

# The Shift Invariant Discrete Wavelet Transform (SIDWT) with Inflation Time Series Application

Suparti<sup>1</sup>, Rezy Eko Caraka<sup>2</sup>, Budi Warsito<sup>1</sup>, & Hasbi Yasin<sup>1</sup>

<sup>1</sup>Department of Statistics, Diponegoro University, Indonesia

<sup>2</sup>Awardee of LPDP Scholarship, Ministry of Finance, Indonesia

Correspondence: Suparti, E-mail: supartisudargo@yahoo.co.id; Rezy Eko Caraka, E-mail: rezyekocaraka@gmail.com

Received: April 28, 2016 Accepted: May 16, 2016 Online Published: July 25, 2016

doi:10.5539/jmr.v8n4p14

URL: <http://dx.doi.org/10.5539/jmr.v8n4p14>

## Abstract

Analysis of time series used in many areas, one of which is in the field economy. In this research using time series on inflation using Shift Invariant Discrete Wavelet Transform (SIDWT). Time series decomposition using transformation wavelet namely SIDWT with Haar filter and D4. Results of the transformation, coefficient of drag coefficient wavelet and scale that is used for modeling time series. Modeling done by using Multiscale Autoregressive (MAR). In a certain area, inflation to it is an important that he had made the standard-bearer of economic well-being of society, the factors Directors investors in selecting a kind of investment, and the determining factor for the government to formulate policy fiscal, monetary, as well as non-monetary that will be applied. Inflation can be analyzed using methods Shift Invariant Discrete Wavelet Transform (SIDWT) which had been modeled for them to use Multiscale Autoregressive (MAR) with the  $R^2$  value 93.62%.

**Keywords:** Shift Invariant Discrete Wavelet Transform (SIDWT), Time Series, Multiscale Autoregressive (MAR), Inflation.

## 1. Introduction

Accord In a certain area, inflation to it is an important that had made the standard-bearer of economic well-being of society, the factors Directors investors in selecting a kind of investment, and the determining factor for the government to formulate policy fiscal, monetary, as well as non-monetary that will be applied. In general, inflation can lead to less investment in a country, encouraging increase in interest rate, to encourage investment that is speculative, failure execution of development, the instability, economic balance of payments and a decline, life and welfare of the people. Understanding investors will impact of inflation in high rate of return or profits investment is needed at the time investors will choose the kind of investment that will be done. This is because inflation has an impact on the value of the money that was invested by investors. High inflation will increase the risk investment projects in the long term [Prahutama et al., 2014]

In the mandate prescribed in the Bank Indonesia Law, the goal of Bank Indonesia focuses on achievement of a single objective, that of achieving and maintaining stability in the value of the rupiah. There are two aspects to stability in the value of the rupiah, namely stability of the currency in relation to goods and services and stability in relation to the currencies of other nations. The first aspect is reflected in the inflation rate, while the second is reflected in the rupiah exchange rate against foreign currencies. The purpose in formulating this single objective is to clarify the targets to be achieved by Bank Indonesia and the limits of Bank Indonesia's responsibility. This provides for easy measurement of whether Bank Indonesia has achieved this objective. In working towards this objective, Bank Indonesia understands that achievement of economic growth and inflation control need to be brought into consistent alignment for the sake of optimum, sustainable results in the long-term.

The indicator commonly used to measure the level of inflation is the Consumer Price Index (CPI). Changes in the CPI over time are indicative of price movements for packages of goods and services consumed by the public. Since July 2008, the packages of goods and services in the CPI basket have been based on the 2007 Cost of Living Survey conducted by the Statistics Indonesia (BPS). Following this, BPS monitors price movements for these goods and services in selected cities and towns each month, using information from traditional markets and modern retail outlets on specific categories of goods and services in each location. The inflation measured in the CPI in Indonesia is divided into 7 expenditure categories (based on the Classification of Individual Consumption by Purpose - COICOP). These are: Food Stuffs ; Processed Foods; Beverages and Tobacco ;Housing ;Clothing ;Health ; Education and Sports and ; Transportation and

Communications.

Monetary Policy is not just to react to inflationary pressure is going on now, but should respond to inflationary pressures that will come. Inflation that is relatively high and to draw up a policy that was able to respond to inflationary pressure in the future, so it needs to be the prediction to inflation. The prediction that accurate will give important role in determining the policy of the government had an impact on people's welfare, and investment world. This research aims to transform using The Shift Invariant Discrete Wavelet Transform (SIDWT). It is hoped that this method can be one of the alternative to the government in predicting inflation

**2. The Basic Method**

Wavelet is a name for a small waves that up and down in the same time. For example waves sinus function that move up and down the plots of  $\sin(u)$  with  $u \in (-\infty, \infty)$  [Percival et al., 2000]

**2.1 Wavelet Function**

Wavelet function is distinguished as two types, namely wavelet father ( $\phi$ ) and wavelet mother ( $\psi$ ) did not mention any kind, said wavelet pointed to wavelet mother [Bruce et al., 1996]. Function wavelet have characteristics:

$$\int_{-\infty}^{\infty} \phi(x) dx = 1 \text{ and } \int_{-\infty}^{\infty} \psi(x) dx = 0 \tag{1}$$

By dilatation dyadic and translation integer, family wavelet namely

$$\phi_{j,k}(x) = 2^{\frac{j}{2}} \phi(2^j x - k) \text{ and } \psi_{j,k}(x) = 2^{\frac{j}{2}} \psi(2^j x - k) \tag{2}$$

Function  $\phi_{j,k}(x)$  and  $\psi_{j,k}(x)$  That ortogonal have characteristics of

$$\int_{-\infty}^{\infty} \phi_{j,k}(x) \phi_{j,k'}(x) dx = \delta_{k,k'}, \int_{-\infty}^{\infty} \psi_{j,k}(x) \phi_{j,k'}(x) dx = 0, \int_{-\infty}^{\infty} \psi_{j,k}(x) \psi_{j',k'}(x) dx = \delta_{j,j'} \delta_{k,k'} \tag{3}$$

With  $\delta_{i,j} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

Wavelet function can form a base in the  $L^2(\mathbb{R})$  with  $L^2(\mathbb{R}) = \{f \mid \int_{-\infty}^{\infty} f^2(x) dx < \infty\}$ . As a result every  $f \in L^2(\mathbb{R})$  can be declared as a combination in linear a base that was built by wavelet [Suparti, 2000]

$$f(x) = \sum_{k \in \mathbb{Z}} c_{j,k} \phi_{j,k}(x) + \sum_{j < J} \sum_{k \in \mathbb{Z}} d_{j,k} \psi_{j,k}(x) \tag{4}$$

with

$$c_{j,k} = \langle f, \phi_{j,k} \rangle = \int_{-\infty}^{\infty} f(x) \phi_{j,k}(x) dx$$

$$d_{j,k} = \langle f, \psi_{j,k} \rangle = \int_{-\infty}^{\infty} f(x) \psi_{j,k}(x) dx$$

The function  $f$  produces the form of infinite series , but the function  $f$  can be approximated by either using a limited sum up the index  $J$  , with  $J$  large , it can be expressed as the sum of components  $S$  scale and detail components  $D$

$$F_j(x) = \sum_{k \in \mathbb{Z}} c_{j,k} \phi_{j,k}(x) + \sum_{k \in \mathbb{Z}} d_{j-1,k} \psi_{j-1,k}(x) + \sum_{k \in \mathbb{Z}} d_{j-2,k} \psi_{j-2,k}(x) + \dots + \sum_{k \in \mathbb{Z}} d_{1,k} \psi_{1,k}(x) = S_j + D_{j-1} + D_{j-2} + \dots D_1 \tag{5}$$

to  $J$  approaching, then  $F_j(x)$  approaches  $f(x)$ .

**2.2 Discrete Wavelet Transform (Dwt)**

Transformation wavelet is divided in two parts of it, which is a continue wavelet transform (CWT) and discrete wavelet tranform (DWT) [Lindsay et al., 1996]. Transformation wavelet malar from a function  $f(x) \in L^2(\mathbb{R})$  is defined as follows:

$$T^{wav}f(a, b) = \int_{-\infty}^{\infty} f(x)\overline{\Psi_{a,b}(x)} dx \tag{6}$$

There are other forms of the transformation wavelet namely tranformasi discrete wavelet (DWT). For transformation discrete wavelet signals that analyzed assumed that they have been sampled with long interval period [Renauld, 2000]. Main purpose of transformation wavelet is changing signals

2.3 Shift Invariant Discrete Wavelet Transform (SIDWT)

Shift Invariant Discrete Wavelet Transform (SIDWT) in various literature has some pronunciation for example, Shift invariant frames, Maximal Overlap Discrete Wavelet Transform (MODWT), wavelet translation DWT, undecimated-Discrete Wavelet Transform. SIDWT have an advantage can be used for each sample size  $N$  [Warsito et al., 2013]. In addition, SIDWT could eliminate the declining data. Then in SIDWT there are  $N$  coefficient wavelet and becomes coefficient at every level SIDWT. For example, there is a data of time series  $X$ , and transformation SIDWT will produce vector column  $W_1, W_2, \dots, W_{j_0}$  and  $V_{j_0}$  with each measuring  $N$ . Smoothing coefficient derived from the  $X$  data derived from repeated multiplication of  $X$  with filter scale( $\tilde{g}$ ) and wavelet filter( $\tilde{g}$ ).

The main objective in the formulation SIDWT is to define the transformation that is as DWT .By defining a  $\tilde{V}$  which is an  $N \times N$  matrix that contains filter  $g$  and  $\tilde{W}$  is  $N \times N$  matrix containing filter  $\tilde{h}$ . Suppose for the first level , known  $L = 4$  and  $N > 4$  , then the matrix  $\tilde{W}_1$  ordered as follows

$$\tilde{W}_{1,t} \equiv \sum_{l=0}^{L-1} \tilde{h}_l X_{t-l \bmod N} \tag{7}$$

$t = 0$ , gained:  $\tilde{W}_{1,0} \equiv \tilde{W}_0^T X = \sum_{l=0}^{N-1} \tilde{h}_{-l \bmod N} X_l$ ,

so,

$$\tilde{W}_0^T = [\tilde{h}_0^\circ, \tilde{h}_{N-1}^\circ, \tilde{h}_{N-2}^\circ, \dots, \tilde{h}_1^\circ]$$

$L \leq N$ , Filter periodic matter takes the form simple

$$\tilde{h}_l^\circ = \begin{cases} \tilde{h}_l, & 0 \leq l \leq L - 1; \\ 0, & L \leq l \leq N - 1 \end{cases}$$

So the line of the first  $\tilde{W}_1$  is

$$\tilde{W}_0^T = [\tilde{h}_0, 0, \dots, 0, \tilde{h}_{L-1}, \dots, \tilde{h}_1]$$

Because  $L = 4$ , thus  $\tilde{h}_{L-1} = \tilde{h}_3$  and  $\tilde{h}_L$  high-zero until  $\tilde{h}_{N-1}$ , with the number of elements that high-zero is  $N - L$ . So the first line matrix  $\tilde{W}_1$

$$[\tilde{h}_0 \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \tilde{h}_3 \quad \tilde{h}_2 \quad \tilde{h}_1]$$

Second Period when line  $\tilde{W}_1$  when  $t = 1$ , gained  $\tilde{W}_{1,0} \equiv \tilde{W}_1^T X = \sum_{l=0}^{N-1} \tilde{h}_{1-l \bmod N} X_l$ , so inventory

$$\tilde{W}_1^T = [\tilde{h}_1^\circ, \tilde{h}_0^\circ, \tilde{h}_{N-1}^\circ, \dots, \tilde{h}_2^\circ]$$

The second line from  $\tilde{W}_1$  is

$$[\tilde{h}_1 \quad \tilde{h}_0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \tilde{h}_3 \quad \tilde{h}_2]$$

Applied goes on to  $t = N - 1$ , and matrix filter wavelet with the structure

As follows:

$$\tilde{W}_1 = \begin{bmatrix} \tilde{h}_0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \tilde{h}_3 & \tilde{h}_2 & \tilde{h}_1 \\ \tilde{h}_1 & \tilde{h}_0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_3 & \tilde{h}_2 \\ \tilde{h}_2 & \tilde{h}_1 & \tilde{h}_0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_3 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & \tilde{h}_3 & \tilde{h}_2 & \tilde{h}_1 & \tilde{h}_0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \tilde{h}_3 & \tilde{h}_2 & \tilde{h}_1 & \tilde{h}_0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \tilde{h}_3 & \tilde{h}_2 & \tilde{h}_1 & \tilde{h}_0 \end{bmatrix}$$

$\tilde{V}_1$  drawn up  $\tilde{V}_{1,t} \equiv \sum_{l=0}^{L-1} \tilde{g}_l X_{t-l \bmod N}$ ,  $\tilde{h}_l$  will be changed to  $\tilde{g}_l$ .

$$\tilde{V}_1 = \begin{bmatrix} \tilde{g}_0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \tilde{g}_3 & \tilde{g}_2 & \tilde{g}_1 \\ \tilde{g}_1 & \tilde{g}_0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{g}_3 & \tilde{g}_2 \\ \tilde{g}_2 & \tilde{g}_1 & \tilde{g}_0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{g}_3 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & \tilde{g}_3 & \tilde{g}_2 & \tilde{g}_1 & \tilde{g}_0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \tilde{g}_3 & \tilde{g}_2 & \tilde{g}_1 & \tilde{g}_0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \tilde{g}_3 & \tilde{g}_2 & \tilde{g}_1 & \tilde{g}_0 \end{bmatrix}$$

So the first of SIDWT can be written in the equation:

$$\begin{bmatrix} \tilde{W}_1 \\ \tilde{V}_1 \end{bmatrix} = \begin{bmatrix} \tilde{W}_1 \\ \tilde{V}_1 \end{bmatrix} x = \tilde{P}_1 X, \quad \text{with} \quad \tilde{P}_1 = \begin{bmatrix} \tilde{W}_1 \\ \tilde{V}_1 \end{bmatrix}$$

and  $P_1^T$  is orthonormal matrix. Thus, to reconstruct the data X of coefficients SIDWT if decomposition is done on the first level, ie :

$$X = \tilde{P}_1^{-1} \begin{bmatrix} \tilde{W}_1 \\ \tilde{V}_1 \end{bmatrix} = \tilde{P}_1^T \begin{bmatrix} \tilde{W}_1 \\ \tilde{V}_1 \end{bmatrix} = \begin{bmatrix} \tilde{W}_1 \\ \tilde{V}_1 \end{bmatrix}^T \begin{bmatrix} \tilde{W}_1 \\ \tilde{V}_1 \end{bmatrix} \tag{15}$$

because P orthogonal matrix [Kingsbury, 2000], then  $\tilde{P}_1^{-1} = \tilde{P}_1^T$

Second level of matrix  $\tilde{W}_2$  with size  $N \times N$  (applied also to matrix  $\tilde{V}_2$  with replacing  $\tilde{h}$  with  $\tilde{g}$ )

$$\tilde{W}_2 = \begin{bmatrix} \tilde{h}_0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_3 & 0 & \tilde{h}_2 & 0 & \tilde{h}_1 & 0 \\ 0 & \tilde{h}_0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_3 & 0 & \tilde{h}_2 & 0 & \tilde{h}_1 \\ \tilde{h}_1 & 0_1 & \tilde{h}_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_2 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \tilde{h}_3 & 0 & \tilde{h}_2 & 0 & \tilde{h}_1 & 0 & \tilde{h}_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{h}_3 & 0 & \tilde{h}_2 & 0 & \tilde{h}_1 & 0 & \tilde{h}_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{h}_3 & 0 & \tilde{h}_2 & 0 & \tilde{h}_1 & 0 & \tilde{h}_0 \end{bmatrix}$$

2.4 Multiscale Autoregressive (MAR)

With the *multiscale decomposition* like wavelet, there are benefits provided by the automatically components separate data, such as components trend and irregular component on the data. But this method can be used to perform the prediction on the data remains stationer and those who do not heavy duty. For example a signal  $X = (X_1, X_2, \dots, X_t)$  and assumed that they will be rediction values  $X_{t+1}$ .

$$W_{j,t-2^j(k-1)} \text{ and } V_{j,t-2^j(k-1)} \text{ With } k = 1, 2, \dots, A_j \text{ and } j = 1, 2, \dots, J$$

First Point that must be understood is to know how many and drag coefficient of drag coefficient wavelet which is used in every scale. A Model that prediction that used in this research focuses on the model Autoregressive (AR) [Renauld et al., 2000]. A process autoregressive with orders p known as AR(p) can be written down

$$\hat{X}_{t+1} = \sum_{k=1}^p \hat{\phi}_k X_{t-(k-1)} \tag{9}$$

By using coefficient of decomposition wavelet [Gencay et al., 2001] explained that the prediction AR model to *Multiscale Autoregressive*.

$$\hat{X}_{t+1} = \sum_{j=1}^J \sum_{k=1}^{A_j} \hat{a}_{j,k} w_{j,t-2^j(k-1)} + \sum_{k=1}^{A_j} \hat{a}_{j+1,k} v_{j,t-2^j(k-1)} \tag{10}$$

with:

$a_{j,k}$  = Coefficient MAR ( j=1,2, ...,J and k=1,2, ..., A<sub>j</sub>)

$A_j$  = Order from the model MAR  
 $w_{j,t}$  = Coefficient wavelet of data  
 $v_{j,t}$  = Scale of data

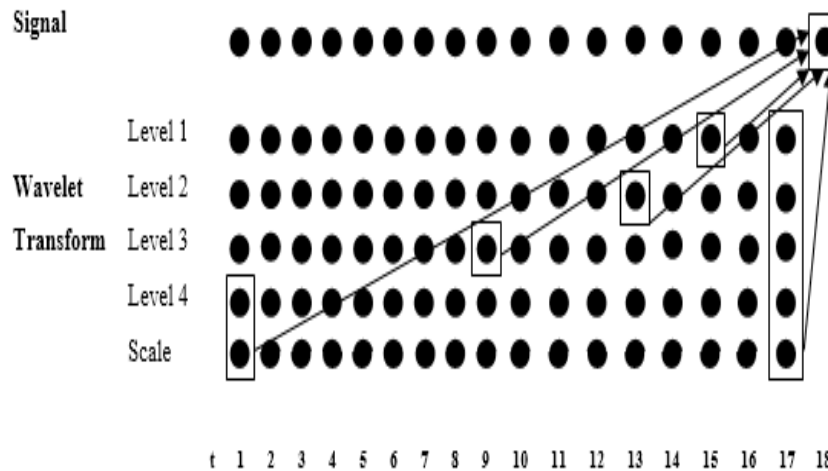


Figure 1. Illustration modeling wavelet  $J=4$  and  $A_j=2$

### 3. Research Methodology

The data used in this research is year-on-year (yoy) of the Indonesian inflation data, since December 2006 up to February 2015. The data is divided into two parts, from December 2006 up to December 2013 as in-sample and the remaining is out-of-sample. The analysis is started with SIDWT decomposition to find the wavelet coefficients and scale coefficients. SIDWT decomposition is done to the stationary data. The data is modeled by using MAR model with the appropriate procedure. The choosing of the best MAR model is based on the out-of-sample criteria.

### 4. Results

The first step is to make a decomposition SIDWT inflation data using using filter Haar and Daublets 4 (D4) with level ( $j$ ) = 4. Decomposition process SIDWT will produce coefficient of wavelet ( $w$ ) and scale ( $v$ ) that consists of  $w_1, w_2, w_3, w_4$ , and  $v_4$  [Caraka et al., 2015]. Coefficient can be set up to a plot for each filter [Suhartono et al., 2010]. It shows how plots through decompositions SIDWT, transformation wavelet able to separate the trend of data [Mallat, 1989]. Can be seen from Figure.2 the coefficient coefficient wavelet ( $w_1, w_2, w_3, w_4$ ) is about zero while coefficient becomes the coefficient ( $v_4$ ) following the movement a trend data [Suhartono et al., 2010]. Processing will be done by using the syntax software R.

Table 1. Summary of variables SIDWT D4 model

Predictor Variable	Coefficients	Pr(> t )	Result
X1	1.81846	8.53e-07	No Significant
X2	-0.34954	0.3725	Significant
X3	0.56626	0.0381	No Significant
X4	-0.30860	0.2944	Significant
X5	1.59386	4.98e-09	No Significant
X6	-0.06183	0.7954	Significant
X7	0.86083	4.76e-06	No Significant
X8	-0.00780	0.9292	Significant
X9	0.96278	2e-16	No Significant
X10	0.02724	0.5431	Significant

First model that has been level ( $j$ ) = 4 established to include ten variables are as follows:

$$\hat{X}_{t+1} = \hat{a}_{1,1}w_{1,t} + \hat{a}_{1,2}w_{1,t-2} + \hat{a}_{2,1}w_{2,t} + \hat{a}_{2,2}w_{2,t-4} + \hat{a}_{3,1}w_{3,t} + \hat{a}_{3,1}w_{3,t-8} + \hat{a}_{4,1}w_{4,t} + \hat{a}_{4,1}w_{4,t-16} + \hat{a}_{5,1}v_{4,t} + \hat{a}_{5,2}v_{4,t-16}$$

$$\hat{X}_{n+1} = 1.81846 X1 - 0.34954 X2 + 0.56626 X3 - 0.30860 X4 + 1.59386 X5 - 0.06183 X6 + 0.86083 X7 - 0.00780 X8 + 0.96278 X9 + 0.02724 X10$$

Regression model after testing can be concluded model suitable used and There are five variables that are not significant, because it will be done analysis to include the variables that significantly X2, X4, X6, X8 and X10 .

Table 2. Summary of significant variables sidwt D4 model

Predictor Variable	Coefficients	Pr(> t )	Result
X2	2.64119	0.02059	Significant
X4	2.74219	1.18e-07	Significant
X6	2.46093	0.00294	Significant
X8	0.88701	0.01202	Significant
X10	0.92311	< 2e-16	Significant

Model which is composed of five variables are:

$$\hat{X}_{n+1} = 2.64119 X2 + 2.74219 X4 + 2.46093 X6 + 0.88701 X8 + 0.92311 X10$$

In the formula MAR can be written as follows

$$\hat{X}_{n+1} = 2.64119 w_{1,n} + 2.74219 w_{2,n} + 2.46093 w_{2,n-4} + 0.88701 w_{3,n} + 0.92311 w_{4,n-16}$$

Regression model after testing can be concluded model suitable used and all parameters significant. Testing assumption normality error using tests Shapiro-Wilk can be concluded that the assumed normality error did not follow normal distribution. *Variance Inflation Factors* (VIF) for all the variables less than 10 so that it can be concluded that there was no multicollinearity testing assumption heteroscedasticity using Breusch-Pagan tests be concluded that the assumed homoscedasticity are met. Testing assumption independency error using Durbin Watson tests can be concluded that error independent.

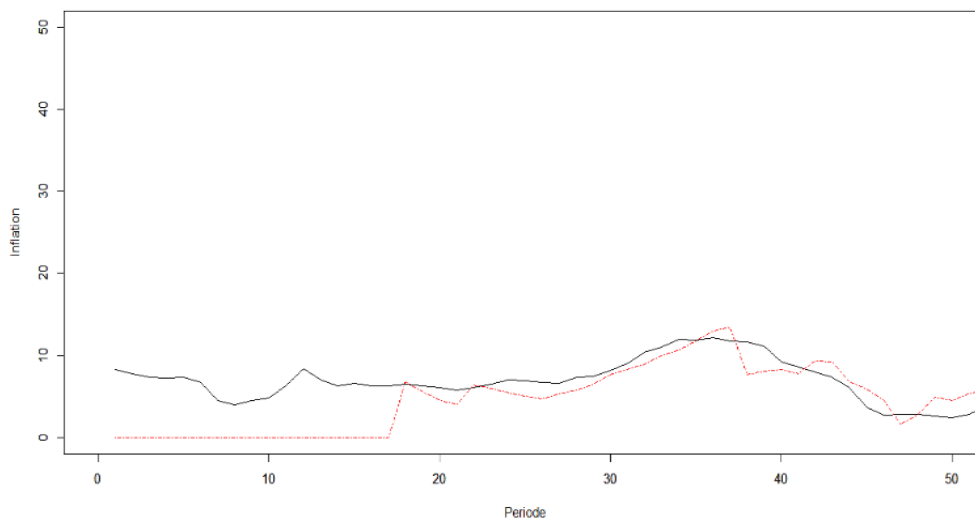


Figure 2. Simulation SIDWT with Inflation Time Series

Based on Figure.2 simulation SIDWT seen that the training network has his prediction that is quite accurate information that is shown by the close target line with the output. R<sup>2</sup> value is 0.9362 or 93.62%.

### 5. Conclusion

Inflation can be analyzed using methods that Shift Invariant Discrete Wavelet Transform (SIDWT) which had been modeled for them to use Multiscale Autoregressive (MAR) with the R<sup>2</sup> value 93.62%. With knew the prediction inflation, it is expected to provide a solution to the government to overcome lag response government policy that during this to happen. That can be arranged best combination of several policies that are able to respond to inflationary pressures that will come.

## Acknowledgement

This research is supported by Directorate of Research and Community Services, The Ministry of Research, Technology, and Higher Education Republic of Indonesia

## References

- Bruce, A., & H.-Y. Gao. (1996). *Applied Wavelet Analysis with S-PLUS*, Springer: New York.
- Caraka, R. E, Yasin, H. & Suparti. (2015). Pemodelan Tinggi Pasang Air Laut di Kota Semarang Menggunakan Maximal Overlap Discrete Wavelet Transform (MODWT). *Climatological and Geophysics (BMKG)*, 2(2), 104-114.
- Gencay, R., Selcuk, F., & Whitcher, B. (2001). *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics*, Academic Press.
- Kingsbury, N. G. (2000). A dual-tree complex wavelet transform with improved orthogonality and symmetry properties, *Proceedings of the IEEE Int. Conf. on Image Proc. (ICIP)*. <http://dx.doi.org/10.1109/icip.2000.899397>
- Lindsay, R. W., Percival, D. B., & Rothrock, D. A. (1996). The discrete wavelet transform and the scale analysis of the surface properties of sea ice, *IEEE Transactions on Geoscience and Remote Sensing*, 34(3), 771-787. <http://dx.doi.org/10.1109/36.499782>
- Mallat, S. G. (1989). A theory for multiresolution signal decomposition: the wavelet representation, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 11(7), 674-693. <http://dx.doi.org/10.1109/34.192463>
- Percival, D. B., & Walden, A. T. (2000). *Wavelet Methods for Time Series Analysis*, Cambridge. <http://dx.doi.org/10.1017/CBO9780511841040>
- Prahutama, A., Utama, T. W., Caraka, R. E., & Zumrohtuliyosi, D. (2014). Pemodelan Inflasi Berdasarkan Harga-Harga Pangan Menggunakan Spline Multivariable. *Jurnal Media Statistika*, 7(2), 89-94.
- Renauld, O., Starck, J. L., & Murtagh, F. (2003). Prediction based on a multiscale Decomposition. *Int. J. Wavelets Multiresolut. Inform. Process.*, 217-232. <http://dx.doi.org/10.1142/S0219691303000153>
- Renauld, O., Starck, J. L., & Murtagh, F. (2000). *Wavelet-based Forecasting of Short and Long Memory Time Series*. Universite de Geneve. Geneve.
- Suhartono, B.S.S. Ulama & Endhart, A.J. (2010). Seasonal Time Series Data Forecasting by Using Neural Network Multiscale Autoregressive Model. *American Journal of Applied Sciences*, 7(10), 1373-1378.
- Suparti, Subanar, H. (2000). Estimasi Regresi dengan Metode Wavelet Shrinkage. *Jurnal Sains dan Matematika*. 105-113.
- Warsito, B., Subanar., & Aburakhman. (2013). Pemodelan Time Series dengan Maximal Overlap Discrete Wavelet Transform. *Proceedings of Seminar Nasional Statistika*, ISBN:9788-602-14387-0-1

## Copyrights

Copyright for this article is retained by the author(s), with first publication rights granted to the journal.

This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (<http://creativecommons.org/licenses/by/3.0/>).