The Effect of Initial Imperfection on Buckling Strength for Straight and Curved Pipe Under Pure Bending Load

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ABSTRACT

The effect of initial imperfection on the buckling strength of a straight pipe under bending has been investigated by nonlinear *FEA*, considering the effect of a cross sectional oval deformation by changing the variables of pipes, that is, *L/D* varying from about 2.5 to 20 and *D/t* varying from about 50 to 200. Furthermore also the buckling strength of curved pipes (*D/t* = 200) have been investigated by changing *R/D* from 50 to 200. Not only the elastic buckling but also the elasto-plastic buckling was investigated. From the numerical results, the followings were found. The buckling strength reduced more in longer pipes than shorter pipes due to pre-buckling oval deformation. But, the reduction of buckling strength due to imperfection is larger in shorter pipes than in longer pipes. Moreover, the buckling strength reduces more by the imperfection of buckling mode than by that of oval mode. The effect of imperfection on buckling strength in plastic region was smaller than in elastic region. The buckling moment of curved pipes reduces more when the curvature of pipe increases in both original and imperfect pipes, especially for longer pipes.

KEY WORDS: buckling strength; initial imperfection; straight pipe; curved pipe; oval deformation; elastic buckling; elasto-plastic buckling.

NOMENCLATURE

D : *Pipe diameter*
E : *Young's modulus*
M : *Applied Moment*
M_a : *Maximum moment in elasto-plastic analysis*
M_b : *Buckling moment obtained by nonlinear calculation in elastic analysis.*
M_{cr} : *Critical bending moment under axial compression*
M_o : *Maximum moment in elastic analysis pipe without imperfection*
M_p : *Ultimate (plastic) moment*
M_y : *Yield moment*
r : *Pipe radius*
R : *Radius of curvature in curved pipe*
t : *Nominal wall thickness*
δ : *Oval deformation*
δ_o : *Maximum imperfection amplitude*
ν : *Poisson's ratio*
σ_{cr} : *Critical buckling stress under axial compression*
σ_y : *Yield stress*

INTRODUCTION

To predict the buckling strength of straight and/or curved pipes is important for designing the offshore pipelines. The various kinds of loads, such as axial compression, external pressure and bending are applied to the offshore pipelines. The buckling strength of a pipe is much reduced due to the initial imperfections. The effects of initial imperfections on the buckling strength of pipes under axial compression and pressure are investigated by many researchers, but the research for buckling strength of pipes under bending is still limited. Therefore, in this study, the comprehensive study for the buckling strength of pipes under bending is performed by utilizing *FEA*.

First, the previous researches for estimating the buckling strength of pipe under pure bending moment are briefly explained.

The maximum bending stress of cylinder under the critical (buckling) moment M_{cr} can be expressed by equation (1).

$$\sigma_{cr} = \frac{M_{cr}}{\pi r^2 t} \tag{1}$$

Where, r is the pipe radius, and t is the wall thickness.

If the critical buckling stress of a cylinder under bending is same as the buckling stress of a cylinder under uniform compression, the critical stress can be expressed by Eq. 2.

$$\sigma_{cr} = \frac{E}{\sqrt{3(1-\nu^2)}}\left(\frac{t}{r}\right) \tag{2}$$

From Eqs. 1~2, the critical moment can be estimated as follow.

$$M_{cr1} = 0.605\pi Ert^2 \quad (3)$$

Timoshenko and Gere (1961) gave the statement that the maximum compressive stress at critical buckling moment is about 30 % higher than the stress obtained from Eq. 2. Then, the buckling moment is given by Eq. 4.

$$\begin{aligned} M_{cr3} &= 1.3 \times M_{cr1} \\ &= 0.787\pi Ert^2 \end{aligned} \quad (4)$$

The calculated results of previous investigation by authors (Yudo and Yoshikawa, 2012) for straight pipes, in which *L/D* varies from 5 to 20, and *D/t* varies from 50 to 200, showed that the critical bending moment in linear calculation is expressed by Eq. 5 .

$$\begin{aligned} M_{cr3} &= 1.1 \times M_{cr1} \\ &= 0.666\pi Ert^2 \end{aligned} \quad (5)$$

Brazier (1927) investigated the stability of long cylindrical shells under bending. If a long cylinder is subjected to bending, its cross section flatters. Consequently, its bending stiffness deteriorates with increasing bending moment as a function of applied curvature, and exhibits a maximum. Brazier performed a somewhat approximate analysis and found that the buckling moment is given by Eq. 6 with $\nu = 0.3$

$$M_{\max} = (2\sqrt{2}/9)\pi Ert^2 / \sqrt{1-\nu^2} = 0.329\pi Ert^2 \quad (6)$$

The maximum stress caused by this moment is computed as follow.

$$\sigma_{\max} = 0.33Et/r \quad (7)$$

Chwalla (1933) derived the equation (8) as the maximum moment considering the sectional oval deformation by energy method. Moreover, he also showed that the maximum stress occurs at another bending curvature and it can be expressed by Eq. 9.

$$M_{\max} = M_k = 0.378\pi Ert^2 \quad (8)$$

$$\sigma_{\max} = 0.51Et/r \quad (9)$$

Seide and Weingarten (1961) solved as a bifurcation buckling problem. Assuming that the pre-buckling behavior can be defined with sufficient accuracy by a linear membrane solution, they found that the critical buckling stress is only 1.5% higher than Eq. 2, for a cylinder with *r/t* = 100.

Odland (1978) calculated that the collapse moment of cylinder with *R/t*=100 and showed that the collapse moment of cylinder with *L/D*=5, 10, and infinity is ab.70, 60, and 50% of M_{cr1}, respectively, due to the oval deformation.

Ju and Kyriakides (1992) also mentioned that the oval deformation reduces the bending rigidity and leads to a limit load instability associated with local collapse.

In the previous research by authors (Yudo and Yoshikawa, 2014) for straight pipes under pure bending moment, the nonlinear *FEA* considering the effect of the cross sectional oval deformation was performed by changing the scantling of pipes. In this study, the length-to-diameter ratio *L/D* varies from 5 to 20, and *D/t* varies 50 to 200. From the numerical results, the maximum moment for long pipe considering oval deformation is shown as follow:

$$M_{\max} = 0.52M_{cr1} = 0.314\pi Ert^2 \quad (10)$$

The initial yield moment of pipe under bending is shown below.

$$M_y = \sigma_y \pi \cdot r^2 \cdot t \quad (11)$$

Where, σ_y is the yield stress.

SUPERB (1996) declared the geometrical imperfections (excluding corrosion) that are normally allowed in pipeline design will not significantly influence on the moment capacity under bending, and the buckling moment in plastic region can be calculated as:

$$M_p = \left(1.05 - 0.003 \cdot \frac{r}{t}\right) \cdot \sigma_y \cdot D^2 \cdot t \quad (12)$$

If r/t equals to 50, Eq. 11 becomes

$$M_p = 1.14 \cdot \sigma_y \pi \cdot r^2 \cdot t \quad (13)$$

The moment capacity in Eq. 12 gives only 15% higher than the initial yielding moment, and this seems to be considerably conservative value.

Gellin (1980) has been investigated the effect of nonlinear material behavior on the buckling of an infinitely long cylindrical shell under pure bending. The maximum moment and associated curvature is determined as a function of material and geometric parameters. The curvature at which short wave-length bifurcations occur is also determined

Zhang and Li (1999) had already develop a set of new governing equations to solve the nonlinear bending and instability of imperfect cylindrical tubes, furthermore formulating the effects of initial out-of-roundness on the bending and stability.

Elchalakani - Zhao and Grzebieta (2002) present a theoretical treatment to predict the moment-rotation response of circular hollow steel tubes of varying D/t ratios under pure bending. In extensional deformation and rigid plastic material behaviour were assumed in the derivation of the deformation energy. The plasticity observed in the tests was assumed to spread linearly along the length of the tube. Two local plastic mechanisms (Star and Diamond shapes) were studied to model the behaviour observed in the tests especially during the unloading stage

For the previous studies, the buckling strength of straight and curved pipe under bending has been investigated. But, the comprehensive study the effect of initial imperfection on the buckling strength of pipe under bending has not been found. In this study, the series calculations of buckling and collapse strength of straight and curved pipe under bending are performed by utilizing nonlinear *FE* software.

PROCEDURE OF CALCULATION

Parameters for Calculation

In calculation of straight pipe, the ratio between pipe length and diameter *(L/D)* and the ratio between diameter and thickness *(D/t)* are taken as the calculation parameters. *L/D* varies from 2.5 to 20, in which the diameter is changing from 1000 to 4000 mm. Furthermore *D/t* varies from 50 to 200. Where, *D* is the pipe diameter, *t* is the wall thickness, and *L* is the pipe length.

Table 1. Calculation Parameters for Straight Pipe

CASES (Nmm)	D (mm)	D/t	L/D
I M = 8.0 x 10^{10}	1000	50	2.5 to 20
II M = 1.6 x 10^{11}	2000	100	2.5 to 20
III M = 3.2 x 10^{11}	4000	200	2.5 to 20

In calculation of a curved pipe, the pipe diameter is originally fixed to 4000 mm, and the thickness is fixed to 20 mm. *L/D* varies from 2.5 to 20 by changing the pipe length. *(R/D)* varies from 50 to 200 as shown in table 2. Where, *R* is radius of curvature in curved pipe.
Fig. 1 shows the caption of straight and curved pipe, respectively. δ shows the oval deformation at mid span section.

Table 2. Calculation Parameters for Curved Pipe

M (Nmm)	D (mm)	t (mm)	L/D	R/D
3.2 x 10^{11}	4000	20	2.5 to 20	Straight Pipe (∞)
				Curved Pipe (50)
				Curved Pipe (100)
				Curved Pipe (200)

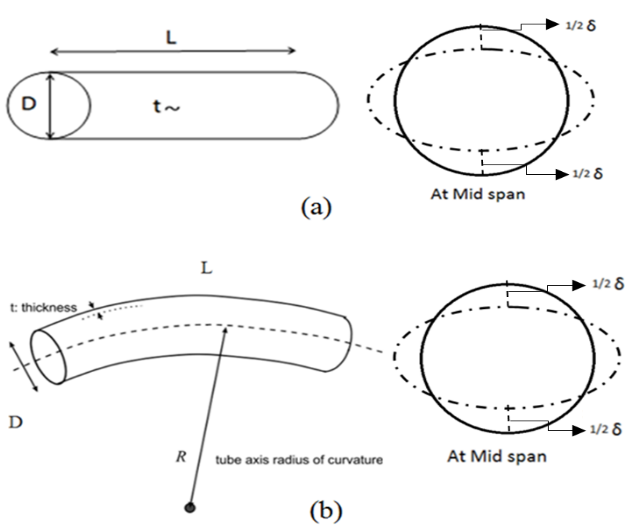


Fig. 1 Pipe geometry (a) straight pipe and (b) curved pipe.

Model for Calculation and Calculation Program

The full length models of straight and curved pipes with initial imperfection are used in *FEA*, see Fig. 2. The nonlinear buckling calculations of pipe under pure bending are performed. In this model, the initial imperfection of buckling mode, which can be obtained by linear buckling calculation for perfect pipe, is introduced.

General purpose *FE* software Msc Marc is used for the nonlinear buckling analysis in which the oval deformation of cross section before buckling is taken account. The quadrilateral 4 nodes element (No.75) is used. The calculating region is divided into 36 elements in circumferential direction. The element number in length is basically 120 elements and more elements are used to maintain the calculation accuracy in the case of long cylinder. The mesh convergence study had done in ref. (Yudo and Yoshikawa, 2012).

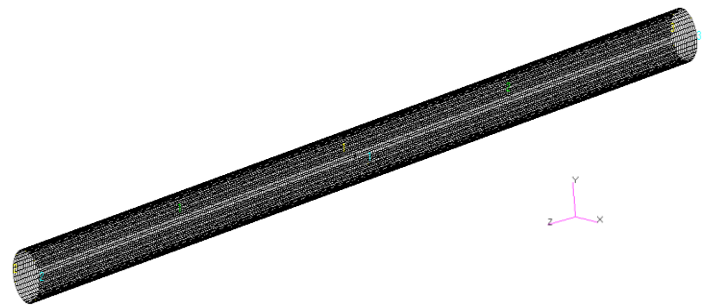


Fig. 2 Finite element modeling for full Pipe

Boundary Condition and Loading Condition

Cylindrical coordinates are used. The following boundary conditions are given at the mid span of cylinder, except at four points shown in Fig. 3.

$$\begin{array}{ll} U_R \neq 0 & R_\Theta \neq 0 \\ U_\Theta \neq 0 & \Theta_\Theta \neq 0 \\ U_Z = 0 & Z_\Theta \neq 0 \end{array} \qquad (14)$$

The rigid body elements *(RBE)* are inserted at both end section in order to connect the center of circle and the points on circle shown in Fig. 3. The bending moment is loaded at the centre of circle at both ends. The *RBE* prevent the oval deformation of both end section, and keep the section in plane under rotational deformation by bending moment.

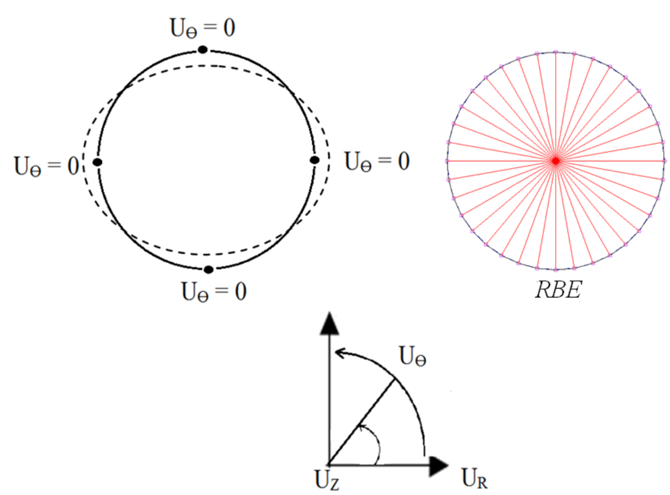


Fig. 3 Mid span section & *RBE* at both end section

Method to Obtain Bifurcation Moment in Nonlinear Calculation

To obtain the bifurcation buckling moment and its mode, the eigen-values are calculated at each load increment considering the pre-buckling deformation. The buckling load is judged as follows. When the value of eigen-value is larger than the applied load, the buckling does not happen. Furthermore when the value of eigen-value is smaller than the applied load, the buckling has already happened.

The cross section point at which the applied moment coincides with the eigen-value (see Fig. 4) is treated as the buckling load.

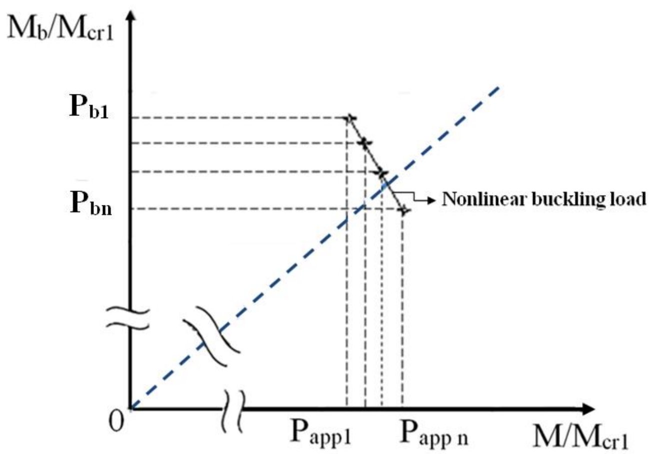


Fig. 4 Schematic diagram for estimation of bifurcation moment

NUMERICAL RESULTS FOR STRAIGHT PIPE

Fig. 5 shows the relationship between the applied bending moment and the oval deformation at mid span when the pipe behaves elastically. The vertical axis shows the non-dimensional moment *(M/Mcr1)*. The red lines are numerical results for the pipes without imperfection and the black lines are numerical results for the pipes with imperfection of buckling mode, δ_o/t = 0.05. The buckling strength for the imperfect pipe is lower than the original pipe.

In longer pipes, such as *L/D* = 10, 15, and 20, the oval deformation increases much with increasing the applied load. On the other hand, in shorter pipes, such as *L/D* =2.5 and 5.0, the oval deformation is small by the restriction of sectional deformation at both ends. The maximum moment of each curve shows the bifurcation buckling moment explained above.

Fig. 6 shows the deformation just before buckling at mid span for the original pipe of *D/t* = 200. Where, Fig.(a) and (b) are for the pipe of *L/D* = 2.5 and *L/D* = 20, respectively. The oval deformation at pre-buckling stage is more remarkable in a long pipe (*L/D* = 20) than in a short pipe (*L/D* = 2.5). Due to the oval deformation before buckling at mid span, the maximum moments of long pipes reduced more compared to the short pipes.

Fig. 7 shows the deformation just before buckling at mid span for the pipe of *D/t* = 200 with the imperfection of buckling mode δ_o/t = 0.05. Where, Fig.(a) and (b) are for the pipe of *L/D* = 2.5 and *L/D* = 20, respectively.

In a long pipe, the oval deformation is dominant. On the other hand, the deformation of buckling mode grows up in a short pipe. Therefore, the reduction of buckling strength due to imperfection is larger in shorter pipes than in longer pipes.

Fig. 8 shows the relationship between the buckling moment and *L/D*. The red lines are the numerical results for pipe without imperfection and the black lines are the numerical results for pipe with imperfection of buckling mode δ_o/t = 0.05. The dotted lines are the elastic numerical results and the solid lines are the elasto-plastic numerical results. In elasto-plastic analysis, the yield stress of material is set to 621MPa.

In elastic analysis, the buckling moment decreases with increasing the length of pipe and becomes constant at the longer pipes. As mentioned above, the buckling moments reduces more in shorter pipes than longer pipes due to imperfection. In the longer pipes, the buckling moment is only 3 % lower than the original pipe due to the imperfection of δ_o/t = 0.05, where the maximum moment for the original straight pipe as shown by Eq. 10.

In elasto-plastic analysis, the effect of imperfection for the straight pipes is smaller than in elastic analysis. Moreover, the buckling moments of all length of pipes with same *D/t* are almost same, because the maximum buckling moments are limited by yielding.

Fig. 9 shows the comparison of shape deformation just before buckling at mid span for original straight pipe *L/D* = 2.5, *D/t* = 200) between (a) elastic analysis and (b) elastic-plastic analysis. The figure showed that there weren't significant difference shapes for both analyses.

Fig. 10 shows the relationship between the maximum buckling moment *(Mb/Mo)* and the imperfection *(δ_o/t)* for the straight pipes when the pipes behave elastically. It is shown that the buckling moment decreases with increasing the imperfection.

The effect of imperfection for shorter pipes such as *L/D* = 2.5, and 5.0 is large. On the other hand the imperfection affects small on the buckling moment for longer pipes. The buckling moment becomes constant when large imperfections.

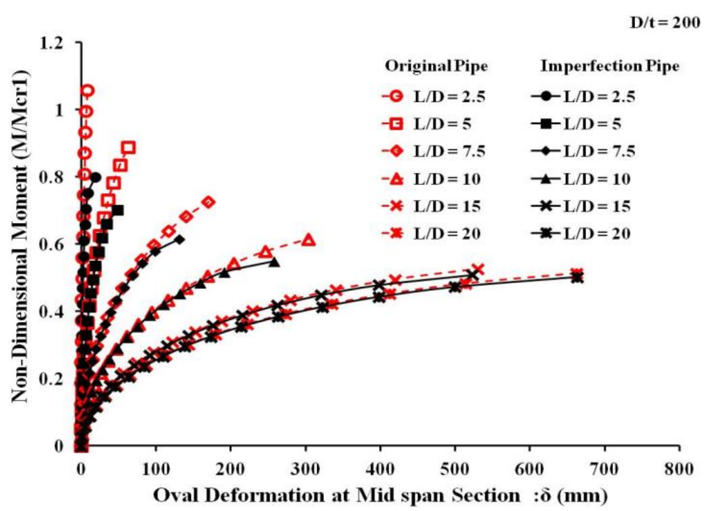
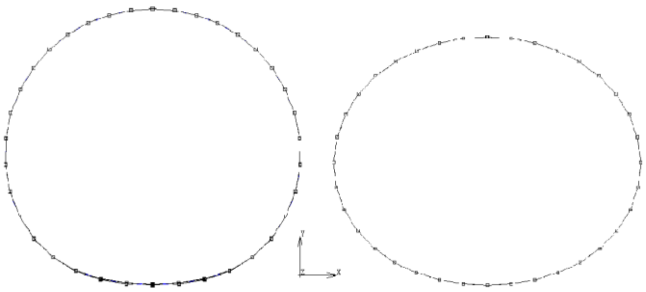
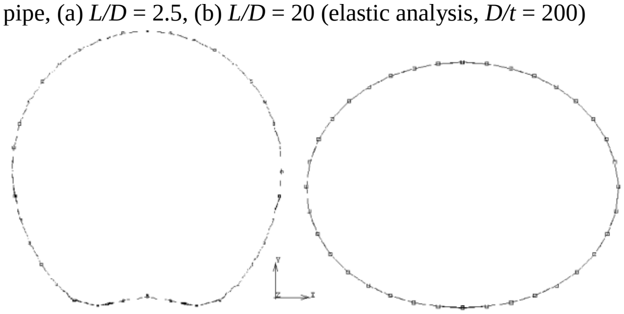


Fig. 5 Oval deformation at mid span for the original perfect pipe and the imperfect pipe of buckling mode, δ_o/t = 0.05 (elastic analysis, *D/t* = 200)



(a) (b)

Fig. 6 Deformation just before buckling at mid span for original straight pipe, (a) L/D = 2.5, (b) L/D = 20 (elastic analysis, D/t = 200)



(a) (b)

Fig. 7 Deformation just before buckling at mid span for straight pipe with imperfection of buckling mode, δ_o/t = 0.05 (a) L/D = 2.5, (b) L/D = 20 (elastic analysis, D/t = 200)

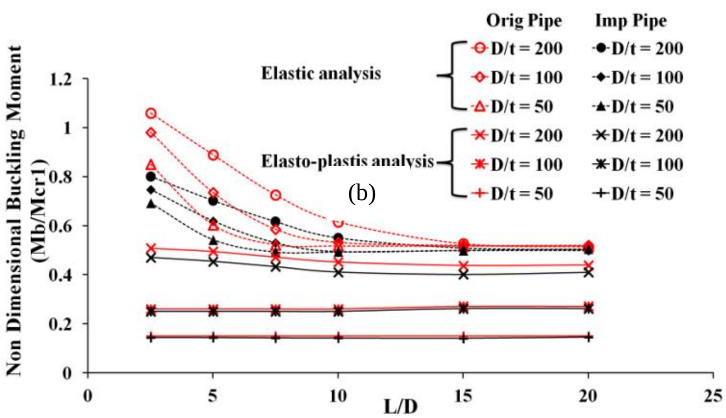
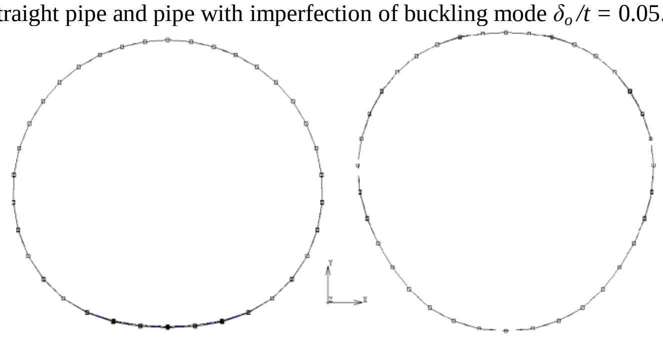


Fig. 8 Relationships between the buckling moment and L/D for original straight pipe and pipe with imperfection of buckling mode δ_o/t = 0.05.



(a) (b)

Fig. 9 Deformation just before buckling at mid span for original straight pipe L/D = 2.5, D/t = 200), (a) elastic analysis, (b) elasto-plastic analysis

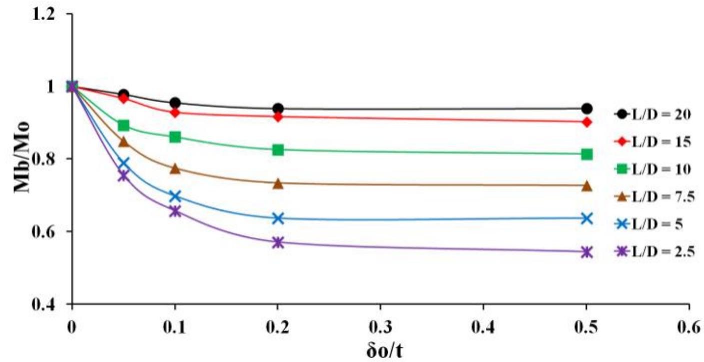


Fig. 10 Relationship between the maximum buckling moment and the imperfection of buckling mode for straight pipe (elastic analysis, D/t = 200)

In order to investigate the yielding effect on the buckling strength of pipe in bending, the non-dimensional parameter *(β)* shown below is considered.

$$\beta = (D/t)(\sigma_y / E) \propto \left(M_{cr1} / M_y\right) \tag{15}$$

This parameter is proportion to the ratio of the linear buckling moment and the initial yielding moment. The elasto-plastic buckling strength is examined by changing the yield stress and the diameter.

Figs. 11~12 show the relationship between the buckling moment and the imperfection *(δ_o/t)* for the pipes of L/D =2.5 and 20, respectively. The maximum moment is expressed non-dimensionally by dividing with the initial yielding moment *(My)*. The solid lines with triangle, diamond and rectangular marker are the numerical results for the pipes of D/t = 200, whose yield strength are σy = 157.5MPa, 315MPa, and 621MPa, where β equals to 0.15, 0.3 and 0.6, respectively.

The dotted lines with triangle, diamond and rectangular marker are the numerical results for the pipes of D/t = 50, whose yield stress is σy = 621MPa, 1260MPa, and 2520MPa, where β equals to 0.15, 0.3 and 0.6, respectively.

When β is large, the pipe buckles elastically. Furthermore when β is small, the pipe buckles plastically. The non-dimensional buckling moment decreases with increasing the value of β. Moreover, the non-dimensional buckling moments is almost the same for the same value of β. Therefore, the parameter of β is possible to be the parameter which represents the yielding effect on buckling strength of pipe under bending. The effect of imperfection on buckling in plastic range is smaller than in elastic range.

Fig. 13 shows the effect of imperfection on the buckling moment comparing the imperfection of the buckling mode and the oval mode. In this figure, the elastic numerical results are plotted with dotted lines, and the elasto-plastic numerical results are plotted with solid lines.

The line with rectangular marker is original pipe, the line with diamond marker is pipe with the imperfection of oval mode δ_o/t = 0.2 and the line with triangle marker is pipe with the imperfection of buckling mode δ_o/t = 0.2.

The effects of the imperfections of buckling modes are larger than the imperfection of oval mode. The imperfection of buckling mode reduces more the buckling strength than the imperfection of oval mode.

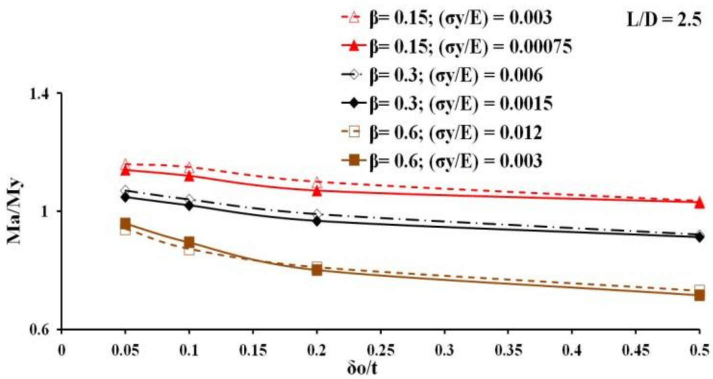


Fig. 11 Relationship between the maximum buckling moment and the imperfection of buckling mode for straight pipes (D/t = 200, L/D = 2.5) by elasto-plastic analysis

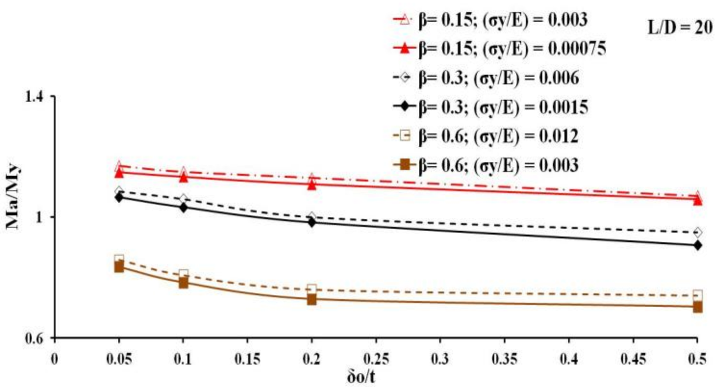


Fig. 12 Relationship between the maximum buckling moment and the imperfection of buckling mode for straight pipes (D/t = 200, L/D = 20) by elasto-plastic analysis

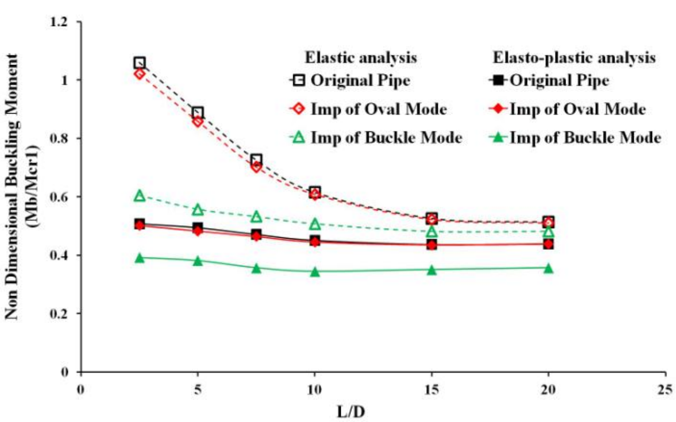


Fig. 13 The effect of imperfection on the buckling moment - comparison of the buckling mode and oval mode - for straight pipe (D/t = 200, L/D = 20) by elasto-plastic analysis

NUMERICAL RESULTS FOR CURVED PIPE

Fig. 14 shows the relationship between the applied bending moment and the oval deformation at mid span for the curved pipe when the pipe behaves elastically. The vertical axis shows the non-dimensional moment *(M/Mcr1)*. The red lines are numerical results for the pipes without imperfection and the black lines are numerical results for the pipes with the imperfection of buckling mode, δ_o/t = 0.05.

In longer pipes, such as L/D = 10, 15, and 20, the oval deformation increases much with increasing the applied load. On the other hand, in shorter pipes, such as L/D = 2.5 and 5.0, the oval deformation is small by the restriction of sectional deformation at both ends. The maximum moment of each curve show the bifurcation buckling moment explained above.

Fig. 15 shows the deformation just before buckling at mid span for the original pipe of D/t = 200 and R/D = 200. Where, Fig.(a) and (b) are for the pipe of L/D = 2.5 and L/D = 20, respectively. Same as the straight pipes, the oval deformation in a long pipe (L/D = 20) is remarkable than in a short pipe (L/D = 2.5). Due to the oval deformation before buckling at mid span, the maximum moments of the long pipes reduced more compared to the short pipes.

Fig. 16 shows the deformation just before buckling at mid span for the pipe of D/t = 200 and R/D = 200 with imperfection of buckling mode δ_o/t = 0.05. Where, Fig.(a) and (b) are for the pipe of L/D = 2.5 and L/D = 20, respectively. Same as the straight pipes, the oval deformation is dominant in a long pipe. On the other hand, the deformation of buckling mode grows up in a short pipe. Therefore, the reduction of buckling strength due to imperfection is larger in shorter pipes than in longer pipes.

Fig. 17 shows the relationship between the buckling moment and L/D for the straight pipe and the curved pipes of D/t = 200. The line with circle marker is calculation result for a straight pipe. The lines with diamond marker, rectangular marker, and triangle marker are the numerical results for the curved pipes of R/D = 200, 100 and 50, respectively. The red lines are the numerical results for original pipes and the solid lines are the numerical results for the imperfect pipes with δ_o/t = 0.05 when the pipes behave elastically.

The buckling moment of curved pipes reduces more when the curvature of pipe increases in both original and imperfect pipes, especially for longer pipes.

The buckling moments for the imperfect pipes are smaller than the original pipes. However, the reduction rate of buckling moment due to imperfection decreases with increasing L/D.

For a short pipe such as L/D = 2.5 the effect of imperfection are larger than a long pipe.

Fig. 18 shows the relationship between the buckling moment and L/D for a straight pipe and curved pipes with imperfection, δ_o/t = 0.05. In this figure, the elastic numerical results are plotted with dotted lines, and the elasto-plastic numerical results are plotted with solid lines. In elasto-plastic analysis, the yield strength of material is set to 621MPa

In elastic analyses, the buckling moment decreases with increasing value L/D and becomes constant in longer pipes. The buckling moments of pipes with small value of L/D are almost the same for each value of R/D. It is supposed that the effect of curvature of curved pipe on buckling strength is reduced by the constraint effect at support end. Similarly, the buckling moments of small L/D in elasto-plastic analysis are almost the same in regardless with value of R/D

The buckling moments for elasto-plastic analysis are smaller than elastic analysis, and the effects of imperfection on buckling moment in elasto-plastic analysis are smaller than in elastic analysis.

For a short curved pipe such as L/D = 2.5 the difference of buckling moment in elastic analysis and elasto-plastic analysis are so large because the short pipe buckles more plastically than the long pipe.

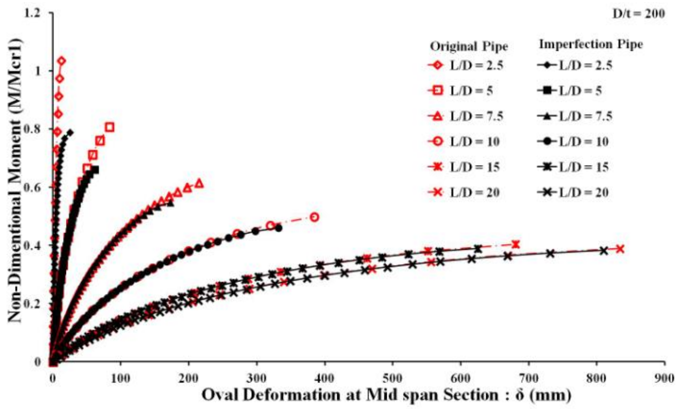
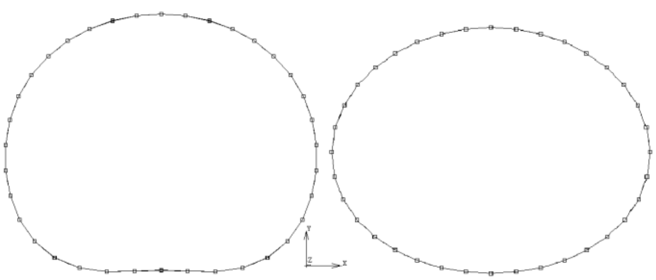
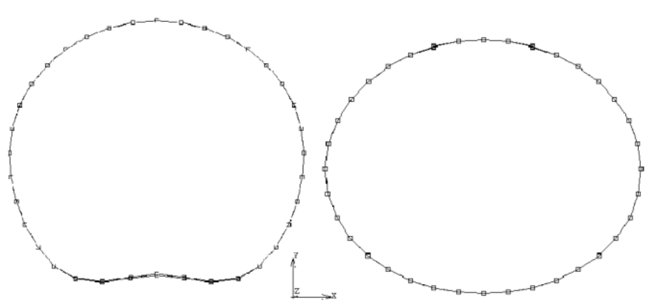


Fig. 14 Oval deformation at mid span for original curved pipe and pipe with imperfection of buckling mode, δ_o/t = 0.05. (elastic analysis, D/t = 200, R/D = 200)



(a) (b)

Fig. 15 Oval deformation at mid span for original pipe, (a) L/D = 2.5, (b) L/D = 20 (elastic analysis, curved pipe, D/t = 200, R/D = 50)



(a) (b)

Fig. 16 Oval deformation at mid span for pipe with imperfection of buckling mode δ_o/t = 0.05 (a) L/D = 2.5, (b) L/D = 20 (elastic analysis, curved pipe D/t = 200, R/D = 50)

Fig. 19 shows the relationship between the maximum buckling moment *(Mb/Mo)* and the imperfection (δ_o/t) for the curved pipes when pipes behave elastically. The buckling moment decreases with increasing imperfection. The effect of imperfection on buckling moment for a short pipe such as L/D = 2.5 is large, on the other hand the effect is small for a long pipe. The buckling moment becomes constant when the large imperfection exists.

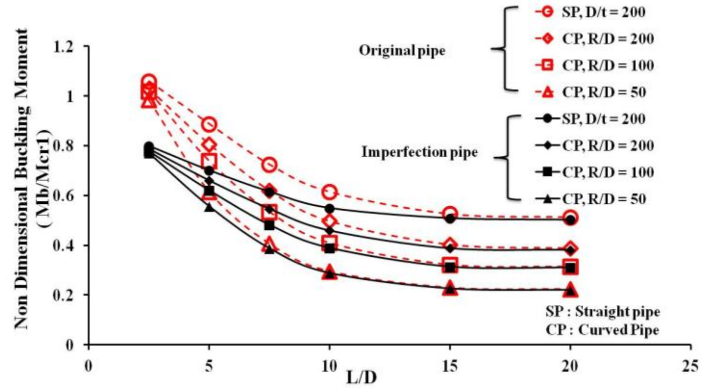


Fig. 17 Relationship between buckling moment and L/D for original curved pipe and pipe with imperfection of buckling mode δ_o/t = 0.05 (elastic analysis, D/t = 200)

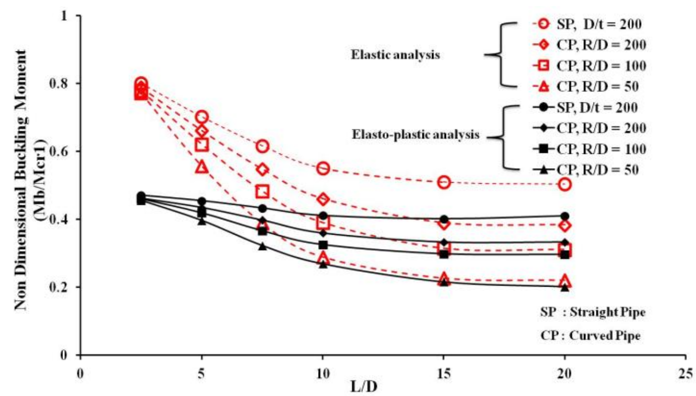


Fig. 18 Relationship between buckling moment and L/D for curved pipe with imperfection of buckling mode δ_o/t = 0.05 (D/t = 200)

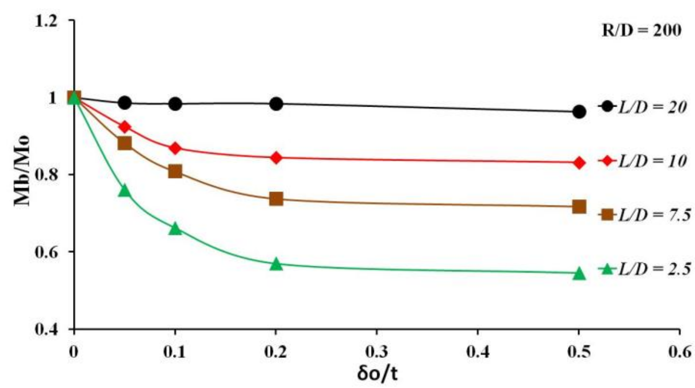


Fig. 19 Relationship between maximum buckling moment and imperfection of buckling mode for curved pipe (elastic analysis, D/t = 200, R/D = 200)

The non-dimensional parameter $(\beta = (D/t)(\sigma y/E))$ shown by Eq. 14 is also considered for the curved pipe in order to investigate the yielding effect on the buckling strength in bending. Figs. 20~21 show the relationship between the buckling moment and imperfection (δ_o/t). The vertical axis shows the non-dimensional maximum moment divided by the initial yielding moment *(My)*.

The lines with diamond, rectangular and triangle marker are the numerical results for the pipe of D/t = 200, whose yield stress is σy = 157.5MPa, 315MPa, and 621MPa, where β equals to 0.15, 0.3 and 0.6

respectively. The non-dimensional buckling moment decreases with increasing the value of β.

Similarly for curved pipe, the parameter of β is possible to be the parameter which represents the yielding effect on the buckling strength of pipe under bending.

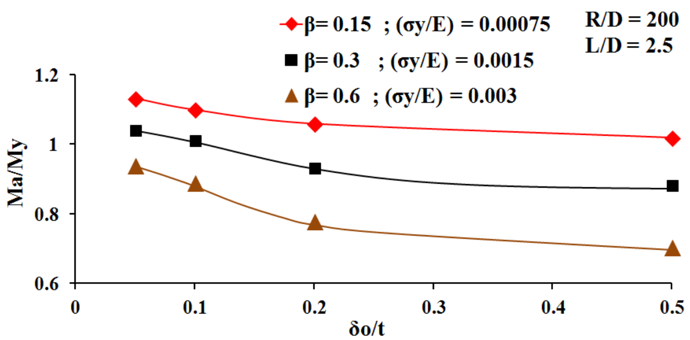


Fig. 20 Relationship between maximum buckling moment and imperfection for straight pipe (*D/t* = 200, *L/D* = 2.5) by elasto-plastic analysis

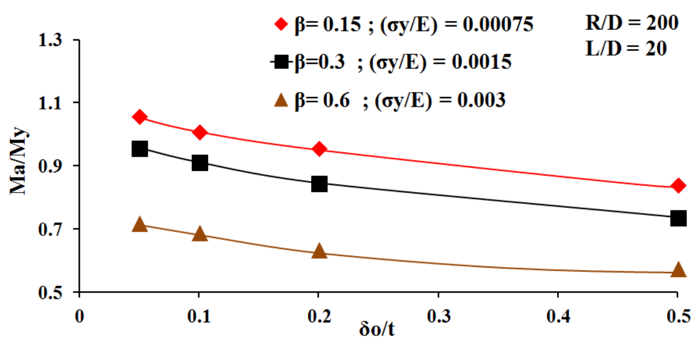


Fig. 21 Relationship between maximum buckling moment and imperfection for straight pipe (*D/t* = 200, *L/D* = 20) by elasto-plastic analysis

CONCLUSIONS

In this investigation, the calculations of buckling and collapse strength of straight and curved pipes with initial imperfection under bending are performed by utilizing nonlinear *FE* software. The followings are clarified by the numerical calculation.

1) The buckling strength reduced more in longer pipes than in shorter pipes due to pre-buckling oval deformation.
2) The reduction of buckling strength due to imperfection is larger in shorter pipes than in longer pipes.
3) The effect of the imperfection of buckling modes on buckling moment is larger than that of oval mode.
4) The parameter of $\beta = (D/t)(\sigma y/E)$ is possible to be the parameter which represents the yielding effect on buckling strength of pipe under bending. When β is large, the pipe buckles elastically. Furthermore when β is small, the pipe buckles plastically.
5) The effect of imperfection on buckling moment in plastic range is smaller than in elastic range.
6) The buckling moment of curved pipes reduces more when the curvature of pipe increases in both original and imperfect pipes, especially for longer pipes.

REFERENCES

Brazier, LG (1927). "On The Flexure of Thin Cylindrical Shells and Other Thin Sections," *Proc. Royal. Society*. Series. A 116.104–114

Chwalla, VE (1933). "Reine Biegung Schlanker, Dünnwandiger Rohre Mit Gerader Achse," *Zeitschrift für Angewandte Mathematik und Mechanik* , Band 13, Heft 1.

Elchalakani, M, Zhao, XL, and Grzebieta, RH (2002). "Plastic mechanism analysis of circular tubes under pure bending," *International Journal of Mechanical Sciences*, Volume 44, Issue 6, June 2002, pp 1117-1143

Gellin, S (1980). "The plastic buckling of long cylindrical shells under pure bending," *Int. J. Solids and Structures*, 16 (1980), pp. 397–407

Ju, GT, and Kyriakides, S (1992). "Bifurcation and Localization Instabilities in Cylindrical Shells under Bending," part II predictions, *Int. J. Solids Struct*, Vol. 29, pp1143–117

Odland, J (1978). *Buckling Resistance of Unstiffened and Stiffened Circular Cylindrical Shell Structures*, Norweigian Maritime Research, Vol. 6 No.3.

Seide, P, and Weingarten, VI (1961). "On The Buckling of Circular Cylindrical Shells under Pure Bending," *Transaction. ASME, J. Appl. Mech*. Vol. 28, pp112–116.

SUPERB (1996). *Buckling and Collapse Limit State*, December 1996.

Timoshenko, S, and Gere, J (1961). *Theory of Elastic Stability*, 2nd Edition, McGraw-Hill International Book Company.

Yudo, H, and Yoshikawa, T (2012). "Mechanical Behaviour of Pipe under Pure Bending Load," *Proceedings of the 26th Asian Technical Exchange and Advisory Meeting on Marine Structures*, JAPAN, TEAM, pp 359 -364

Yudo, H, and Yoshikawa, T (2014). "Buckling Phenomenon for Straight and Curved Pipe under Pure Bending," *J Mar Sci Technol, DOI 10.1007/s00773-014-0254-5.*

Zhang, WG, and Li, DX (1999). " On nonlinear bending and instability of imperfect cylindrical tubes ", *International Journal of Pressure Vessels and Piping*, Volume 76, Issue 1, pp 43-47.