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Application of Active-set Optimization Method to Solve Stochastic Optimization Problem of Supplier Selection and Inventory Control Problem

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Sutrisno, Widowati, Sunarsih and Kartono

Department of Mathematics, Diponegoro University
Tembalang, Semarang, Indonesia

Corresponding author: kartonoundip@gmail.com

Abstract. A stochastic optimization problem that solved by using scenario approach will have a large scale deterministic equivalent optimization. In this paper, we apply the active-set optimization method to solve a stochastic optimization problem of integrated supplier selection and inventory control problem which it solved by using scenario approach or wait and see solution approach. We have solved two numerical experiments which are a small scale problem to illustrate how to determine the solution and a large scale problem to observe how the problem is solved. From the results, the active-set method is applicable for this large scale supplier selection problem considering inventory control problem from the view of the computational time.

1. Introduction

Mathematical model approaches have been developed by many researchers or practitioners to determine the optimal decision in logistics and supply chain management (LSCM). The problem's complexity in LSCM is growing up due to the problem size is become larger or more complex. In LSCM, supplier selection is a problem to determine the best/optimal supplier(s) to supply some product(s) usually raw material(s) [1]. Many researches have been conducted to solve supplier selection problem and most of them are formulating a mathematical model based on the occurred specifications or assumptions such as linear programming [2], [3].

A related problem in LSCM is inventory controlling which is a problem to decide how many product volume that should be stored in the warehouse so that it can be used for future time periods with minimal total holding cost. Furthermore, the decision maker can decide the inventory level to follow some given trajectory. In control theory, it was commonly called as trajectory tracking control problem. In [4], a mathematical model in a mixed integer quadratic programming was formulated to solve these inventory control problem integrated with supplier selection both for deterministic case and stochastic case. For these cases, a Generalized Reduced Gradient (GRG) method was employed to solve. Special for stochastic case, the corresponding optimal decision was determined by finding the wait and see solution which is an optimal solution that corresponds to the realization of the variable random. To find the wait

and see solution of a stochastic optimization, it can has to be converted into deterministic equivalent optimization model. This conversion obviously will produces a medium to large scale deterministic optimization problem following the size of the random variable's sample. To solve it, an analytical optimization method obviously needs very large computer's memory capacity and very huge of the computational time. A numerical optimization method is a right choice to solve this large scale deterministic optimization problem due that a numerical method is use smaller computer's memory and shorter computational time than an analytical method.

One of the most used numerical optimization method is active-set algorithm [5]. It was used to solve many real problems like portfolio selection problem [6], warehouse location problem [7], optimization problem of extreme learning machines [8] and also it was used in many other theoretical problems like nonlinear system identification [9] and parameter estimation problem in generalized linear models [10].

In this paper, we apply the active-set algorithm to solve numerically the inventory control problem with supplier selection of a single product inventory system considering random demand. An existing mathematical model from our previous works will be used as the problem. Numerical experiment is given to observe how the solution is generated by the active-set algorithm.

2. Problem and Mathematical Model

Suppose a supplier selection problem and inventory control problem considering the random demand. By using the active-set method, we solved the existing model available in [4] i.e.

$$\min \tilde{Z} = E[Z] = E\left[\sum_{t=1}^{T}\sum_{s=1}^{S} U_{ts}X_{ts} + \sum_{t=1}^{T} H_t I_t + \sum_{t=1}^{T}\left(I_t - r_t\right)^2 \right] \tag{1}$$

subject to:

$$I_{t-1} + \sum_{s=1}^{S} X_{ts} - I_t \geq \hat{D}_t, \forall t \in T; \tag{2}$$

$$X_{ts} \leq C_s, \forall t \in T, \forall s \in S; \tag{3}$$

$$I_t \leq M, \forall t \in T; \tag{4}$$

$$X_{ts} \text{ integer and nonnegative}, \forall t \in T, \forall s \in S; \tag{5}$$

where S is the supplier set (number of supplier), X_{ts} is the product volume that will be purchased from supplier s at any time period t, I_t is the inventory level at any time period t, $U_{t,s}$ is the purchase cost for unit product from supplier s at any time period t, H_t is the unit product holding cost at any time period t and $E[\cdot]$ denotes the expectation value. The objective function is the expectation of the total cost including purchase cost and holding cost for all planning time periods. The last term in the objective function presents the trajectory tracking control term for inventory control purposes. The first constraint presented for demand satisfying where $\hat{D}_t$ denotes the variable random for the demand value at time period t, the second constraint presents the supplier capacity satisfying where C_s denotes the maximum capacity of supplier s, the third constraint presents the inventory capacity satisfying (M denotes the maximum capacity of the storage) and the last constraint presents the integer and non-negativity constraint.

Optimization problem (1) is a constrained stochastic quadratic optimization problem which has convex objective function. Since it is solved by determining the wait and see solution, stochastic optimization is converted into deterministic equivalent model that solved by using active-set algorithm which starts by producing a guess of the optimal set called active set. If the saved active set turns out to be incorrect, it will be repeated by using gradient and Lagrange multiplier to produce a new active set [5].

3. Numerical Experiment

We have applied the active-set algorithm to solve (1) with three suppliers S1, S2 and S3. We have determined the optimal decision (wait and see solution) for three time periods. Suppose that the unit product purchasing cost is $50 for S1, $52 for S2 and $50 for S3. The supplier capacity is 300 units for S1, 500 units for S2 and 250 units for S3. The storage capacity is 300 units where the holding cost is $4 per unit product. We assume that these parameter value are same for all time periods. In this example, the uncertain parameter occurred in the problem is only demand value. Let the demand value for each time period is approached by variable random $D_t, t = 1, 2, 3$ where these random variables are identically and independent with the following probability distribution

$$f_{D_t}(D) = \begin{cases} 0.3, & \text{if } D = 100 \\ 0.3, & \text{if } D = 150 \\ 0.4, & \text{if } D = 200 \end{cases}$$

for all t. We have used LINGO 17.0 by LINDO SYSTEMS INC. to input the problem then we generate the deterministic equivalent model and export it into MPS format model which is an industry standard format developed by IBM. The original problem (stochastic problem) has only 12 decision variables. Since each time period there is one random variable with three realization values, then the number of the scenario is 27. Hence, the deterministic equivalent model of this example has 324 decision variables. By using LINDO API, we import the MPS format model in MATLAB then by using 'active-set' algorithm included in MATLAB, we solve the problem to find the optimal decision. The used stopping criteria of active-set algorithm is variable tolerance 1e-6 or max function evaluation 32400 or max iteration 10000. We have done all computation in common personal computer with AMD A5 2.7 GHz of Processor, 2 GB of memory and Windows 8 of Operating System. Since it has 27 scenarios, we do not show the optimal decision for all scenarios. We only discussed the optimal decision for scenario-1 ($D_t = 100$ for all t) and scenario 12 ($D_1 = 150, D_2 = 100,$ and $D_3 = 200$) . The optimal decision for this problem for scenario-1 and the corresponding optimal decision generated by LINGO 17.0 which employs Generalized Reduced Gradient (GRG) are given in Fig. 1.
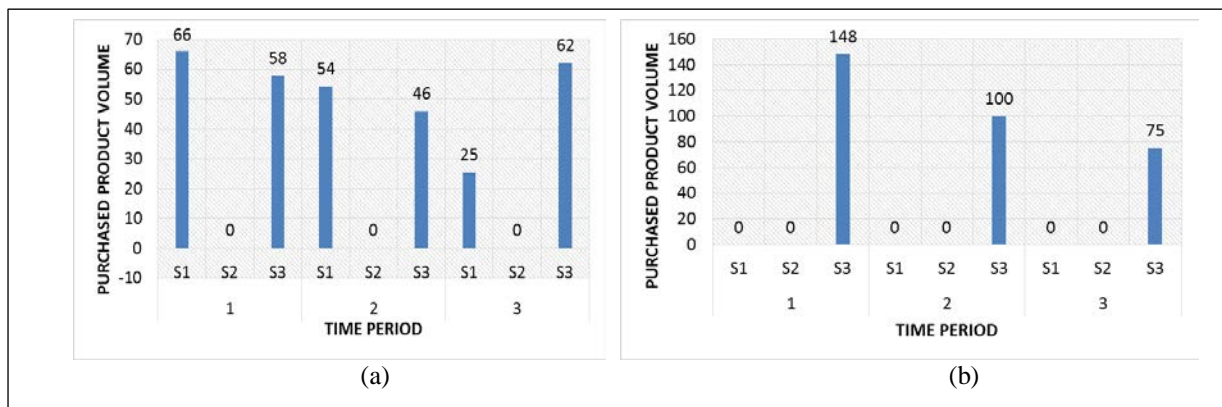
Figure 1. The optimal decision for scenario-1 generated by (a) active-set algorithm (b) LINGO 17.0

From Fig. 1.a, it can be observed that for time period-1, the active-set method generates 66 units purchased from S1 and 58 units purchased from S3 where no unit purchased from S2. From Fig. 1.b, the GRG method in LINGO generates 148 units purchased from S3. For time periods 2 and 3, it can be interpreted analogously. From these results, it seems that these two methods generate strongly differ. But if we compare the total purchased unit for each time periods and the total purchased units for whole time periods, these two methods generate a closely solution. Let we calculate the relative error of the solution generated by active-set to the solution generated by GRG which showed in Fig. 2.
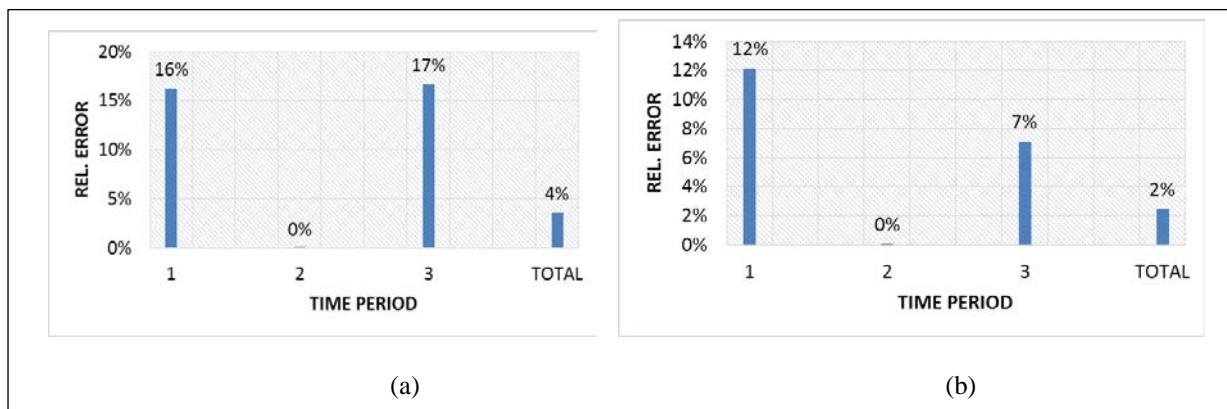
Figure 2. Relative error of active-set's solution to GRG's solution (a) scenario-1 (b) scenario-12

From Fig. 2 (a) and (b), it can be seen that for the total purchased units for whole time periods, the relative error is only 4% for scenario-1 and 2% for scenario-12. If we compared the optimal objective function value which is the expected total cost, the active-set method has produced $28181.50563 whereas the GRG method has produced $25613 and their relative error is 10%. Furthermore, we have compared the computational time between these methods which 42.9 seconds for active-set and 82.34 seconds for GRG. It can be seen that active-set method reduces 48% of computational time.

From these results, it can be observed that the numerical optimization 'active-set' can be used as alternative instead of analytical method to solve the given problem especially for large scale problem. Due to we only used a daily used personal computer with only 2 GB of memory, we cannot give the example for sufficiently large scale problem since it will need a high performance computer to solve. For example, if we have a problem with 10 suppliers and 10 time periods where the demand's probability distribution has 10 of sample size then we have a deterministic equivalent quadratic optimization problem with $(10 \times 10 + 10)^{10} = 110^{10} = 2.6 \times 10^{20}$ variables which is a very large scale quadratic optimization problem.

In [4], the optimization problem (1) was solved by the inside solver in LINGO 15.0 which used GRG method that was generated the optimal decision i.e. the optimal volume of the purchased product for each time period. The active-set method was also successfully generated the optimal decision based on this method i.e. optimal volume of the product that should be purchased from each supplier at each time period although the total cost result between these method is different but the computation time by active-set was less than GRG. It can be derived that for a large scale problem, active-set is a better choice to solve than GRG based on the computation time.

4. Concluding Remark
In this article, an active-set optimization algorithm was used to solve deterministic equivalent optimization problem for stochastic quadratic optimization model of inventory control problem with supplier selection problem considering random demand. From the comparison of the solution generated by active-set algorithm to the solution generated by GRG method, it can be concluded that active-set algorithm can be used as alternative method to solve the problem. Furthermore, active-set algorithm can be used to solve the large scale problem rather than by using some analytical optimization method.

5. Conclusion

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