Emass in, y

- The rate of energy transfer to the control volume by mass in the *x*-direction is :

$$(\dot{E}_{in} - \dot{E}_{out})_{by\ mass,\ x} = (\dot{m}e_{stream})_x - \left[(\dot{m}e_{stream})_x + \frac{\partial(\dot{m}e_{stream})_x}{\partial x}dx\right]$$

$$= -\frac{\partial[\rho u(dy \cdot 1)C_p T]}{\partial x}dx = -\rho C_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy$$

- The net rate of energy transfer to the control volume by mass is determined to be :

$$(\dot{E}_{in} - \dot{E}_{out})_{by\ mass} = -\rho C_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy - \rho C_p\left(v\frac{\partial T}{\partial y} + T\frac{\partial v}{\partial y}\right)dx\,dy$$

$$= -\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)dx\,dy$$

- The net rate of heat conduction to the volume element in the *x*-direction is :

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by heat, } x} = \dot{Q}_x - \left(\dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x} dx\right)$$

$$= -\frac{\partial}{\partial x}\left(-k(dy \cdot 1)\frac{\partial T}{\partial x}\right) dx = k\frac{\partial^2 T}{\partial x^2} dx\, dy$$

- the net rate of energy transfer to the control volume by heat conduction becomes :

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by heat}} = k\frac{\partial^2 T}{\partial x^2} dx\, dy + k\frac{\partial^2 T}{\partial y^2} dx\, dy = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) dx\, dy$$

- The energy equation for the steady two-dimensional flow of a fluid with constant properties and negligible shear stresses is :

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

- which states that *the net energy convected by the fluid out of the control volume is equal to the net energy transferred into the control volume by heat conduction.*

- When the viscous shear stresses are not negligible, their effect is accounted for by expressing the energy equation as :

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\Phi$$

- where the *viscous dissipation function* **Φ** is obtained after a lengthy analysis to be :

$$\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2$$

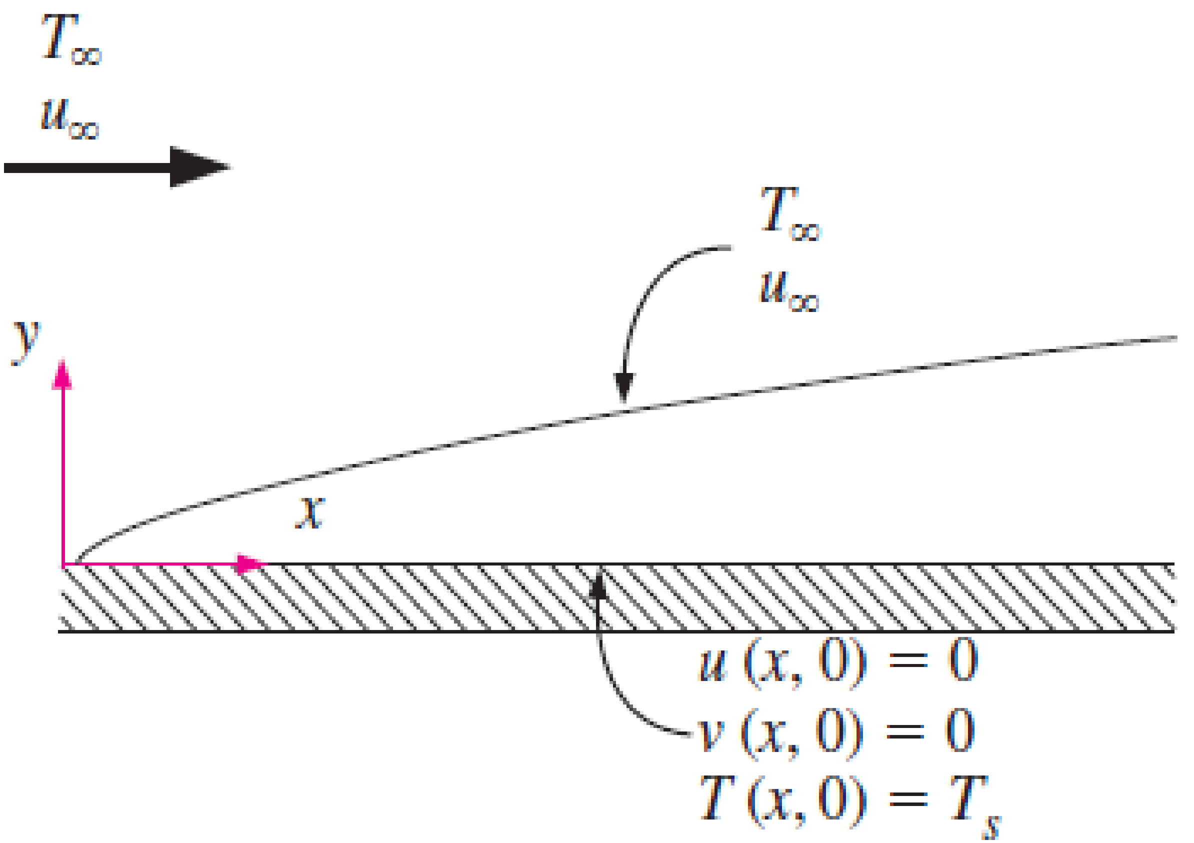
- Viscous dissipation may play a dominant role in high-speed flows, especially when the viscosity of the fluid is high (like the flow of oil in journal bearings).
- This manifests itself as a significant rise in fluid temperature due to the conversion of the kinetic energy of the fluid to thermal energy.
- Viscous dissipation is also significant for high-speed flights of aircraft.

- For the special case of a stationary fluid, *u*=*v*=0 and the energy equation reduces to the steady two-dimensional heat conduction equation :

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

- This is a Fourier Equation of Conduction Heat Transfer

SOLUTIONS OF CONVECTION EQUATIONS FOR A FLAT PLATE



Boundary conditions for flow over a flat plate

- Consider **laminar flow** of a fluid over a ***flat plate.***
- The fluid approaches the plate in the *x*-direction with a uniform upstream velocity, which is equivalent to the free stream velocity u_∞.
- When viscous dissipation is negligible, the continuity, momentum, and energy equations reduce for steady, incompressible, laminar flow of a fluid with constant properties over a flat plate to :

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum: $$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \nu\,\frac{\partial^2 u}{\partial y^2}$$

Energy: $$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2}$$

with the boundary conditions (Fig. 6–26),

At $x = 0$:	$u(0, y) = u_\infty,$	$T(0, y) = T_\infty$
At $y = 0$:	$u(x, 0) = 0,$	$v(x, 0) = 0, T(x, 0) = T_s$
As $y \to \infty$:	$u(x, \infty) = u_\infty,$	$T(x, \infty) = T_\infty$

- When fluid properties are assumed to be constant and thus independent of temperature, the first two equations can be solved separately for the velocity components u and v.
- Once the velocity distribution is available, we can determine the friction coefficient and the boundary layer thickness using their definitions.

- Also, knowing *u* and *v*, the temperature becomes the only unknown in the last equation, and it can be solved for temperature distribution.
- The continuity and momentum equations were first solved in 1908 by the German engineer **H. Blasius**, a student of **L. Prandtl**.
- This was done by transforming the two partial differential equations into a single ordinary differential equation by introducing a new independent variable, called the **similarity variable.**

- The finding of such a variable, assuming it exists, is more of an art than science, and it requires to have a good insight of the problem.
- Noticing that the general shape of the velocity profile remains the same along the plate, Blasius reasoned that the nondimensional velocity profile u/u_∞ should remain unchanged when plotted against the nondimensional distance y/δ, where δ is the thickness of the local velocity boundary layer at a given *x.*

- That is, although both δ and *u* at a given *y* vary with *x,* the velocity *u* at a fixed y/δ remains constant.
- Blasius was also aware from the work of Stokes that δ is proportional to $\sqrt{vx/u_\infty}$, and thus he defined a *dimensionless similarity variable* as:

$$\eta = y\sqrt{\frac{u_\infty}{vx}}$$

and thus u/u_∞ function(η).

- stream function $\Psi(x, y)$

$$u = \frac{\partial \psi}{\partial y} \qquad \text{and} \qquad v = -\frac{\partial \psi}{\partial x}$$

- Blassius defined a function $f(\eta)$ as the dependent variable as

$$f(\eta) = \frac{\psi}{u_\infty \sqrt{\nu x / u_\infty}}$$

- Then the velocity components become

$$u = \frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial y} = u_\infty\sqrt{\frac{\nu x}{u_\infty}}\frac{df}{d\eta}\sqrt{\frac{u_\infty}{\nu x}} = u_\infty\frac{df}{d\eta}$$

$$v = -\frac{\partial\psi}{\partial x} = -u_\infty\sqrt{\frac{\nu x}{u_\infty}}\frac{\partial f}{\partial x} - \frac{u_\infty}{2}\sqrt{\frac{\nu}{u_\infty x}}\,f = \frac{1}{2}\sqrt{\frac{u_\infty \nu}{x}}\left(\eta\frac{df}{d\eta} - f\right)$$

- By differentiating these *u* and *v* relations, the derivatives of the velocity components can be shown to be

$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x}\eta\frac{d^2 f}{d\eta^2}, \qquad \frac{\partial u}{\partial y} = u_\infty\sqrt{\frac{u_\infty}{\nu x}}\frac{d^2 f}{d\eta^2}, \qquad \frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x}\frac{d^3 f}{d\eta^3}$$

- Substituting these relations into the momentum equation and simplifying, we obtain

$$2\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} = 0$$

- which is a third-order nonlinear differential equation.
- Therefore, the system of **two partial differential** equations is transformed into **a single ordinary differential equation** by the use of a similarity variable.

- Using the definitions of f and η, the boundary conditions in terms of the similarity variables can be expressed as

$$f(0) = 0, \qquad \left.\frac{df}{d\eta}\right|_{\eta=0} = 0, \qquad \text{and} \qquad \left.\frac{df}{d\eta}\right|_{\eta=\infty} = 1$$

- The transformed equation with its associated boundary conditions cannot be solved analytically, and thus an alternative solution method is necessary.

- The problem was first solved by Blasius in 1908 using a power series expansion approach, and this original solution is known as the ***Blasius solution***.
- The problem is later solved more accurately using different numerical approaches (Table 6.3).
- The nondimensional velocity profile can be obtained by plotting u/u_∞ against η .
- The results obtained by this simplified analysis are in excellent agreement with experimental results.

η	f	$\frac{df}{d\eta} = \frac{u}{U_\infty}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
∞	∞	1	0

TABLE 6–3

Similarity function *f* and its derivatives for laminar boundary layer along a flat plate.

- Recall that we defined the boundary layer thickness as the distance from the surface for which $u/u_\infty = 0.99$.
- We observe from Table 6–3 that the value of η corresponding to $u/u_\infty = 0.992$ is $\eta = 5.0$.
- Substituting $\eta = 5.0$ and $y = \delta$ into the definition of the similarity variable (Eq. 6-43) gives .

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}} \qquad 5.0 = \delta\sqrt{u_\infty/\nu x}.$$

- Then the velocity boundary layer thickness becomes

$$\delta = \frac{5.0}{\sqrt{u_\infty / vx}} = \frac{5.0x}{\sqrt{\mathrm{Re}_x}}$$

- Note that the boundary layer thickness increases with increasing kinematic viscosity and with increasing distance from the leading edge *x,* but it decreases with increasing free-stream velocity *u*.

- Therefore, a large free-stream velocity will suppress the boundary layer and cause it to b thinner.

- The shear stress on the wall can be determined from its definition and the $\partial u/\partial y$ relation.

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}} \frac{d^2 f}{d\eta^2}\bigg|_{\eta=0}$$

- Substituting the value of the second derivative of f at $\eta = 0$ from Table 6–3 gives

$$\tau_w = 0.332 u_\infty \sqrt{\frac{\rho \mu u_\infty}{x}} = \frac{0.332 \rho u_\infty^2}{\sqrt{\mathrm{Re}_x}}$$

- Then the local skin friction coefficient becomes

$$C_{f,x} = \frac{\tau_w}{\rho \mathcal{V}^2/2} = \frac{\tau_w}{\rho u_\infty^2/2} = 0.664\ \mathrm{Re}_x^{-1/2}$$

- Note that unlike the boundary layer thickness, wall shear stress and the skin friction coefficient decrease along the plate as $x^{-1/2}$.

The Energy Equation

- Knowing the velocity profile, we are now ready to solve the energy equation for temperature distribution for the case of constant wall temperature T_s.
- First we introduce the dimensionless temperature θ as

$$\theta(x, y) = \frac{T(x, y) - T_s}{T_\infty - T_s}$$

- Noting that both T_s and T_∞ are constant, substitution into the energy equation gives

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2}$$

- Temperature profiles for flow over an isothermal flat plate are similar, just like the velocity profiles, and thus we expect a similarity solution for temperature to exist.

- Further, the thickness of the thermal boundary layer is proportional to $\sqrt{vx/u_\infty}$ just like the thickness of the velocity boundary layer, and thus the similarity variable is also η, and θ = θ (η).

- Using the chain rule and substituting the *u* and *v* expressions into the energy equation gives

$$u_\infty \frac{df}{d\eta}\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial x} + \frac{1}{2}\sqrt{\frac{u_\infty v}{x}}\left(\eta\frac{df}{d\eta} - f\right)\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial y} = \alpha\frac{d^2\theta}{d\eta^2}\left(\frac{\partial\eta}{\partial y}\right)^2$$

- Simplifying and noting that Pr = υ/α give

$$2\frac{d^2\theta}{d\eta^2} + \Pr f\frac{d\theta}{d\eta} = 0 \qquad (6.58)$$

- with the boundary conditions $\theta(0) = 0$ and $\theta(\infty) = 1$.
- Obtaining an equation for θ as a function η of alone confirms that the temperature profiles are similar, and thus a similarity solution exists.
- Again a closed-form solution cannot be obtained for this boundary value problem, and it must be solved numerically.

- It is interesting to note that for Pr = 1, this equation reduces to Eq. 6-49,

$$2\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} = 0$$

- when θ is replaced by *df*/*dη*, which is equivalent to *u*/*u*∞ (see Eq. 6-46),

$$u = \frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial y} = u_\infty\sqrt{\frac{vx}{u_\infty}}\frac{df}{d\eta}\sqrt{\frac{u_\infty}{vx}} = u_\infty\frac{df}{d\eta}$$

- The boundary conditions for θ and $df/d\eta$ are also identical.
- Thus we conclude that the velocity and thermal boundary layers coincide, and the non dimensional velocity and temperature profiles (u/u_∞ and θ) are identical for steady, incompressible, laminar flow of a fluid with constant properties and Pr 1 over an isothermal flat plate (Fig. 6-27).

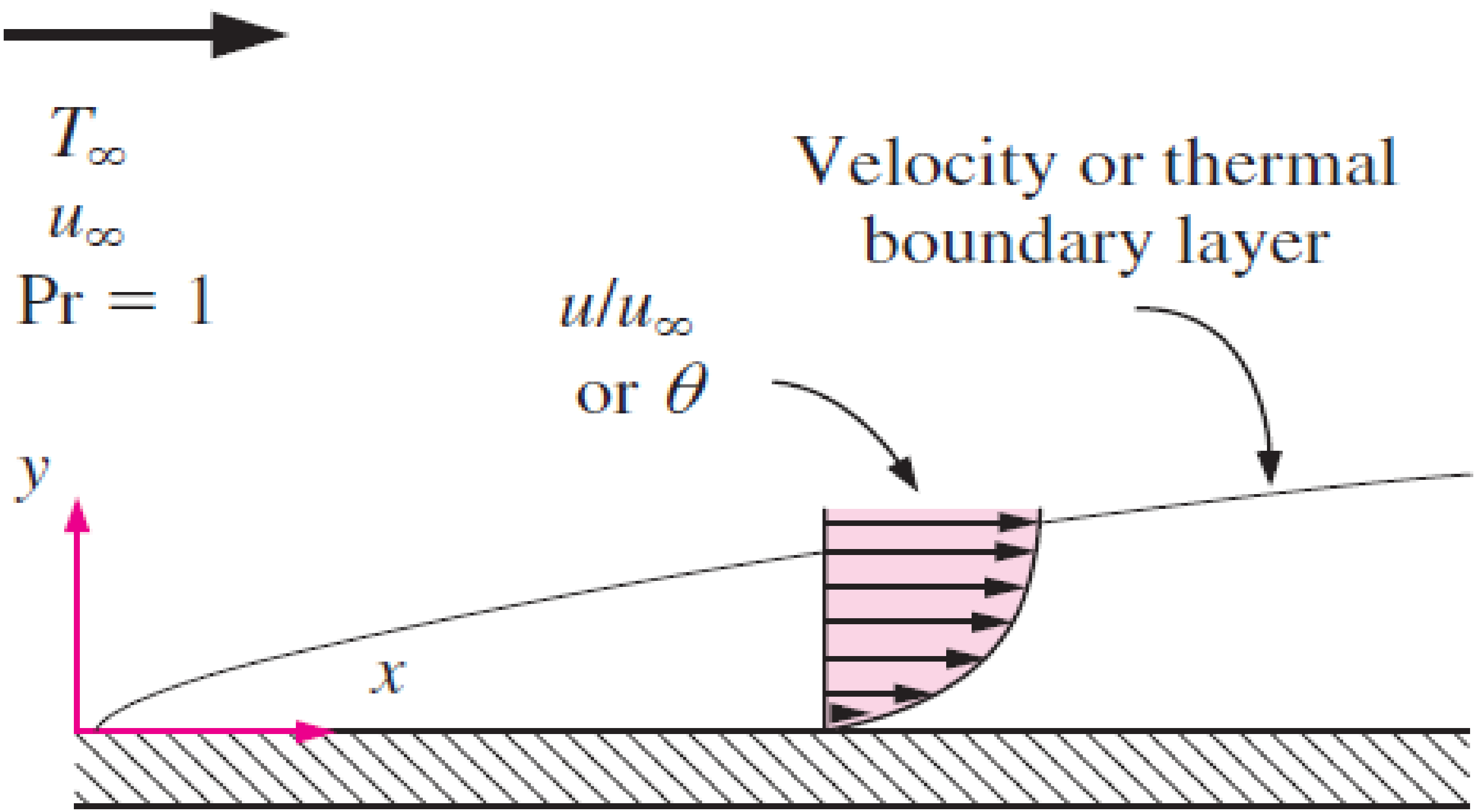


FIGURE 6–27

When Pr =1, the velocity and thermal boundary layers coincide, and the nondimensional velocity and temperature profiles are identical for steady, incompressible, laminar flow over a flat plate.

- The value of the temperature gradient at the surface (y = 0 or η = 0) in this case is, from Table 6–3,

$$d\theta/d\eta = d^2f/d\eta^2 = 0.332$$

- Equation 6-58 is solved for numerous values of Prandtl numbers.
- For Pr > 0.6, the nondimensional temperature gradient at the surface is found to be proportional to Pr $^{1/3}$, and is expressed as

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=0} = 0.332\,\mathrm{Pr}^{1/3}$$

- The temperature gradient at the surface is

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left.\frac{d\theta}{d\eta}\right|_{\eta=0}\left.\frac{\partial \eta}{\partial y}\right|_{y=0}$$

$$= 0.332\,\mathrm{Pr}^{1/3}(T_\infty - T_s)\sqrt{\frac{u_\infty}{\nu x}}$$

- Then the local convection coefficient and Nusselt number become

$$h_x = \frac{q_s}{T_s - T_\infty} = \frac{-k(\partial T/\partial y)\big|_{y=0}}{T_s - T_\infty} = 0.332\ \mathrm{Pr}^{1/3} k \sqrt{\frac{u_\infty}{\nu x}}$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\ \mathrm{Pr}^{1/3} \mathrm{Re}_x^{1/2} \qquad \mathrm{Pr} > 0.6$$

- The Nu_x values obtained from this relation agree well with measured values.

- Solving Eq. 6-58 numerically for the temperature profile for different Prandtl numbers, and using the definition of the thermal boundary layer, it is determined that $\delta/\delta t \cong \mathrm{Pr}^{1/3}$.
- Then the thermal boundary layer thickness becomes

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{5.0x}{\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}}$$

- Note that these relations are valid only for laminar flow over an isothermal flat plate.
- Also, the effect of variable properties can be accounted for by evaluating all such properties at the film temperature defined as $T_f = (T_s + T_\infty)/2$.
- The Blasius solution gives important insights, but its value is largely historical because of the limitations it involves.
- Nowadays both laminar and turbulent flows over surfaces are routinely analyzed using numerical methods.

NONDIMENSIONALIZED CONVECTION EQUATIONS AND SIMILARITY

- When viscous dissipation is negligible, the continuity, momentum, and energy equations for steady, incompressible, laminar flow of a fluid with constant properties are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(6-21)}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \qquad \text{(6-28)}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad \text{(6-35)}$$

- These equations and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by relevant and meaningful constant quantities:

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{\mathcal{V}}, \quad v^* = \frac{v}{\mathcal{V}},$$

$$P^* = \frac{P}{\rho \mathcal{V}^2}, \quad \text{and} \quad T^* = \frac{T - T_s}{T_\infty - T_s}$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \qquad \textbf{(6-64)}$$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{dP^*}{dx^*} \qquad \textbf{(6-65)}$$

$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L \mathrm{Pr}}\frac{\partial^2 T^*}{\partial y^{*2}} \qquad \textbf{(6-66)}$$

with the boundary conditions

$$u^*(0, y^*) = 1, \quad u^*(x^*, 0) = 0, \quad u^*(x^*, \infty) = 1, \quad v^*(x^*, 0) = 0,$$
$$T^*(0, y^*) = 1, \quad T^*(x^*, 0) = 0, \quad T^*(x^*, \infty) = 1$$

FUNCTIONAL FORMS OF FRICTION AND CONVECTION COEFFICIENTS

- The nondimensionalized boundary layer equations (Eqs. 6-64, 6-65, and 6-66) involve three unknown functions u^*, v^*, and T^*, two independent variables x^* and y^*, and two parameters Re_L and Pr.
- The pressure $P^*(x^*)$ depends on the geometry involved (*it is constant for a flat plate*), and it has the same value inside and outside the boundary layer at a specified x^*.
- Therefore, it can be determined separately from the free stream conditions, and dP^*/dx^* in Eq. 6-65 can be treated as a known function of x^*.

- For a given geometry, the solution for *u** can be expressed as

$$u^* = f_1(x^*, y^*, \mathrm{Re}_L)$$

- Then the shear stress at the surface becomes

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\mu \mathcal{V}}{L} \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} = \frac{\mu \mathcal{V}}{L} f_2(x^*, \mathrm{Re}_L)$$

$$C_{f,x} = \frac{\tau_s}{\rho \mathcal{V}^2/2} = \frac{\mu \mathcal{V}/L}{\rho \mathcal{V}^2/2} f_2(x^*, \mathrm{Re}_L) = \frac{2}{\mathrm{Re}_L} f_2(x^*, \mathrm{Re}_l) = f_3(x^*, \mathrm{Re}_L)$$

- Similarly, the solution of Eq. 6-66 for the dimensionless temperature *T** for a given geometry can be expressed as

$$T^* = g_1(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr})$$

- Using the definition of *T**, the convection heat transfer coefficient becomes

$$h = \frac{-k(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} = \frac{-k(T_\infty - T_s)}{L(T_s - T_\infty)} \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = \frac{k}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

$$\mathrm{Nu}_x = \frac{hL}{k} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} = g_2(x^*, \mathrm{Re}_L, \mathrm{Pr})$$

- Note that the Nusselt number is equivalent to the *dimensionless temperature gradient at the surface*

- The average friction and heat trnsfer coefficients are determined by integrating $C_{f,x}$ and Nu_x over the surface of the given body with respect to x^* from 0 to 1.

- Integration will remove the dependence on *x**, and the average friction coefficient and Nusselt number can be expressed as

$$C_f = f_4(\mathrm{Re}_L) \qquad \text{and} \qquad \mathrm{Nu} = g_3(\mathrm{Re}_L, \mathrm{Pr})$$

Local Nusselt number:

$$\mathrm{Nu}_x = \text{function}\ (x^*, \mathrm{Re}_L, \mathrm{Pr})$$

Average Nusselt number:

$$\mathrm{Nu} = \text{function}\ (\mathrm{Re}_L, \mathrm{Pr})$$

A common form of Nusselt number:

$$Nu = C\,\mathrm{Re}_L^m \mathrm{Pr}^n$$

ANALOGIES BETWEEN MOMENTUM AND HEAT TRANSFER

- In forced convection analysis, we are primarily interested in the determination of the quantities C_f and Nu.
- Therefore, it is very desirable to have a relation between C_f and Nu so that we can calculate one when the other is available.
- Such relations are developed on the basis of the similarity between momentum and heat transfers in boundary layers, and are known as *Reynolds analogy* and *Chilton–Colburn analogy*.

- Reconsider the nondimensionalized momentum and energy equations for steady, incompressible, laminar flow of a fluid with constant properties and negligible viscous dissipation (Eqs. 6-65 and 6-66).
- When Pr = 1 (which is approximately the case for gases) and $\partial P^*/\partial x^* = 0$ (which is the case when, $u = u_\infty =$ *constant* in the free stream, as in flow over a flat plate), these equations simplify to

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 T^*}{\partial y^{*2}}$$

- which are exactly of the same form for the dimensionless velocity u^* and temperature T^*.
- The boundary conditions for u^* and T^* are also identical.

- Therefore, the functions u^* and T^* must be identical, and thus the first derivatives of u^* and T^* at the surface must be equal to each other,

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

- Then

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x \qquad (\mathrm{Pr} = 1)$$

- which is known as the **Reynolds analogy**

Profiles: $u^* = T$

Gradients: $$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

Analogy: $$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x$$

When Pr = 1 and $\partial P^*/\partial x^* \approx 0$, the nondimensional velocity and temperature profiles become identical, and Nu is related to C_f by Reynolds analogy.

- This is an important analogy since it allows us to determine the heat transfer coefficient for fluids with Pr 1 from a knowledge of friction coefficient which is easier to measure.
- Reynolds analogy is also expressed alternately as

$$\frac{C_{f,x}}{2} = \mathrm{St}_x \qquad (\mathrm{Pr} = 1)$$

- where

$$\mathrm{St} = \frac{h}{\rho C_p \mathcal{V}} = \frac{\mathrm{Nu}}{\mathrm{Re}_L \mathrm{Pr}}$$

- is the Stanton number.

- Reynolds analogy is of limited use because of the restrictions Pr = 1 and $\partial P^*/\partial x^* = 0$ on it, and it is desirable to have an analogy that is applicable over a wide range of Pr.
- This is done by adding a Prandtl number correction.
- The friction coefficient and Nusselt number for a flat plate are

$$C_{f,x} = 0.664\ \mathrm{Re}_x^{-1/2} \qquad \text{and} \qquad \mathrm{Nu}_x = 0.332\ \mathrm{Pr}^{1/3}\ \mathrm{Re}_x^{1/2}$$

- Taking their ratio and rearranging give the desired relation, known as the **modified Reynolds analogy** or **Chilton–Colburn analogy**,

$$C_{f,x}\frac{Re_L}{2} = \mathrm{Nu}_x \, \mathrm{Pr}^{-1/3}$$

- Or

$$\frac{C_{f,x}}{2} = \frac{h_x}{\rho C_p \mathcal{V}} \mathrm{Pr}^{2/3} \equiv j_H$$

- for 0.6 < Pr < 60. j_H is called the *Colburn j- factor*

- Although this relation is developed using relations for laminar flow over a flat plate (for which $\partial P^*/\partial x^* = 0$), experimental studies show that it is also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients.
- For laminar flow, the analogy is not applicable unless $\partial P^*/\partial x^* = 0$.
- **It does not apply to laminar flow in a pipe.**

- Analogies between C_f and Nu that are more accurate are also developed, but they are more complex and beyond the scope of this book.
- The analogies given above can be used for both local and average quantities.

EXAMPLE 6–2 Finding Convection Coefficient from Drag Measurement

- A 2-m x 3-m flat plate is suspended in a room, and is subjected to air flow parallel to its surfaces along its 3-m-long side. The free stream temperature and velocity of air are 20°C and 7 m/s. The total drag force acting on the plate is measured to be 0.86 N. Determine the average convection heat transfer coefficient for the plate (Fig. 6–33).

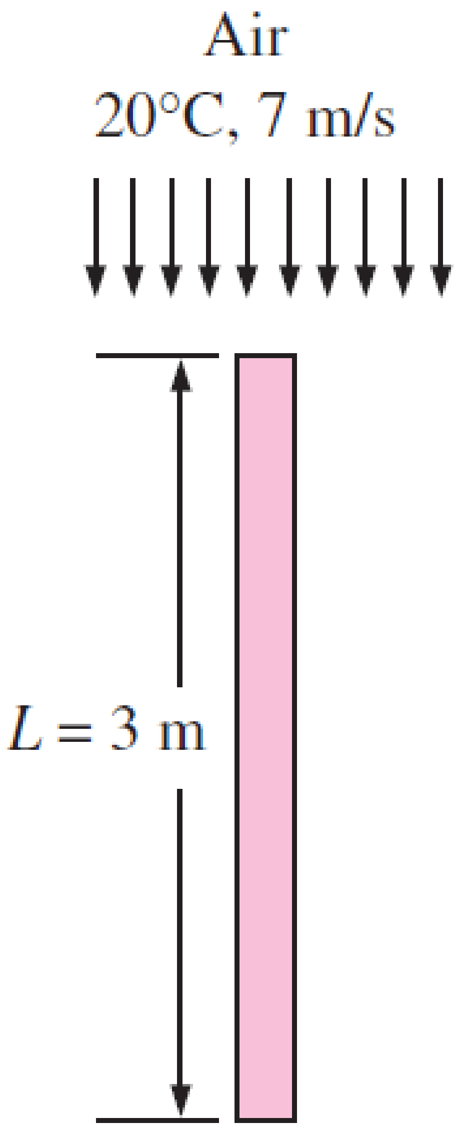


FIGURE 6–33
Schematic for Example 6–2

SOLUTION

A flat plate is subjected to air flow, and the drag force acting on it is measured. The average convection coefficient is to be determined.

Assumptions

1 Steady operating conditions exist.

2 The edge effects are negligible.

3 The local atmospheric pressure is 1 atm.

Properties The properties of air at 20°C and 1 atm are (Table A-15):

$$\rho = 1.204 \text{ kg/m}^3, \quad C_p = 1.007 \text{ kJ/kg} \cdot \text{K}, \quad \text{Pr} = 0.7309$$

Analysis The flow is along the 3-m side of the plate, and thus the characteristic length is $L = 3$ m. Both sides of the plate are exposed to air flow, and thus the total surface area is

$$A_s = 2WL = 2(2 \text{ m})(3 \text{ m}) = 12 \text{ m}^2$$

For flat plates, the drag force is equivalent to friction force. The average friction coefficient C_f can be determined from Eq. 6-11,

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2}$$

Solving for C_f and substituting,

$$C_f = \frac{F_f}{\rho A_s \mathcal{V}^2/2} = \frac{0.86 \text{ N}}{(1.204 \text{ kg/m}^3)(12 \text{ m}^2)(7 \text{ m/s})^2/2}\left(\frac{1\text{kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 0.00243$$

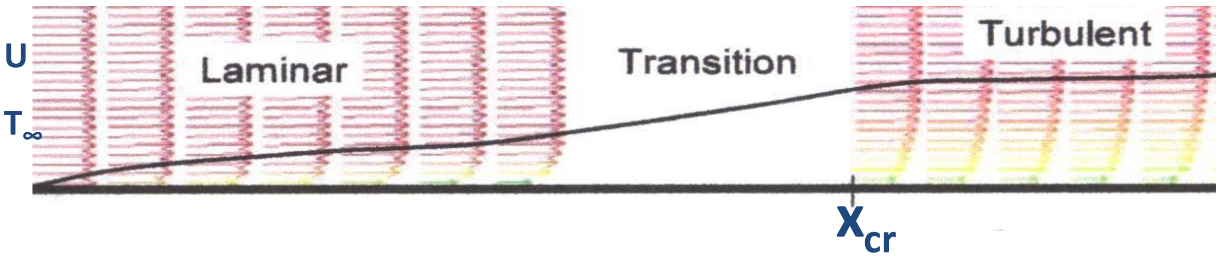
Then the average heat transfer coefficient can be determined from the modified Reynolds analogy (Eq. 6-83) to be

$$h = \frac{C_f}{2}\frac{\rho \mathcal{V} C_p}{\Pr^{2/3}} = \frac{0.00243}{2}\frac{(1.204 \text{ kg/m}^3)(7 \text{ m/s})(1007 \text{ J/kg}\cdot{}^\circ\text{C})}{0.7309^{2/3}} = 12.7 \text{ W/m}^2\cdot{}^\circ\text{C}$$

Discussion This example shows the great utility of momentum-heat transfer analogies in that the convection heat transfer coefficient can be determined from a knowledge of friction coefficient, which is easier to determine.

7-3 Forced convection over a flat plate

- Laminar and turbulent



In general, near the leading edge, the flow is laminar. However, laminar flow is not stable. Beyond a certain point the flow becomes turbulent. This point is called critical point.

- The critical Reynolds number is defined

$$\mathrm{Re}_{cr} = \frac{\rho U x_{cr}}{\mu} = 5x10^5 \qquad \mathrm{Re} \le 5x10^5 \text{ **Laminar** , } \qquad \mathrm{Re} \ge 5x10^5 \text{ **Turbulent**}$$

- The general form of Nusselt numbers correlation equations

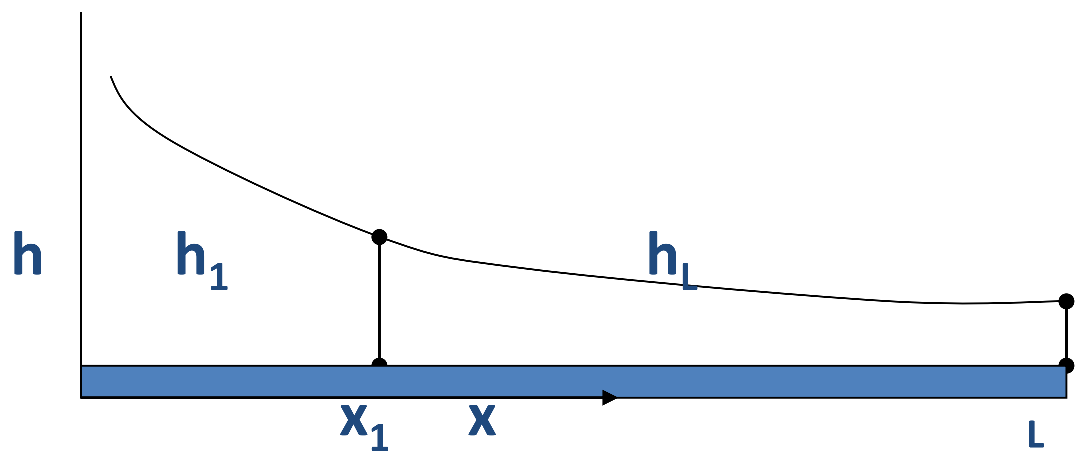
$$Nu_L = c\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

c, m, and *n* are constants depend on the flow and boundary conditions

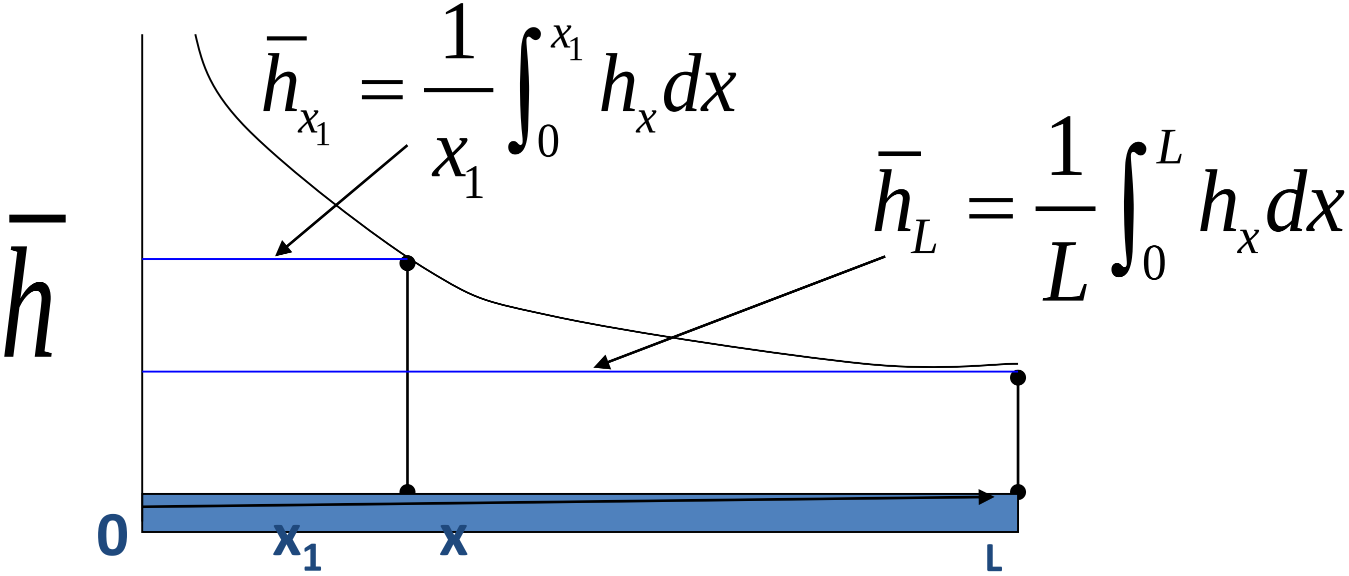
- Fluid properties are evaluated at mean film temperature

$$T_f = \frac{T_s + T_\infty}{2}$$

- **The local and average heat transfer coefficients**
  - The local heat transfer coefficient, h_x, is heat transfer coefficient at the point x measured from the leading edge of the surface.
  - The average convection heat transfer coefficient is the average value from the leading edge to point x.



- **The average convection heat transfer coefficient** also depends on the position, x. It is the average value from the leading edge to point x.



Nusselt number correlations for parallel flow over a flat plate

(1) Constant surface temperature

- **Laminar flow, $Re_L \leq 5 \times 10^5$**

  - The local Nusselt number

$$Nu_x = \frac{h_x x}{k} = 0.332 \, Re_x^{1/2} \, Pr^{1/3}$$

  - The average Nusselt number

$$\bar{N}u_x = \frac{\bar{h}_x x}{k} = 0.664 \, Re_x^{1/2} \, Pr^{1/3}$$

Calculation the average convection heat transfer coefficient for laminar flow along a flat plate (constant surface temperature)

$$Nu_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$h_x = \frac{k}{x}(0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}) = \frac{k}{x}(0.332(\frac{Ux}{\nu})^{\frac{1}{2}}\,\mathrm{Pr}^{1/3}) = 0.332k(\frac{U}{\nu})^{\frac{1}{2}}\,\mathrm{Pr}^{1/3}\,x^{-\frac{1}{2}}$$

$$\bar{h}_x = \frac{1}{x}\int_0^x 0.332k(\frac{U}{\nu})^{\frac{1}{2}}\,\mathrm{Pr}^{1/3}\,x^{-\frac{1}{2}}dx = 0.332k(\frac{U}{\nu})^{\frac{1}{2}}\,\mathrm{Pr}^{1/3}\,x^{-1}\int_0^x x^{-\frac{1}{2}}dx$$

$$= 0.332k(\frac{U}{\nu})^{\frac{1}{2}}\,\mathrm{Pr}^{1/3}\,\frac{1}{-\frac{1}{2}+1}x^{-\frac{1}{2}+1} = 0.332k(\frac{U}{\nu})^{\frac{1}{2}}\,\mathrm{Pr}^{1/3}\,x^{-1}(2x^{\frac{1}{2}})$$

$$= 0.664k(\frac{Ux}{\nu})^{\frac{1}{2}}\,\mathrm{Pr}^{1/3}\,x^{-1}$$

$$\frac{\bar{h}_x x}{k} = 0.664(\frac{Ux}{\nu})^{\frac{1}{2}}\,\mathrm{Pr}^{1/3}$$

(1) Constant surface temperature

- **Turbulent flow starting at x = 0**, with artificial transition devices

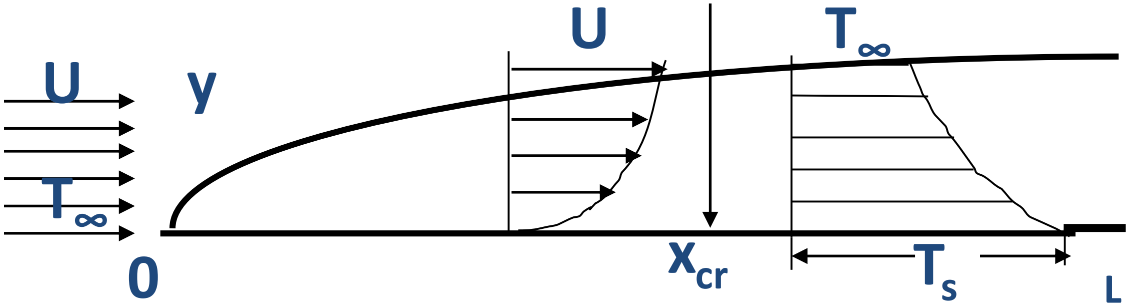
  - The local Nusselt number

$$Nu_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$$

  - The average Nusselt number

$$\bar{N}u_x = \frac{\bar{h}_x x}{k} = 0.037\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$$

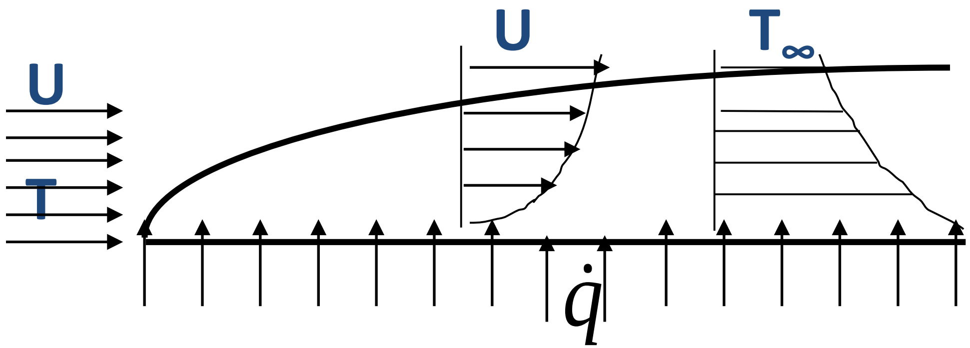
- **Combined laminar and turbulent flow**



$$\bar{N}u_L = \frac{\bar{h}_L L}{k} = (0.037\,\mathrm{Re}_L^{4/5} - 871)\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re}_L = \frac{\rho U L}{\mu} \succ 5x10^5$$

➢ For x smaller than x_{cr}, the flow is laminar. Use laminar equation.

(2) Constant surface heat flux



- Local Nusselt number for laminar flow : $Re_L \le 5 \times 10^5$

$$Nu_x = \frac{h_x x}{k} = 0.453 \, Re_x^{1/2} \, Pr^{1/3}$$

- Local Nusselt number for turbulent flow

$$Nu_x = \frac{h_x x}{k} = 0.0308 \, Re_x^{4/5} \, Pr^{1/3}$$

The surface temperature is not constant, It is obtained as follows:

$$\dot{q} = h_x (T_s - T_\infty) \Rightarrow T_s = T_\infty + \frac{\dot{q}}{h_x}$$

- By applying the definition of average heat transfer coefficient, the average Nusselt number correlations can also be obtained.

- Note

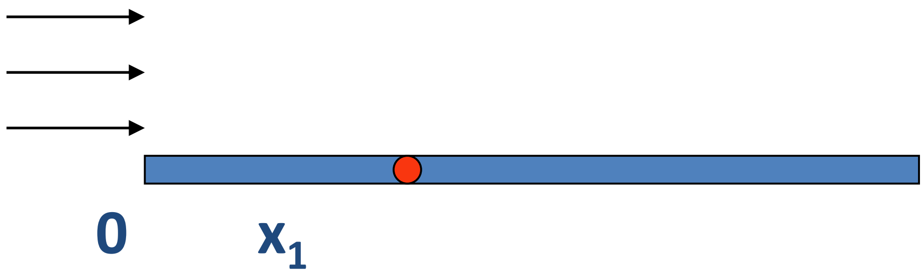
$$h_x \propto \frac{1}{\sqrt{x}}, lam.....h_x = \frac{1}{x^{0.2}}, Turbulent$$

$$x \rightarrow 0, h \rightarrow \infty \Rightarrow T_s \rightarrow T_\infty$$

for constant surface heat flux, T_s increases with x.

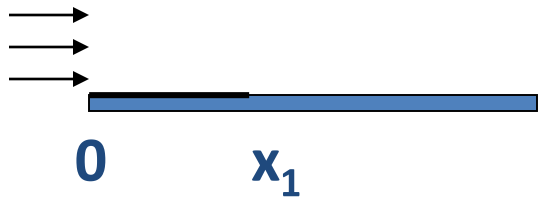
Examples to select Nusselt number correlation equations

- To calculate the heat transfer rate at the point x_1



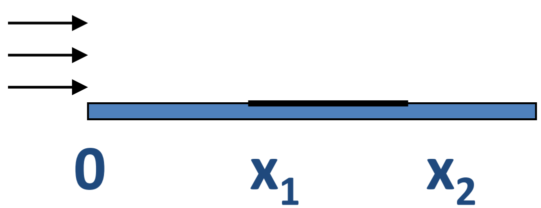
1. No artificial transition device, Re_x smaller than $5x10^5$
2. With artificial transition device at the leading edge

- To calculate the heat transfer rate of a board from 0 to x_1



1. No artificial transition device, Re_x smaller than $5x10^5$
2. With artificial transition device at the leading edge

- To calculate the heat transfer rate in the region between x_1 to x_2



Steps to calculate convection heat transfer rate — external flow

1. Use boundary condition to determine whether the problem is constant temperature or constant heat flux.

2. Calculate the film temperature :

$$T_f = \frac{T_s + T_\infty}{2}$$

3. In general, the problem will ask you to calculate the heat transfer rate or one of the two temperatures.

4. If one of the two temperatures is required to be determined, its value is not given, assume one.

5. Get the physical properties of the coolant using the film temperature

6. Calculate the Reynolds number

$$\mathrm{Re} = \frac{\rho U L}{\mu}$$

- with artificial transition device — use turbulent equation if

$$\mathrm{Re} \geq 5x10^5$$

- without artificial transition device use laminar if

$$\mathrm{Re} \leq 5x10^5$$

7. Choose the correct equation & calculate the Nusselt number
    - Local value
    - Average value over a distance

8. Calculate the heat transfer coefficient,

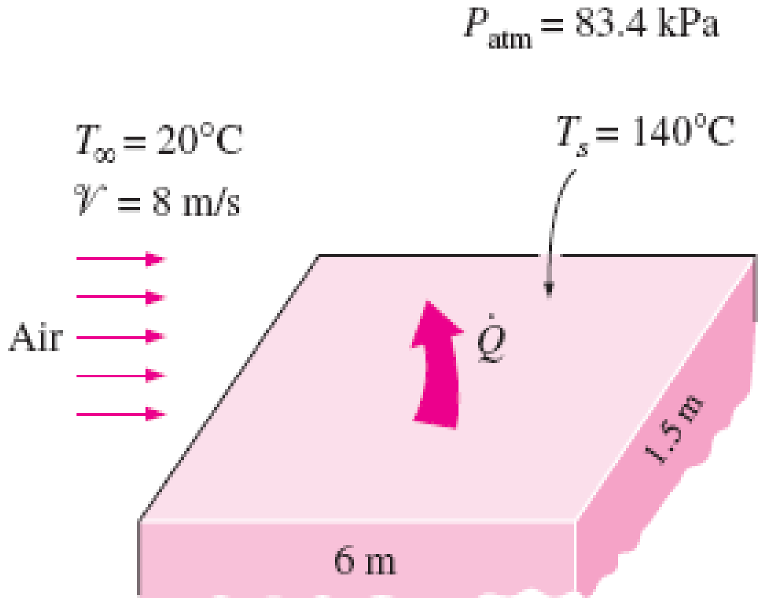
$$\dot{Q} = \bar{h}A(T_s - T_\infty) \qquad \bar{h}_L = \frac{k}{L} Nu_L$$

9. Calculate the heat transfer rate or temperature. If the problem is to determine one of the two temperatures, compare the calculated value with the assumed one. If the difference between the two is large, reassume one and repeat the calculation.

10. Other form of external surfaces—empirical equations

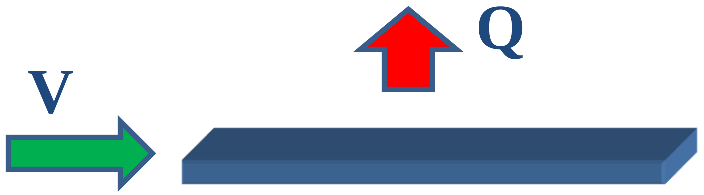
Example 1 Cooling a hot block at high elevation by forced convection

The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5 m x 6 m flat plate whose temp. is 140°C. Determine the rate of heat transfer from the plate if the air flows parallel to the (*a) 6-m-long* side and (*b) the 1.5-m side.*



Example 7-1 Cooling a hot block at high elevation by forced convection

- Given : A = 1.5m x 6m, T_∞ = 20 ^oC, T_s = 140 ^oC, U = 8m/s, p =83.7kPa,



- Find : the rate of heat transfer

(a) L = 6m

- The temperature for properties evaluation is:

$$T_f = \frac{T_s + T_\infty}{2} = 80^o C$$

Example 7-1 Cooling a hot block at high elevation by forced convection

- Properties of air , (see Table 15)

  k= 0.02953W/mK, Pr =0.7154, μ = 2.096x10^{-5} (kg/m.s),

Note : the air density depends strongly on pressure, Table 15 is good only for pressure at 1 bar = 101 kPa.

Other air properties is almost independent of pressure.

The kinematic viscosity requires to be corrected.

$$p = \rho RT \Rightarrow \rho = \frac{p}{RT} = \frac{837000}{287x(273+80)} = 0.826 kg / m^3$$

$$\nu_p = \frac{\mu_p}{\rho_p} = \frac{2.096}{0.826} = 2.53x10^{-5} (m^2 / s)$$

Properties of air at 1 atm pressure

Temp., T, °C	Density, ρ, kg/m³	Specific Heat, C_p, J/kg · °C	Thermal Conductivity, k, W/m · °C	Thermal Diffusivity, α, m²/s	Dynamic Viscosity, μ, kg/m · s	Kinematic Viscosity, ν, m²/s	Prandtl Number, Pr
−10	1.341	1006	0.02288	1.696×10^{-5}	1.680×10^{-5}	1.252×10^{-5}	0.7387
0	1.292	1006	0.02364	1.818×10^{-5}	1.729×10^{-5}	1.338×10^{-5}	0.7362
5	1.269	1006	0.02401	1.880×10^{-5}	1.754×10^{-5}	1.382×10^{-5}	0.7350
10	1.246	1006	0.02439	1.944×10^{-5}	1.778×10^{-5}	1.426×10^{-5}	0.7336
15	1.225	1007	0.02476	2.009×10^{-5}	1.802×10^{-5}	1.470×10^{-5}	0.7323
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014

- The flow is combined laminar and turbulent flow

$$\mathrm{Re}_L = \frac{UL}{\nu} = \frac{8x6}{2.53x10^{-5}} = 1.9x10^6 \succ 5x10^5$$

- The average Nusselt number

$$\bar{N}u_L = \frac{\bar{h}_L L}{k} = (0.037\,\mathrm{Re}_L^{4/5} - 871)\,\mathrm{Pr}^{1/3} = 2867$$

- The heat transfer coefficient

$$\bar{h}_L = \bar{N}u_L \frac{k}{L} = 2867x\frac{0.02953}{6} = 13.2W/m^2K$$

- The heat transfer rate

$$\dot{Q} = \bar{h}_L A(T_s - T_\infty) = 13.2x6x1.5x(140-20) = 14.3kW$$

(b) L = 1.5m

$$Re_L = \frac{8x1.5}{2.548x10^{-5}} = 4.71x10^5 \prec 5x10^5$$ → **laminar**

$$\bar{N}u_L = \frac{\bar{h}_L L}{k} = 0.644\,Re^{0.5}\,Pr^{\frac{1}{3}} = 408$$

$$\bar{h}_L = \frac{k}{L} Nu_L = 8.03W/m^2K$$

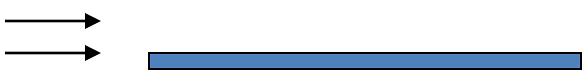
$$\dot{Q} = hA(T_s - T_\infty) = 8.67kW$$

Example 2 Uniform heat flux board

A 15-cm x 15-cm circuit board dissipating 15 W of power uniformly is cooled by air, which approaches the circuit board at 20°C with a velocity of 5 m/s. Disregarding any heat transfer from the back surface of the board, determine the surface temperature of the electronic components (*a) at the leading* edge and (*b) at the end of the board. Assume the flow to be* turbulent since the electronic components are expected to act as turbulators.

Example 7-2 Uniform heat flux board

Given: A = 15cm x 15cm, total power = 15W, the ambient temperature = 20 ^oC, air velocity = 5m/s, The flow is turbulent due to the disturbance of the electronic devices. Consider one side of the board only. Uniform heat flux.



Find : (a) surface temperature at x = 0

(b) surface temperature at x = L

Solution : Firstly we assume surface temperature is 100 ^oC

- $T_f = 60^oC$
  - Properties of fluid from Table A15.

    k = 0.02808W/mK, v = $1.896 \times 10^{-5} m^2/s$, Pr = 0.72

Example 7-2 Uniform heat flux board

(a) $$h_x \propto \frac{1}{x^a}, a \succ 0,,,,,,, x \to 0, h \to \infty \Rightarrow T_s \to T_\infty$$

(b) The flow is turbulent (specified)

- At the end of the board x=0.15m, the average Nusselt number is

$$Nu_x = 0.0308 \mathrm{Re}_L^{0.8} \mathrm{Pr}^{1/3} = 117.4$$

- the heat transfer coefficient at x = 0.15m from the leading edge is

$$h_L = \frac{k}{L} Nu_x = \frac{0.02808}{0.15} 117.4 = 23.2 W / m^2 K$$

Home work problem 7 - 24

- the surface temperature at x = L

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{15}{0.15x0.15} = 666.7W/m^2$$

$$\dot{q} = h(T_s - T_\infty)$$

$$666.7 = 23.2(T_s - 10)$$

$$T_s = 78.7^o C$$

- reassume T_s = 85°C

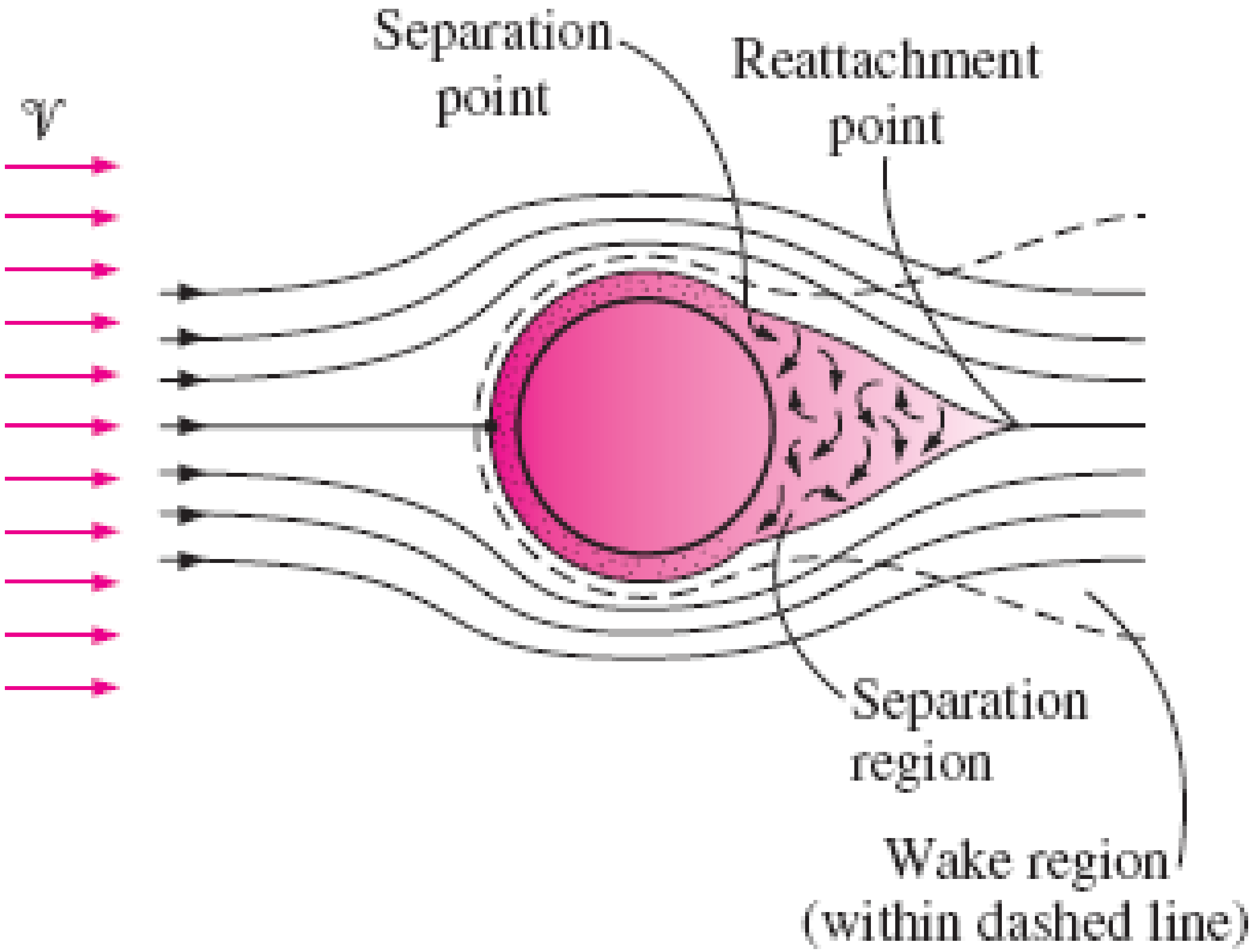
  the two results will be very close

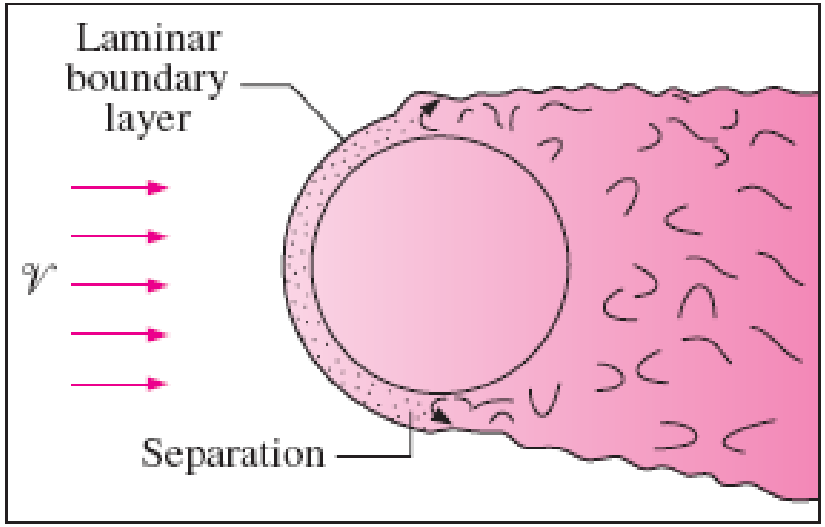
7.4 Flow across cylinders

- The nature of flow

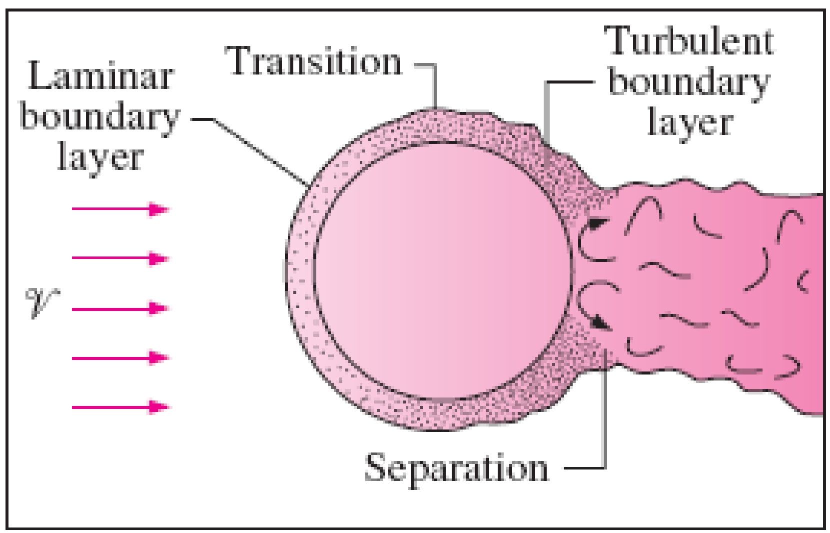
  The flow may involve laminar, transition, turbulent and wake regions. The flow depends, strongly, on the Reynolds number

7.4 Flow across cylinders





(a) Laminar flow (Re $< 2 \times 10^5$)



(b) Turbulence occurs (Re $> 2 \times 10^5$)

Drag Coefficients

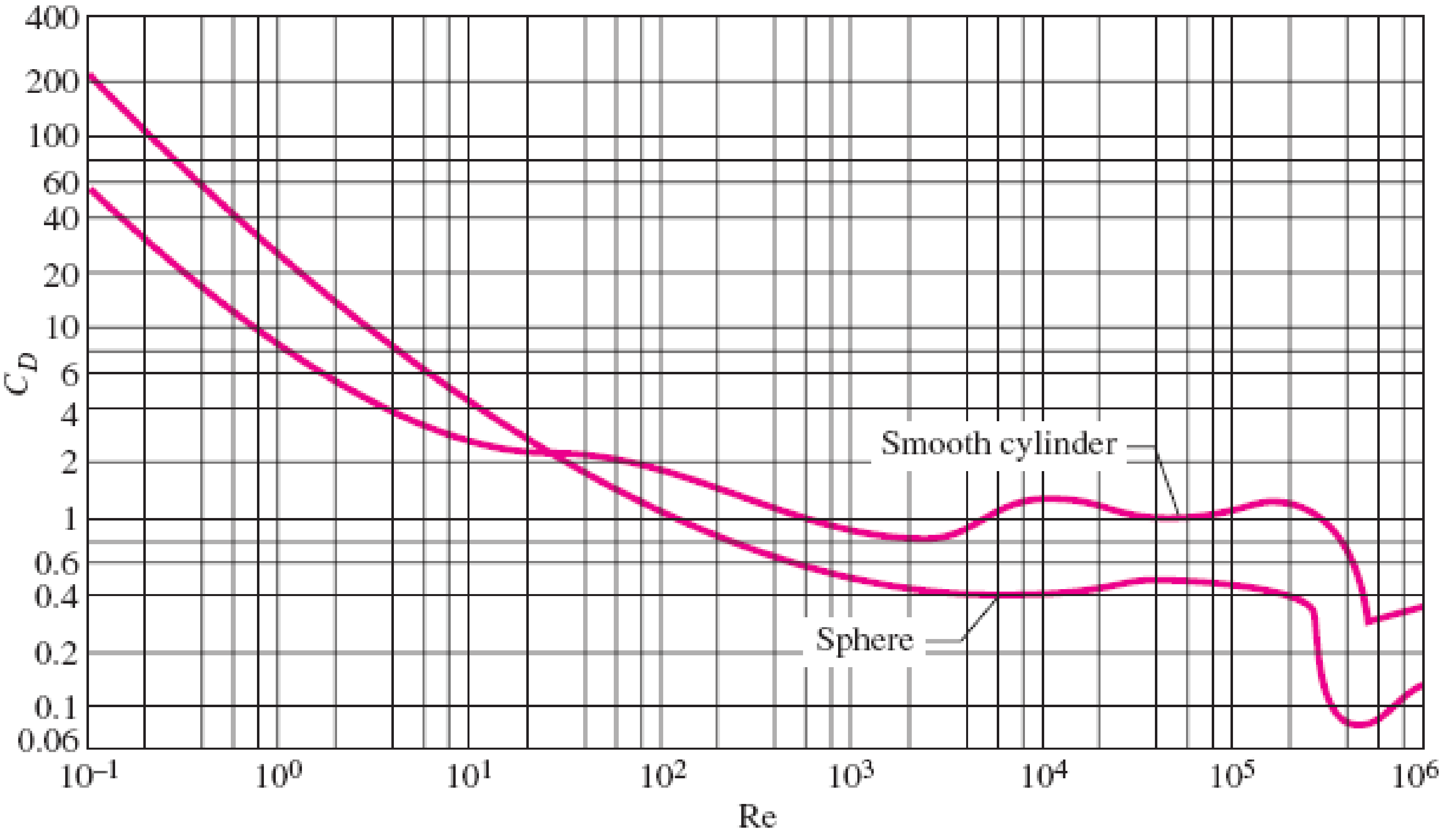


FIGURE 7–17

Average drag coefficient for cross flow over a smooth circular cylinder and a smooth sphere (from Schlichting, Ref. 10).

Effect of Roughness

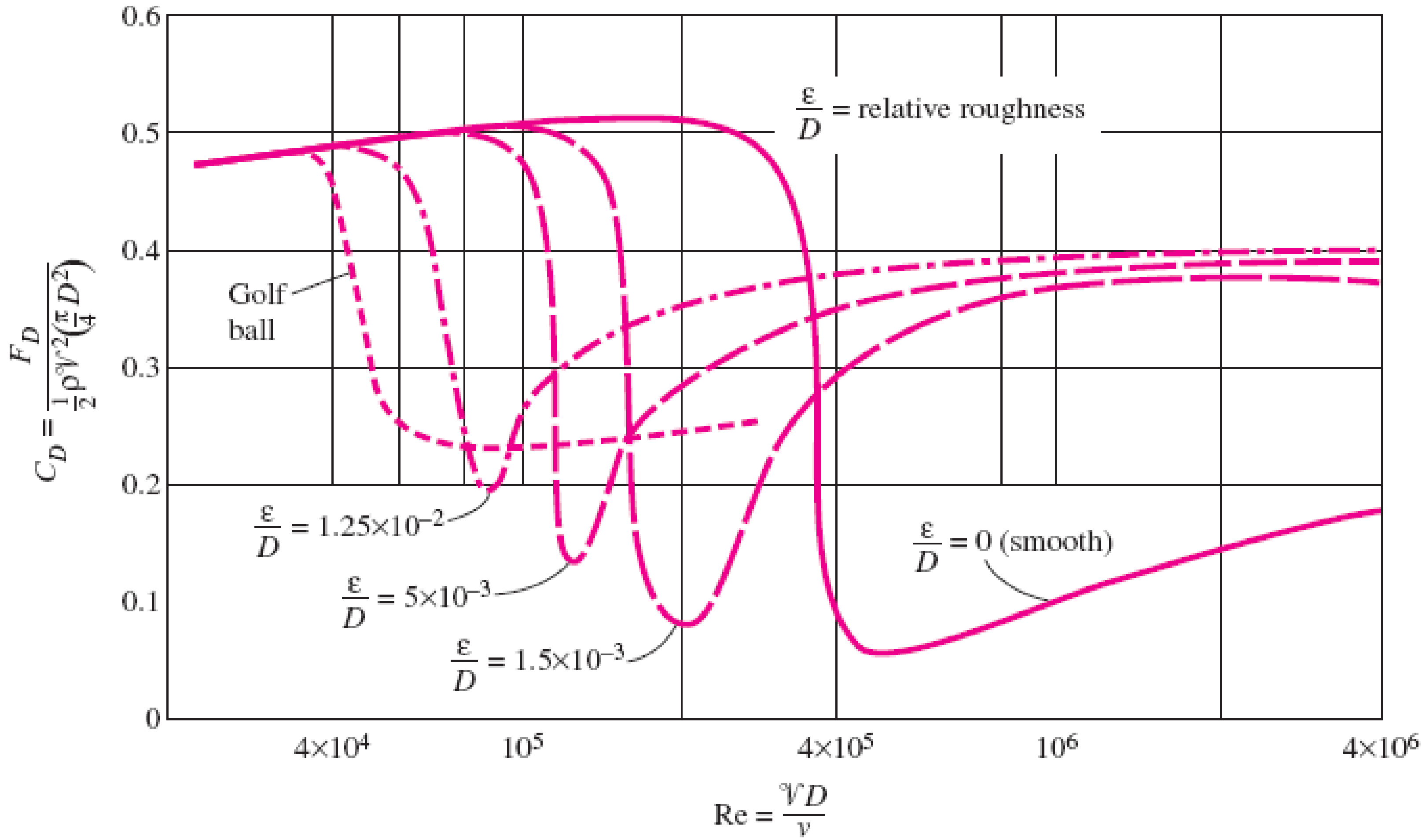
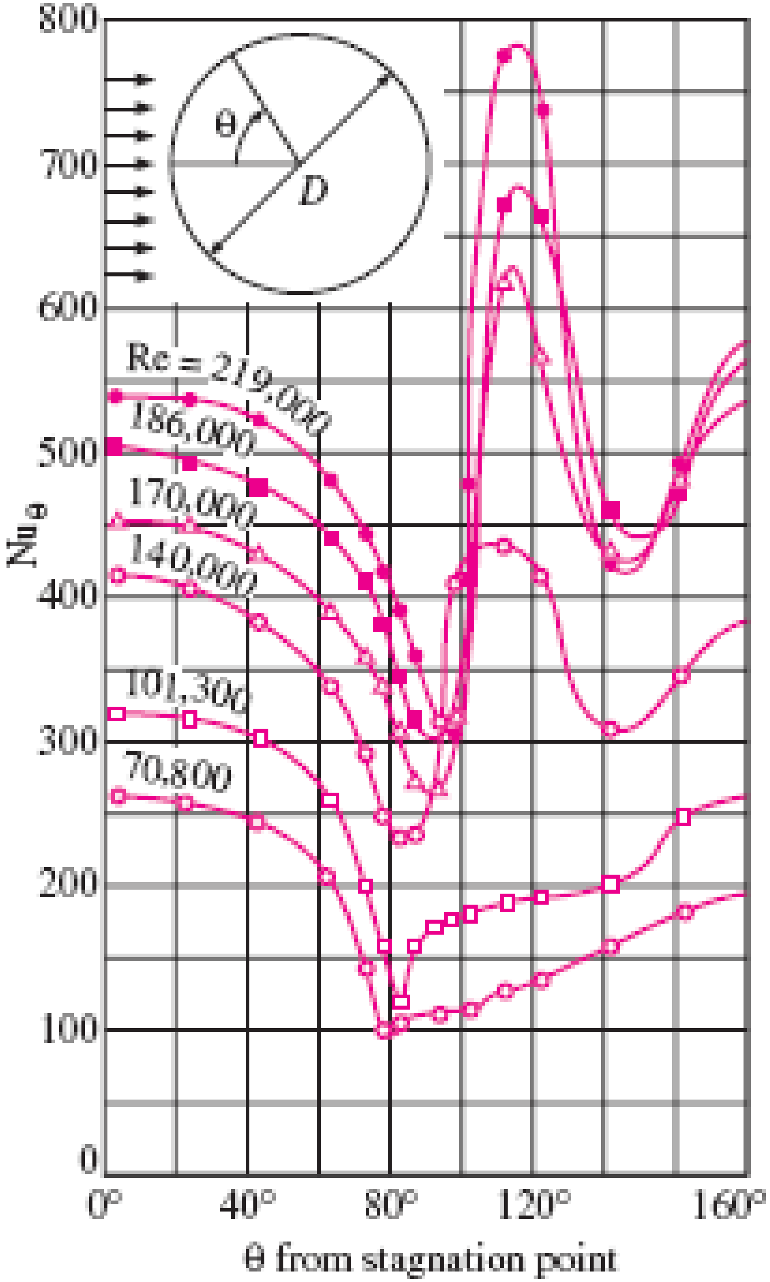


FIGURE 7–19
The effect of surface roughness on the drag coefficient of a sphere (from Blevins, Ref. 1).



Heat Transfer of Cylinder

- The correlation equations of circular cylinder is

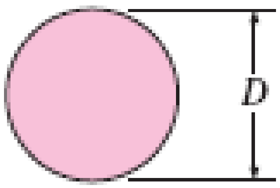
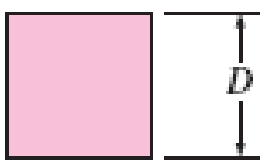
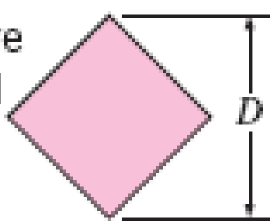
$$Nu_{cyl} = c\,\mathrm{Re}^m\,\mathrm{Pr}^{\frac{1}{3}}$$

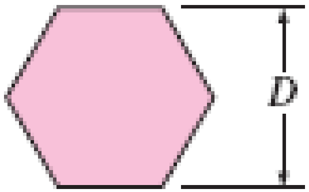
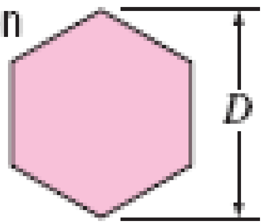
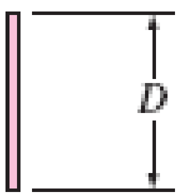
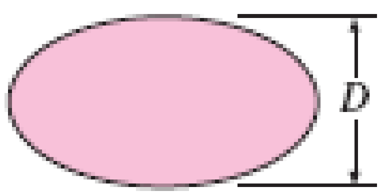
c, and m are constants, depending on the value of Reynolds number. The properties of the fluid is determined at the mean film temperature of the fluid

Re_d	c	m
0.4 – 4	0.989	0.330
4 – 40	0.911	0.385
40 – 40000	0.683	0.466
4000 – 40000	0.193	0.618
40000 – 400000	0.027	0.805

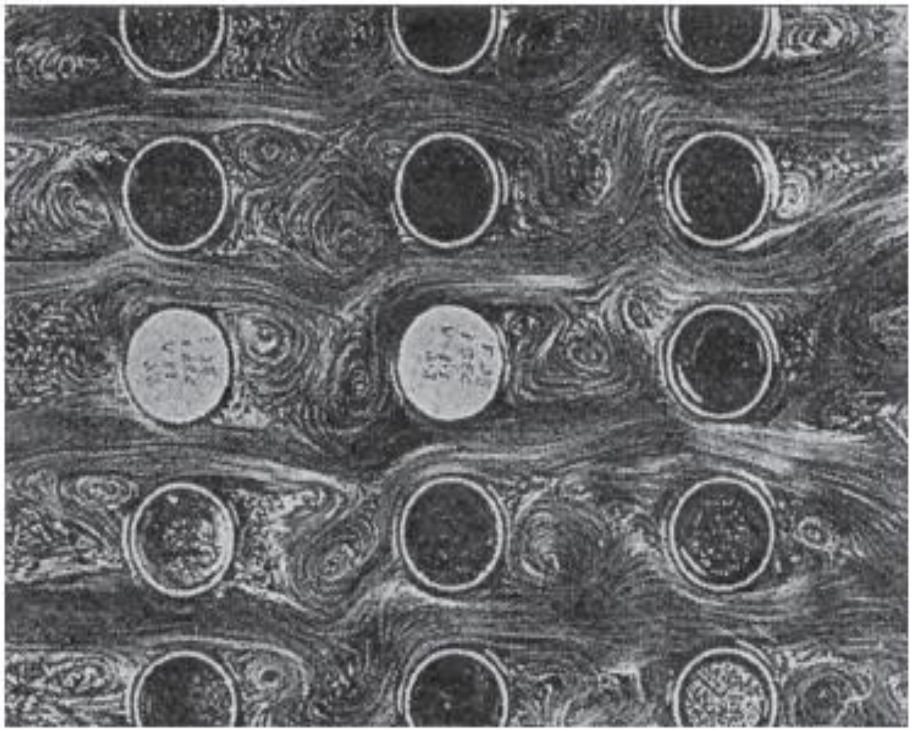
TABLE 7-1

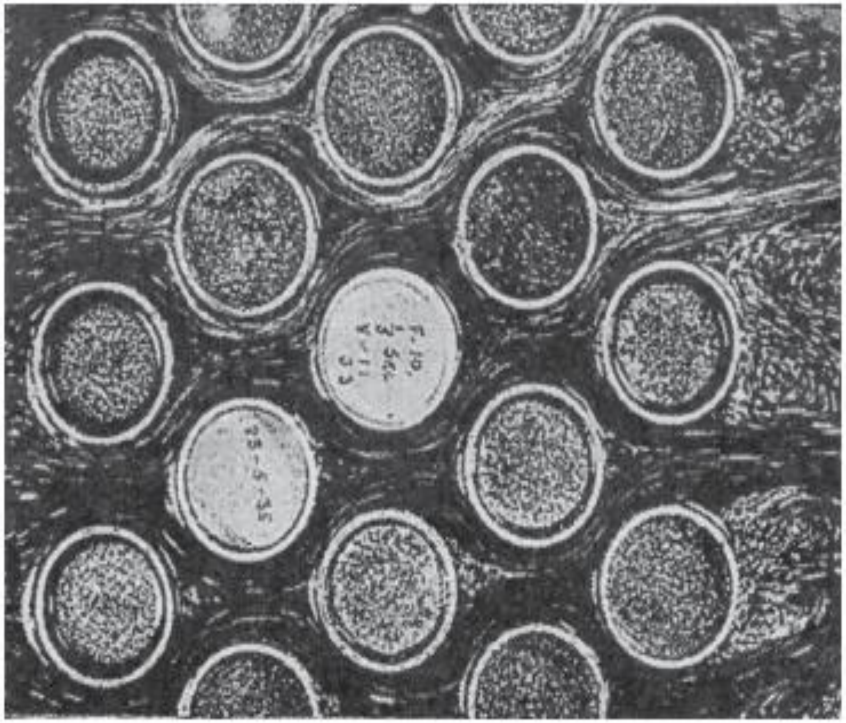
Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

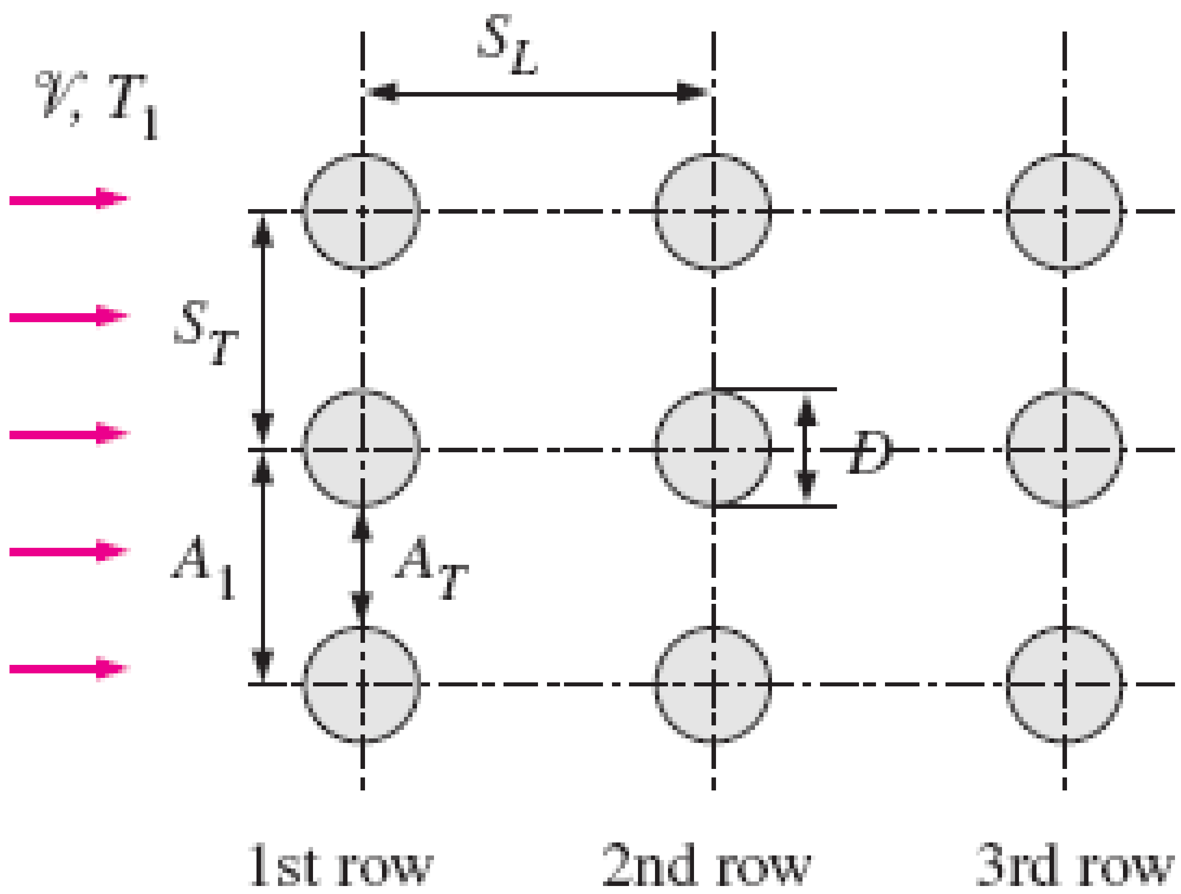
Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\,Pr^{1/3}$ $Nu = 0.911Re^{0.385}\,Pr^{1/3}$ $Nu = 0.683Re^{0.466}\,Pr^{1/3}$ $Nu = 0.193Re^{0.618}\,Pr^{1/3}$ $Nu = 0.027Re^{0.805}\,Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\,Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\,Pr^{1/3}$

Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\,Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\,Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\,Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\,Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\,Pr^{1/3}$

Flow Across Tube Banks

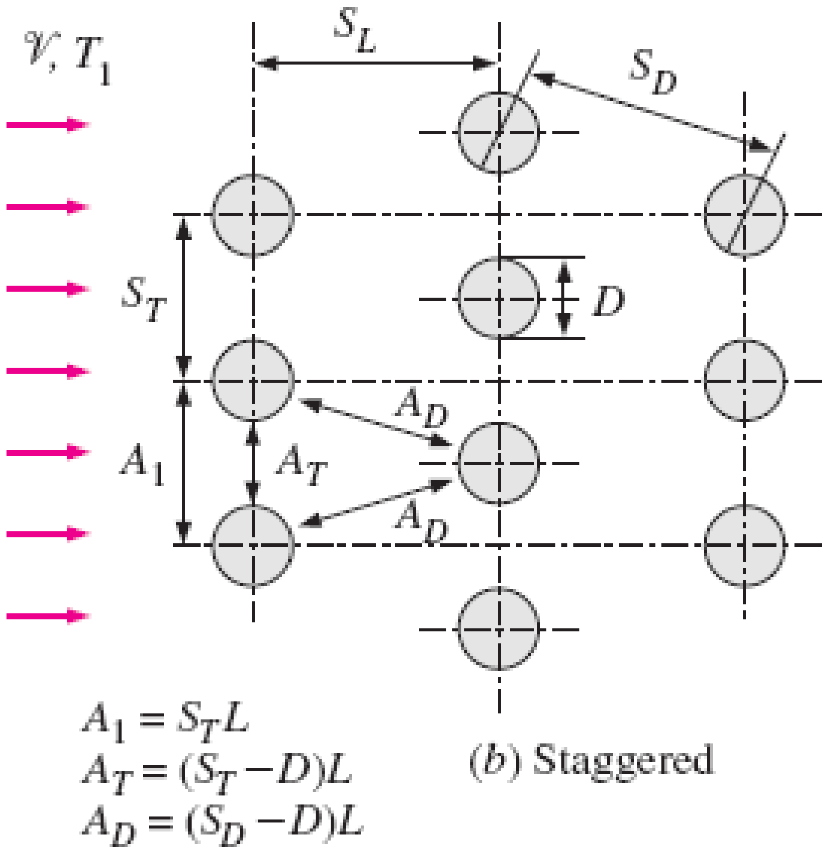






(*a*) In-line

The transverse pitch S_T, longitudinal pitch S_L , and the diagonal pitch S_D



(b) Staggered

$$S_D = \sqrt{S_L^2 + (S_T/2)^2}$$

In tube banks, the flow characteristics are dominated by the **maximum velocity** V_{max} that occurs within the tube bank rather than the approach velocity . Therefore, the Reynolds number is defined on the basis of maximum velocity as

In-line tube banks :

$$\mathcal{V}_{max} = \frac{S_T}{S_T - D}\mathcal{V}$$

Staggered tube banks :

$$\mathcal{V}_{max} = \frac{S_T}{2(S_D - D)}\mathcal{V}$$

Reynolds number is defined as :

$$Re_D = \frac{\rho \mathcal{V}_{max} D}{\mu} = \frac{\mathcal{V}_{max} D}{\nu}$$

Several correlations, all based on experimental data, have been proposed for the average Nusselt number for cross flow over tube banks. More recently, Zukauskas has proposed correlations whose general form is

$$\mathrm{Nu}_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$$

where the values of the constants *C, m,* and *n* depend on value of Reynolds number.

Such correlations are given in Table 7–2 explicitly for $0.7 < Pr < 500$ and $0 < Re_D < 10^6$. Uncertainty in the values of Nusselt number obtained from these relations is 15 percent. Note that all properties except Pr_s are to be evaluated at the arithmetic mean temperature of the fluid determined from

$$T_m = \frac{T_i + T_e}{2}$$

where T_i and T_e are the fluid temperatures at the inlet and the exit of the tube bank, respectively.

TABLE 7–2

Nusselt number correlations for cross flow over tube banks for N > 16 and 0.7 < Pr < 500 (from Zukauskas, Ref. 15, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.27\, Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.033\, Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.35 (S_T/S_L)^{0.2}\, Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.031 (S_T/S_L)^{0.2}\, Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

The average Nusselt number relations in Table 7-2 are for tube banks with 16 or more rows. Those relations can also be used for tube banks with N_L provided that they are modified as

$$\mathrm{Nu}_{D,\, N_L} = F\,\mathrm{Nu}_D$$

where *F is a correction factor F* whose values are given in Table 7-3. For $Re_D > 1000$, the correction factor is independent of Reynolds number.

TABLE 7-3

Correction factor F to be used in $\mathrm{Nu}_{D,\, N_L}$, $= F\mathrm{Nu}_D$ for $N_L < 16$ and $\mathrm{Re}_D > 1000$ (from Zukauskas, Ref 15, 1987).

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

- Once the Nusselt number and thus the average heat transfer coefficient for the entire tube bank is known, the heat transfer rate can be determined from Newton's law of cooling using a suitable temperature difference ΔT.
- The first thought that comes to mind is to use :

$$\Delta T = T_s - T_m = T_s - (T_i + T_e)/2$$

- But this will, in general, **over predict the heat transfer rate**.
- The proper temperature difference for internal flow (flow over tube banks is still internal flow through the shell) is the *Logarithmic mean temperature difference* T_{ln} defined as

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

The exit temperature of the fluid T_e can be determined from :

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m} C_p}\right)$$

Where

A_s = NπDL and $\dot{m} = \rho \mathcal{V} (N_T S_T L)$

Here N is the total number of tubes in the bank, N_T is the number of tubes in a transverse plane, L is the length of the tubes, and is $\mathcal{V}$ the velocity of the fluid just before entering the tube bank. Then the heat transfer rate can be determined from

$$\dot{Q} = hA_s \Delta T_{\ln} = \dot{m} C_p (T_e - T_i)$$

Pressure Drop

$$\Delta P = N_L f \chi \frac{\rho \mathcal{V}_{\max}^2}{2}$$

The pumping power required can be determined from

$$\dot{W}_{\text{pump}} = \dot{V} \Delta P = \frac{\dot{m} \Delta P}{\rho}$$

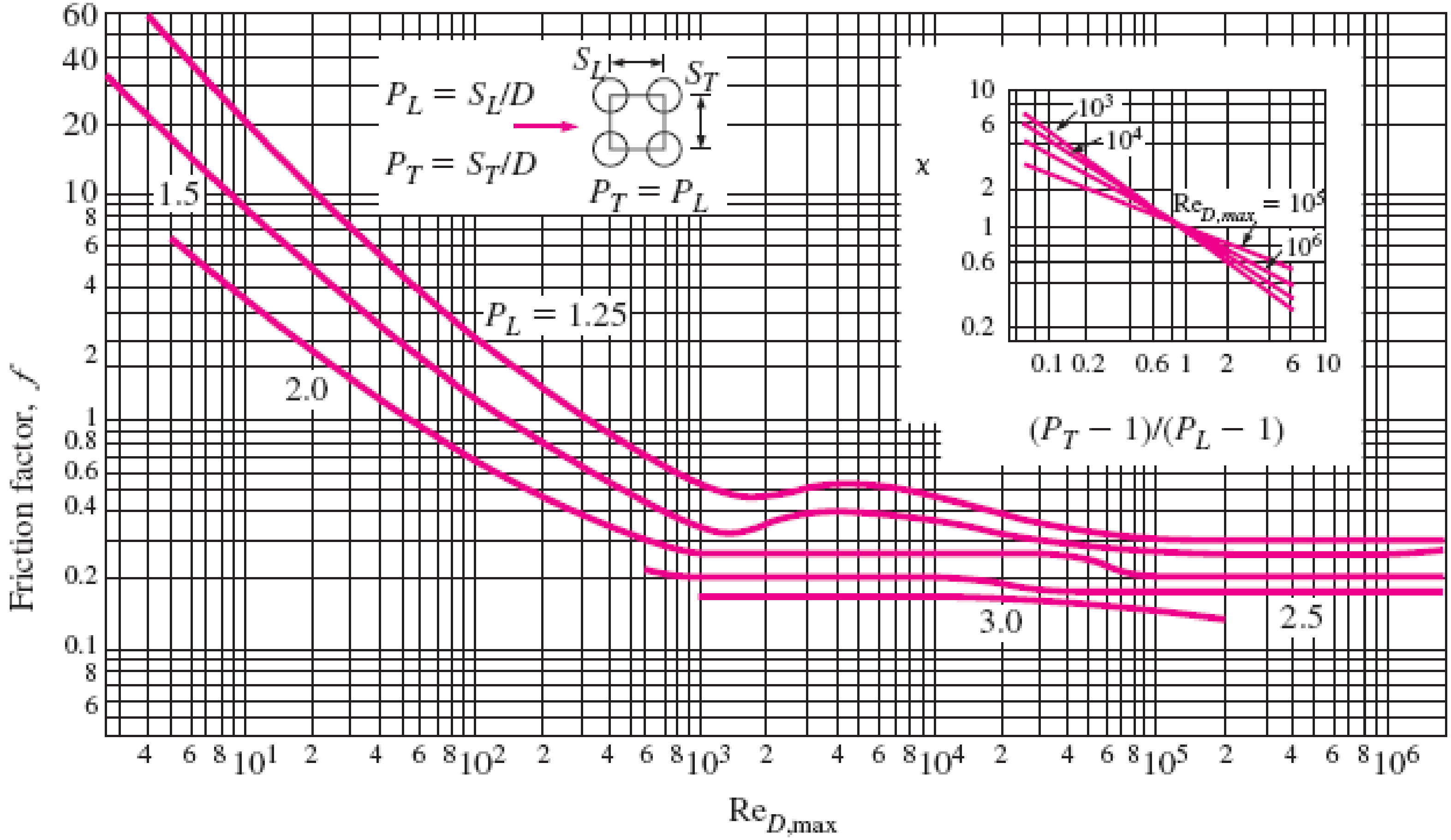
Where

where f is the friction factor and χ is the correction factor

$\dot{V} = \mathcal{V}(N_T S_T L)$ is the volume flow rate

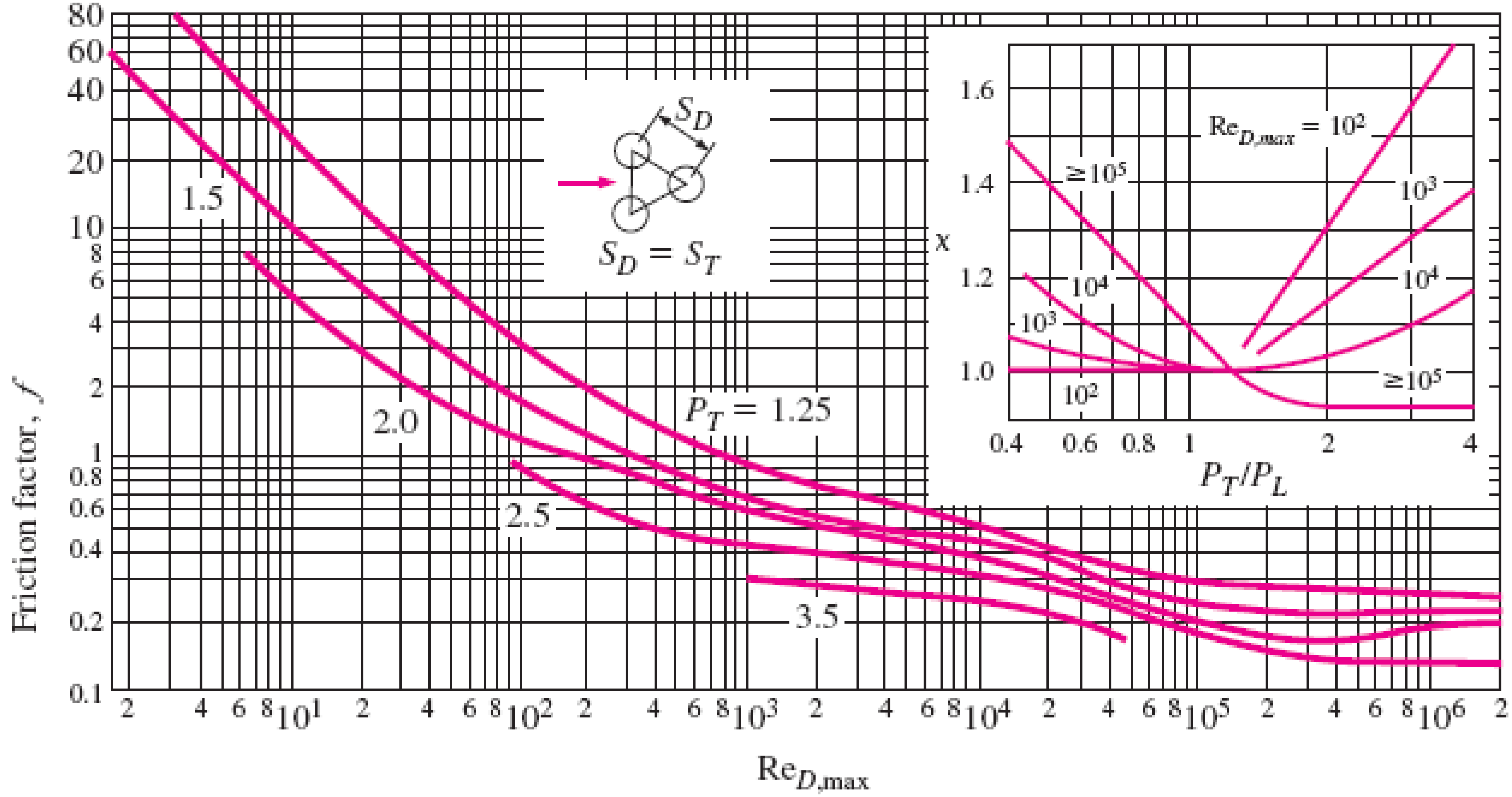
$\dot{m} = \rho \dot{V} = \rho \mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid through the tube bank.

Friction factor & Correction Factor



(*a*) In-line arrangement

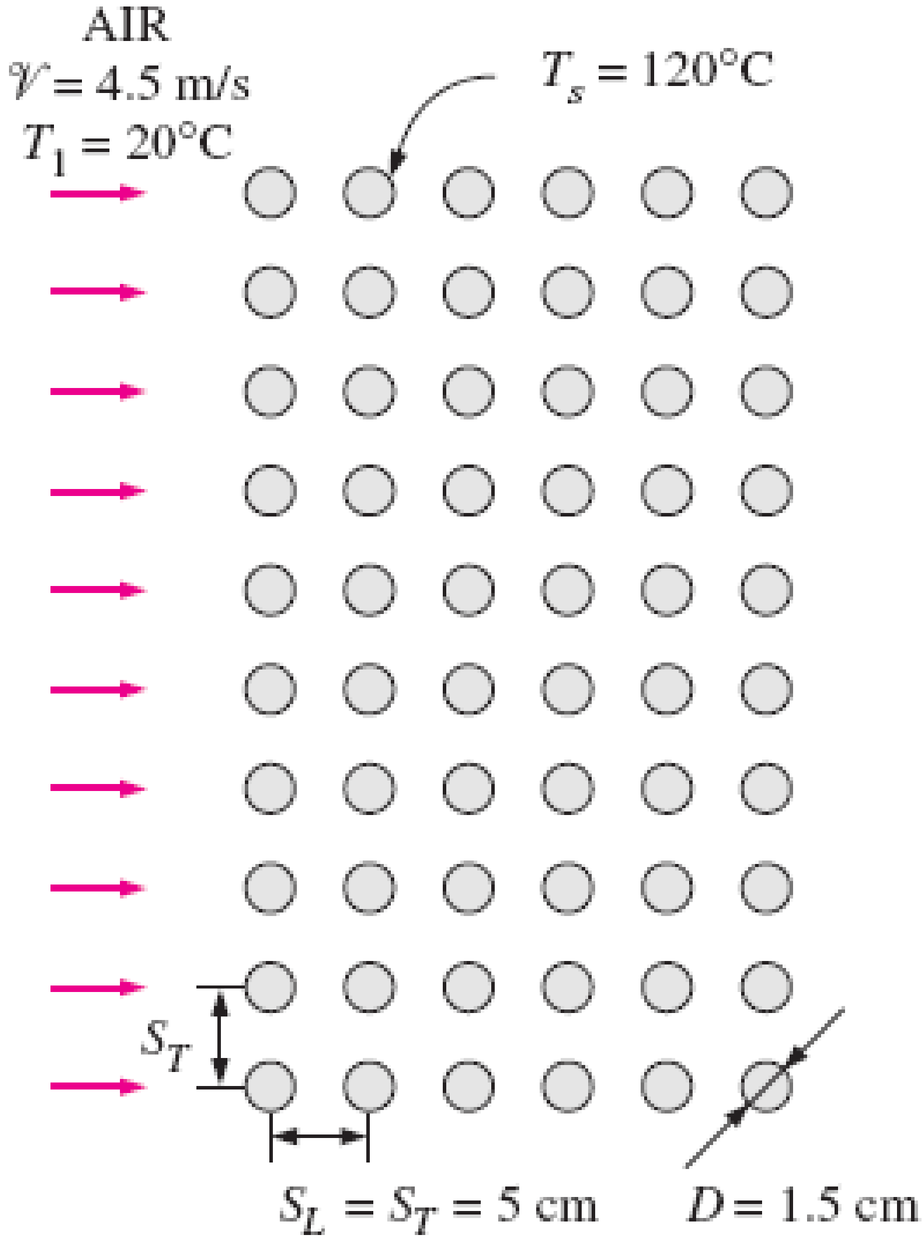
Friction factor & Correction Factor



(*b*) Staggered arrangement

Example : Preheating Air by Geothermal Water in a Tube Bank

In an industrial facility, air is to be preheated before entering a furnace by geothermal water at 120ºC flowing through the tubes of a tube bank located in a duct. Air enters the duct at 20ºC and 1 atm with a mean velocity of 4.5 m/s, and flows over the tubes in normal direction. The outer diameter of the tubes is 1.5 cm, and the tubes are arranged in-line with longitudinal and transverse pitches of $S_L = S_T = 5$ cm. There are 6 rows in the flow direction with 10 tubes in each row, as shown in Figure 7–28. Determine the rate of heat transfer per unit length of the tubes, and the pressure drop across the tube bank.


AIR
𝒱 = 4.5 m/s
T_1 = 20°C
T_s = 120°C
S_T
$S_L = S_T$ = 5 cm
D = 1.5 cm

Solution

Air is heated by geothermal water in a tube bank. The rate of heat transfer to air and the pressure drop of air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of geothermal water.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 60°C (will be checked later) and 1 atm are Table A–15):

$$k = 0.02808 \text{ W/m} \cdot \text{K}, \qquad \rho = 1.06 \text{ kg/m}^3$$

$$C_p = 1.007 \text{ kJ/kg} \cdot \text{K}, \qquad \text{Pr} = 0.7202$$

$$\mu = 2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr}_s = \text{Pr}_{@Ts} = 0.7073$$

Also, the density of air at the inlet temperature of 20°C (for use in the mass flow rate calculation at the inlet) is $\rho_1 = 1.204 \text{ kg/m}^3$

Analysis It is given that $D = 0.015$ m, $S_L = S_T = 0.05$ m, and $\mathcal{V} = 4.5$ m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$\mathcal{V}_{max} = \frac{S_T}{S_T - D}\mathcal{V} = \frac{0.05}{0.05 - 0.015}(4.5 \text{ m/s}) = 6.43 \text{ m/s}$$

$$\text{Re}_D = \frac{\rho \mathcal{V}_{max} D}{\mu} = \frac{(1.06 \text{ kg/m}^3)(6.43 \text{ m/s})(0.015 \text{ m})}{2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = 5091$$

The average Nusselt number is determined using the proper relation from Table 7–2 to be

$$\begin{aligned} \text{Nu}_D &= 0.27 \text{ Re}_D^{0.63} \text{ Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25} \\ &= 0.27(5091)^{0.63}(0.7202)^{0.36}(0.7202/0.7073)^{0.25} = 52.2 \end{aligned}$$

This Nusselt number is applicable to tube banks with $N_L > 16$. In our case, the number of rows is $N_L = 6$, and the corresponding correction factor from Table 7–3 is $F = 0.945$. Then the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\mathrm{Nu}_{D,N_L} = F\mathrm{Nu}_D = (0.945)(52.2) = 49.3$$

$$h = \frac{\mathrm{Nu}_{D,N_L} k}{D} = \frac{49.3(0.02808\ \mathrm{W/m \cdot °C})}{0.015\ \mathrm{m}} = 92.2\ \mathrm{W/m^2 \cdot °C}$$

The total number of tubes is $N = N_L \times N_T = 6 \times 10 = 60$. For a unit tube length ($L = 1$ m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 60\pi(0.015\ \mathrm{m})(1\ \mathrm{m}) = 2.827\ \mathrm{m^2}$$

$$\dot{m} = \dot{m}_1 = \rho_1 \mathcal{V}(N_T S_T L)$$
$$= (1.204\ \mathrm{kg/m^3})(4.5\ \mathrm{m/s})(10)(0.05\ \mathrm{m})(1\ \mathrm{m}) = 2.709\ \mathrm{kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right)$$

$$= 120 - (120 - 20)\exp\left(-\frac{(2.827 \text{ m}^2)(92.2 \text{ W/m}^2\cdot{}^\circ\text{C})}{(2.709 \text{ kg/s})(1007 \text{ J/kg}\cdot{}^\circ\text{C})}\right) = 29.11^\circ\text{C}$$

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{(120 - 29.11) - (120 - 20)}{\ln[(120 - 29.11)/(120 - 20)]} = 95.4^\circ\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (92.2 \text{ W/m}^2\cdot{}^\circ\text{C})(2.827 \text{ m}^2)(95.4^\circ\text{C}) = \mathbf{2.49 \times 10^4 \text{ W}}$$

The rate of heat transfer can also be determined in a simpler way from

$$\dot{Q} = hA_s\Delta T_{\text{in}} = \dot{m}C_p(T_e - T_i)$$

$$= (2.709 \text{ kg/s})(1007 \text{ J/kg}\cdot{}^\circ\text{C})(29.\ 11 - 20)^\circ\text{C} = 2.49 \times 10^4 \text{ W}$$

For this square in-line tube bank, the friction coefficient corresponding to $Re_D = 5088$ and $S_L/D = 5/1.5 = 3.33$ is, from Fig. 7–27*a*, $f = 0.16$. Also, $\chi = 1$ for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f \chi \frac{\rho V_{max}^2}{2}$$

$$= 6(0.16)(1)\frac{(1.06\ kg/m^3)(6.43\ m/s)^3}{2}\left(\frac{1N}{1\ kg \cdot m/s^2}\right) = \mathbf{21\ Pa}$$

Discussion The arithmetic mean fluid temperature is $(T_i + T_e)/2 = (20 + 110.9)/2 = 65.4°C$, which is fairly close to the assumed value of 60°C. Therefore, there is no need to repeat calculations by reevaluating the properties at 65.4°C (it can be shown that doing so would change the results by less than 1 percent, which is much less than the uncertainty in the equations and the charts used).

The End

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