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Article

Stick-Slip Behaviour of a Viscoelastic Flat Sliding along a Rigid Indenter

Budi Setiyana[1,2)*], Rifky Ismail[2)], J. Jamari[2)] and Dirk Jan Schipper[1)]

[1)]Laboratory for Surface Technology and Tribology,
Faculty of Engineering Technology, University of Twente
Drienerlolaan 5, Postbus 217, 7500 AE, Enschede, The Netherlands
[2)]Laboratory for Engineering Design and Tribology,
Department of Mechanical Engineering, Diponegoro University
Jl. Prof. Soedharto SH, Tembalang, Semarang 50275, Indonesia
*Corresponding author: bsetiyana@yahoo.com

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The sliding contact of soft material surface due to a rigid indenter is different from metal and some other polymers. A stick-slip motion is more frequently obtained than a smooth motion. By modeling the soft material as low damping viscoelastic material, this study proposes an analytical model to identify the stick-slip behavior of sliding system. The sliding system is a fixed rigid indenter that slides against on a moving soft material surface. A stick-slip model is developed and the motion of the sliding system is assumed to be in a solely tangential direction. By implementing the simple coulomb friction law, an exact solution is presented in the case of no damping of the sliding system. Results show that the periodic displacement of the stick-slip model is strongly depending on the friction force, sliding velocity and material stiffness. Furthermore, the effect of a viscous damping and velocity-dependent friction on the behaviour of the sliding system are discussed.

Keywords: indentation, periodic displacement, sliding contact, stick-slip

1. Introduction

The sliding contact between a soft indenter and rigid floor was experimentally investigated to find stick-slip phenomena and the induced force of indenter [1]. Experimentally, Coveney et al. [2] performed the sliding contact between rubber surface and a rigid indenter in two modes, i. e. fixed load of indenter and fixed depth of indentation. Experimental investigations showed that along with the sliding contact or abrasion processes, the stick-slip phenomenon and periodic force occurred [2-4]. Consequently, a periodic wear pattern of abraded rubber surface was formed. Moreover, Fukahori et al. [4] stated that the formed wear pattern spacing mainly depended on the frequency of stick-slip oscillation.

The stick-slip phenomenon is often associated with the friction between two contacting bodies. Some analytical models to describe the stick-slip phenomena were proposed. Numerically, Nakano et al. [5-8] studied the stick-slip contact of a sliding system between a soft indenter and a rigid moving floor for preventing stick to occur. The effect of velocity-dependent friction of the sliding system was analyzed that may change the stick-slip behavior, i.e. from stick-slip motion to full slip (steady sliding) motion or vice versa [7-10]. Experimentally, the stick-slip behavior associated with Schallamach wave and bulk deformation of the sliding system was investigated by applying a cylindrical indenter against a soft rubber surface in various driving speed and normal load [11].

The stick-slip contact on rubber sliding is often associated with the compliant behaviour of the rubber. It has been noted that the rubber has compliant behaviour in tangential as well as normal direction. Experimentally, Coveney et al. [2] showed that the stick-slip oscillation of the rubber surface by fixed depth indentation mode was quite different to fixed load mode, especially in moderate and high sliding speed. It showed that there was a difference of the stick-slip oscillation frequency between both modes, therefore, stick-slip amplitude or periodic displacement was also different. The periodic displacement obtained mainly influenced the pattern spacing of abraded rubber surface that had a correlation to the wear rate of rubber along abrasion [3,12].

This study proposes an analytical model of the sliding system with a solely tangential motion. Here, the sliding system is a fixed rigid indenter that slides against a moving soft material surface. The soft material chosen is a low viscous damping viscoelastic material. This model may be closer to the sliding contact of the fixed depth mode than fixed load mode from the rubber sliding. An analytical solution is presented in an exact solution for the sliding system without damping and an approximate solution for the sliding system with a low viscous damping to describe the stick-slip behaviour of that model. Furthermore, the effect of velocity-dependent friction in the behaviour of the sliding system are also discussed.

2. Analytical model and methods

The sliding system considered is shown in Fig. 1(a). It consists of a fixed single indenter and the tangentially driven soft surface. The analytical model Fig. 1(b) representing the sliding system consists of a mass m, damping coefficient c and tangential stiffness k, with the constant of driving velocity V. As shown in Fig. 1(b), the mass m represents the effective oscillating mass that is a part of the soft material around the contact between the indenter and the soft surface.

Based on a fixed depth of the sliding indentation, oscillations in the normal direction are neglected and the normal force F_n is assumed to be constant. The tangential force F_t can be either a static friction force F_s or kinetic friction force F_k. The value of m, c and k are assumed to be constant, so a linear differential equation of motion is applied. By starting the sliding system in the stick phase at time $t = 0$ to $t = t_{od}$, the formulation of driving force F_d and initial sticking time t_{od} can be written as,

$$F_d = kVt + cV \quad (1)$$

$$t_{od} = \frac{F_s - cV}{kV} \quad (2)$$

Boundary condition of the mass m before starting to the slip phase is given as,

$$x(t_{od}) = 0 \text{ and } \dot{x}(t_{od}) = 0 \quad (3)$$

where (•) is time derivative. The slip phase starts if the driving force F_d reaches to the static friction force F_s at $t = t_{od}$ and the governing equation of motion of this phase is expressed as,

$$m\ddot{x} + c\dot{x} + kx = cV + kVt - F_k \quad (4)$$

Because of the resistances during the slip phase, the mass m stopped if the velocity $\dot{x}$ leads to zero at $t = (t_{od} + t_{dslip})$and the stick phase starts again. It is rather complicated to solve Eq. (4) above in an exact solution. Therefore, this study develops an approximate solution for the damped sliding system, on the other side, an exact solution is developed for un-damped sliding system (or no damping system).

In this analysis, some of dimensionless numbers and parameters are introduced as follows,

$$\gamma = \frac{F_s}{F_k} \quad (5)$$

$$\zeta = \frac{c}{2\sqrt{km}} \quad (6)$$

$$\omega_n = \sqrt{k/m} \quad (7)$$

$$\omega = \omega_n\sqrt{1-\zeta^2} \quad (8)$$

$$\lambda = \omega_n \frac{F_s - F_k}{kV} \quad (9)$$

$$\tau = \omega_n t \quad (10)$$

$$\xi = \frac{\Delta x \omega_n}{V} \quad (11)$$

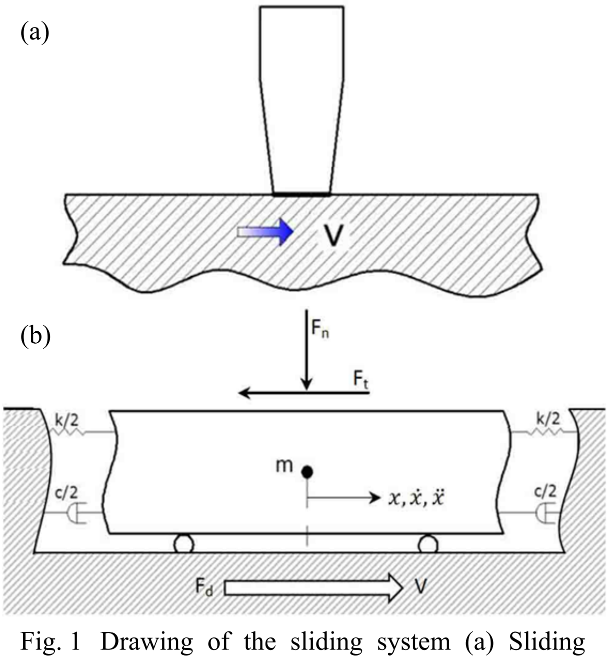

Fig. 1 Drawing of the sliding system (a) Sliding system, (b) Analytical model

3. Results and discussion

3.1. Exact solution for the sliding system without viscous damping

This section develops the exact solution for the un-damped sliding system to describe the stick-slip phenomena, such as: sticking time, slipping time and stick-slip amplitude. This solution may be applied to estimate the stick-slip phenomena of a viscoelastic material with a low damping factor as for a rubber-like material.

Regarding to the tangential indenter forces F_t with the coulomb friction law, the kinetic and static friction force are assumed to be constant. By omitting the damping component, the initial sticking time t_o at the stick phase is,

$$t_o = \frac{F_s}{kV} \quad (12)$$

The general solution at the slip phase in Eq. (4) for $c = 0$ can be given as,

$$x = a\sin\omega_n t + b\cos\omega_n t + Vt - \frac{F_k}{k} \quad (13)$$

By using $A = \sqrt{a^2 + b^2}$ and $\varphi = \arctan\left(\frac{b}{a}\right)$, the above solution can be expressed as,

$$x = A\sin\left(\omega_n t + \varphi\right) + Vt - \frac{F_k}{k} \tag{14}$$

$$\dot{x} = A\omega_n \cos\left(\omega_n t + \varphi\right) + V \tag{15}$$

Some above parameters can be found by inserting the boundary condition $x(t_o) = 0$ and $\dot{x}(t_o) = 0$, that are given as following,

$$a = \frac{1}{\omega_n}\left[-V\cos\left(\omega_n \frac{F_s}{kV}\right) - \omega_n \frac{F_s - F_k}{k}\sin\left(\omega_n \frac{F_s}{kV}\right)\right] \tag{16}$$

$$b = \frac{1}{\omega_n}\left[V\sin\left(\omega_n \frac{F_s}{kV}\right) - \omega_n \frac{F_s - F_k}{k}\cos\left(\omega_n \frac{F_s}{kV}\right)\right] \tag{17}$$

The oscillation amplitude A is given as,

$$A = \frac{1}{\omega_n}\sqrt{V^2 + \left(\omega_n \frac{F_s - F_k}{k}\right)^2} = \frac{V}{\omega_n}\sqrt{1+\lambda^2} \tag{18}$$

Slip phase starts at $t = t_o$ and stops when the velocity of the mass m is zero at $t = (t_o + t_{slip})$ due to the resistance on this phase. By implementing the Eqs. (15,18), the mass m stops at $\dot{x} = 0$,

$$\left(\omega_n t + \varphi\right) = \arccos\left(-\frac{V}{A\omega_n}\right) = \pi \pm \arctan\lambda \tag{19}$$

The velocity of the mass m in the slip phase is always positive. Thus, from Eq. (19), the slipping time t_{slip} is found as,

$$t_{slip} = \frac{2}{\omega_n}\left[\pi - \arctan\lambda\right] \tag{20}$$

At the end of the slip phase, spring force or elastic force in Eq. (4) is $F_e = k(Vt - x) = (2F_k - F_s)$, that is smaller than the static friction force F_s, therefore, the stick phase starts again and finally stops if F_e reachs to F_s. If the analysis is started again at the beginning of this stick phase at $t = 0$, the sticking time t_{stick} replaces the initial sticking time t_0 that is found as follows,

$$t_{stick} = \frac{2\lambda}{\omega_n} \tag{21}$$

This study defines a sticking degree t^*_{st} which is the ratio of the sticking time to stick-slip period T,

$$t^*_{st} = \frac{t_{stick}}{T} = \frac{\lambda}{\pi + \lambda - \arctan\lambda} \tag{22}$$

By using Eq. (14), the stick-slip amplitude Δx or periodic displacement that is a distance covered along the slip phase is,

$$\Delta x = \frac{2\left(F_s - F_k\right)}{k} + \frac{2V}{\omega_n}\left[\pi - \arctan\lambda\right] \tag{23}$$

In special case, if $(F_s - F_k) = 0$ or $\lambda = 0$, the stick-slip amplitude turns out to the periodic displacement under full slip oscillation $\Delta x = VT$. The normalized stick-slip amplitude ξ is defined as,

$$\xi = \frac{\Delta x \omega_n}{V} = 2[\lambda + \pi - \arctan\lambda] \tag{24}$$

Based on Eqs. (20,21), Fig. 2 exhibits the relationship between the sticking time with respect to the dimensionless parameter λ where a linear relationship is found, and the slipping time tends to be constant at larger values of λ. In addition, according to Eqs. (22,24), Fig. 3 shows that the normalized stick-slip amplitude and the sticking degree increase with respect to λ. Based on these results, the dimensionless parameter λ can be regarded as a stick-slip parameter.

According to Eq. (23), Fig. 4 shows a few results for

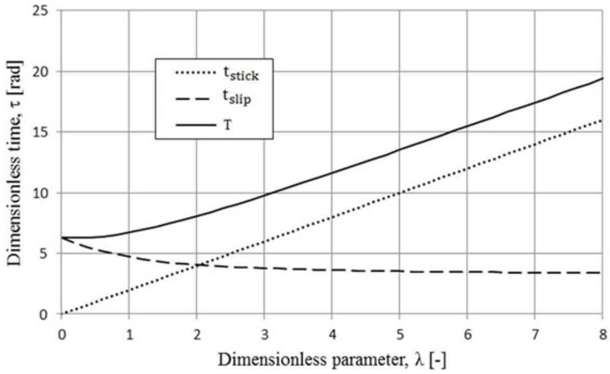

Fig. 2 The relationship among the sticking time, slipping time and stick-slip period with respect to the dimensionless parameter λ

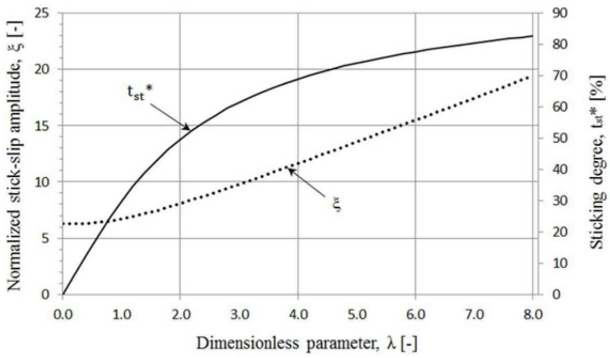

Fig. 3 The relationship between the sticking degree and normalized displacement with respect to the dimensionless parameter λ

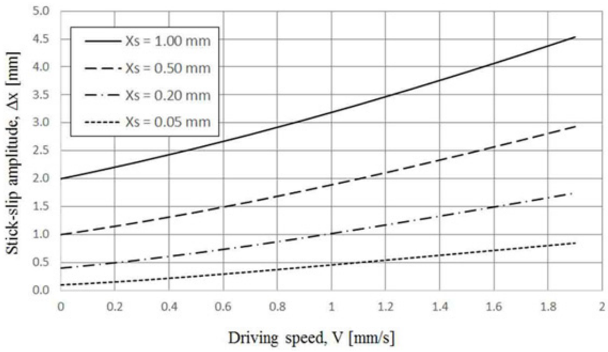

Fig. 4 The relationship between the stick-slip amplitude Δx and the driving speed V (For some values of $X_s = \frac{F_s - F_k}{k}$ at a constant mass m = 0.5 gram and $(F_s - F_k)$ = 5 N)

the stick-slip amplitude Δx with respect to the driving speed V. A low value of X_s represents a high stiffness of the material and vice versa. It can be seen that the large stick-slip amplitude is formed for the low material stiffness. The stick-slip amplitude increase strongly depends on the driving speed increase as stated at the second term of right hand side of Eq. (23), meanwhile, the first term that states the friction force difference determines the initial value of the stick-slip amplitude.

This study defines a velocity ratio v^* that is the ratio of the mass velocity $\dot{x}$ to the driving speed V as follows,

$$v^* = \frac{\dot{x}}{V} = 1 + \sqrt{1+\lambda^2}\cos(\tau + \varphi) \tag{25}$$

Figure 5 shows a comparison of the velocity ratio v^* for several values of the stick-slip parameter λ that starts from the beginning of the slip phase. For high λ values, high amplitude for the velocity ratio, a short slipping time and a long sticking time are found. In general, a high λ results in a long period of the stick-slip mechanism.

By normalizing the spring or elastic force $F_e = k(Vt - x)$ with the friction force difference $(F_s - F_k)$, a normalized elastic force F_e^* during the slip phase is stated as,

$$F_e^* = \frac{F_e}{F_s - F_k} = \frac{1}{\gamma - 1} - \sqrt{1 + \left(\frac{1}{\lambda}\right)^2}\sin(\tau + \varphi) \tag{26}$$

where $\gamma = \dfrac{F_s}{F_k}$ is friction force ratio. On the other side, the normalized elastic force in the stick phase is,

$$F_e^* = \frac{F_e}{F_s - F_k} = \frac{2-\gamma}{\gamma - 1} + \frac{\tau}{\lambda} \tag{27}$$

Based on Eqs. (26,27), the normalized spring force F_e^* for some values of the friction force ratio γ is shown in Fig. 6. These results are evaluated at the stick-slip parameter $\lambda = 2$. An interesting result occurs at a high friction force ratio ($\gamma = 2.5$); negative values for the normalized spring force are found at the end of the slip phase, so it means that the spring in this condition is in under compression.

3.2. The effect of low viscous damping

Sliding system analysis with a low viscous damping

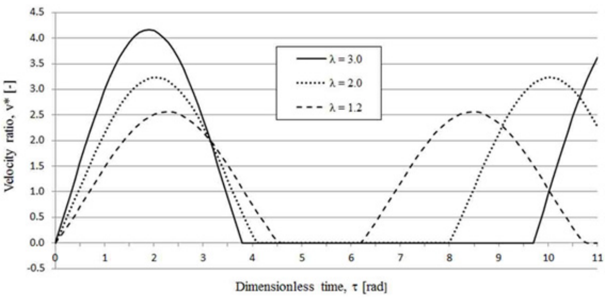

Fig. 5 The velocity ratio of v^* for some values of the stick-slip parameter λ

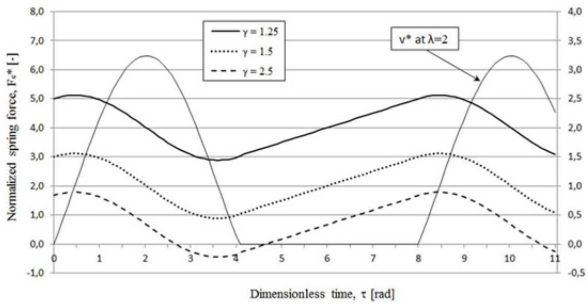

Fig. 6 The normalized spring force F_e^* at $\lambda = 2$ for some friction force ratio $\gamma = \dfrac{F_s}{F_k}$

is presented in an approximate solution that is obtained by neglecting some small terms of the exact solution. By inserting the damping component c in the stick phase, the initial sticking time t_{od} that is expressed in Eq. (2) is smaller than t_o, thus, the damping term reduces the initial sticking time.

By inserting a damping factor ζ at the beginning of the slip phase [5], a general solution of Eq. (4) can be given as,

$$x = A_d \exp\left[-\zeta\omega_n (t - t_{od})\right]\sin(\omega t + \varphi_d) + Vt - \frac{F_k - cV}{k} \tag{28}$$

By using a small damping factor ζ, some approximations can be made. A damped frequency ω is assumed close to the un-damped one $\omega_n \left(\omega = \omega_n\sqrt{1-\zeta^2} \approx \omega_n\right)$ and some small terms of the time derivative of Eq. (28) are not regarded,

$$\dot{x} \approx A_d\omega \exp\left[-\zeta\omega_n (t - t_{od})\right]\cos(\omega t + \varphi_d) + V \tag{29}$$

$$\ddot{x} \approx -A_d\omega^2 \exp\left[-\zeta\omega_n (t - t_{od})\right]\sin(\omega t + \varphi_d) \tag{30}$$

Thus, the stick-slip amplitude A_d is also assumed close to the undamped system A,

$$A_d = \frac{1}{\omega}\sqrt{V^2 + \left(\omega\frac{F_s - F_k}{k}\right)^2} \approx A \tag{31}$$

Referring to Eqs. (29,31) at zero mass velocity ($\dot{x} = 0$), the slip phase starts at $t = t_{od}$ with $(\omega t + \varphi_d) = (\arctan\lambda - \pi)$ and stops at $t = (t_{od} + t_{dslip})$ with

$$(\omega t + \varphi_d) = \arccos\left(-\frac{V}{A\omega\exp\left(-\zeta\omega_n t_{dslip}\right)}\right) = (\pi - \arctan\lambda_d).$$

The parameter λ_d is the stick-slip parameter in the damped sliding system that is found as follows,

$$\lambda_d = \sqrt{\lambda^2 \exp\left(-2\zeta\omega_n t_{dslip}\right) - \left(1 - \exp\left(-2\zeta\omega_n t_{dslip}\right)\right)} \tag{32}$$

It shows that λ_d is smaller than λ. For simplifying the formulation writing, several damping parameters are introduced,

$$\delta_e = \exp\left(-\zeta\omega_n t_{dslip}\right) \tag{33}$$

$$\delta_t = \frac{\arctan\lambda_d}{\arctan\lambda} = \frac{\arctan\sqrt{\delta_e^2\lambda^2 - \left(1-\delta_e^2\right)}}{\arctan\lambda} \tag{34}$$

$$\delta_s = \frac{\sin\left(\omega T_d + \varphi_d\right)}{\sin\left(\omega_n T + \varphi\right)} = \frac{\sqrt{A^2\omega^2 - \left(V^2 / \delta_e^2\right)}}{\sqrt{A^2\omega^2 - V^2}} \tag{35}$$

It should be noted that all of the above damping parameters δ_e, δ_t, δ_s are smaller than unity, and those decrease with increased damping factor ζ. As a result, referring to the previous analysis method in the Eq. (19) to Eq. (22), the slipping time t_{dslip}, sticking time t_{dstick} and sticking degree t^*_{dst} are found as below,

$$t_{dslip} = \frac{2\pi - \left(1+\delta_t\right)\arctan\lambda}{\omega} \tag{36}$$

$$t_{dstick} = \frac{\left(1+\delta_s\delta_e\right)\lambda - 2\zeta}{\omega} \tag{37}$$

$$t^*_{dst} = \frac{\left(1+\delta_s\delta_e\right)\lambda - 2\zeta}{\left(1+\delta_s\delta_e\right)\lambda - 2\zeta + 2\pi - \left(1+\delta_t\right)\arctan\lambda} \tag{38}$$

It is shown that if compared to the undamped sliding system, the viscous damping contributes in an increase in the slipping time, however, it decreases the sticking time and the sticking degree. Therefore, the damping decreases the stick intensity of the sliding system and tends to make the system in full slip. It agrees with Nakano's model [5] that the damping tends to suppress the stick-slip occurrence. A critical condition occurs at zero sticking time,

$$\lambda\left(1+\delta_s\delta_e\right) = 2\zeta \quad \text{or} \quad \frac{\zeta}{\left(1+\delta_s\delta_e\right)} = \frac{\lambda}{2} \tag{39}$$

The critical condition described in Eq. (39) states the limit of stick-slip occurence that occurs at $\zeta \approx \lambda$ for a very small of ζ. If the left hand side of Eq. (38) is larger than the right hand side, the stick-slip phenomenon disappears and the sliding system leads to pure slip (steady sliding). It can be stated that in the damped sliding system, the stick-slip occurrence may disappear although the stick-slip parameter is not zero.

By implementing Eq. (38), effect of the damping term in the sticking degree is described in Fig. 7. It shows that by increasing the damping factor, the sticking degree decreases. There are no value shown of the sticking degree at higher damping factor, especially for low stick-slip parameter. This is due to the difficulty of finding a suitable value of the damping parameters (δ_e, δ_t, δ_s), which are mutually dependent on each other. Therefore, this analytical method is just applicable for low damping factor such as the assumption given earlier. However, the trendlines as described in Fig. 7 can be used to estimate the zero sticking degree that represents the limit of stick-slip occurrence. It can be estimated that the limit of stick-slip occurence is around λ = 0.2 for ζ = 0.015, λ = 0.5 for ζ = 0.050 and λ = 0.9 for ζ = 0.100.

The stick-slip amplitude in the damped system Δx_d is

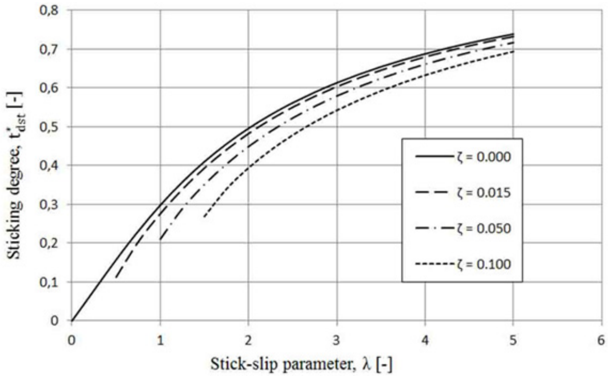

Fig. 7 The sticking degree t^*_{dst} as a function of the stick-slip parameter λ for several damping factor ζ

given as,

$$\Delta x_d = \left(1+\delta_e\delta_s\right)\frac{F_s - F_k}{k} + \frac{V}{\omega}\left[2\pi - \left(1+\delta_t\right)\arctan\lambda\right] \tag{40}$$

If compared to the un-damped system, mathematically, the damping parameter decreases the first term and increases the second term of the right hand side of Eq. (40). However, the second term does not change significantly when compared to the first term, therefore, the damping term tends to decrease the stick-slip amplitude as described in Fig. 8. It is also shown that there is no value of the stick-slip amplitude at high driving speed and high damping factor due to not finding a suitable damping parameter. In general, it can be concluded that the stick-slip amplitude decreases by increasing the damping factor and increases by increasing the driving speed V of the sliding system. It should be noted that in full slip condition (steady sliding), term of the stick-slip amplitude is replaced with the periodic displacement that is a displacement covered along one oscillation. This condition occurs if the limit of stick-slip occurrence is exceeded. Unfortunely, the appearance of this figure can not show the limit of stick-slip occurrence.

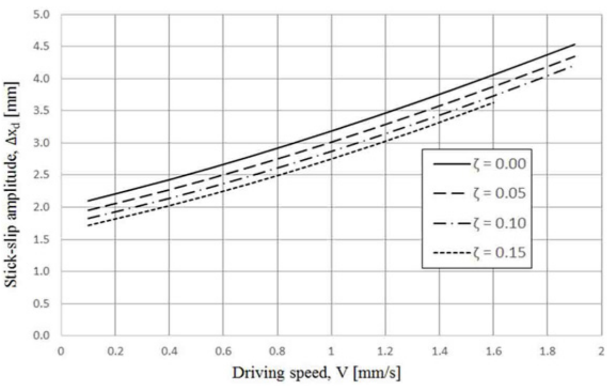

Fig. 8 The stick-slip amplitude Δx_d as a function of driving speed V for several damping factor ζ. (At a constant mass m = 0.5 gram and $X_s = \frac{F_s - F_k}{k} = 1$ mm)

3.3. The effect of velocity-dependent friction

In rubber-like materials, the effect of sliding velocity on friction was investigated [13,14]. As a result, a logarithmic function was determined to describe the relationship between the kinetic friction force and the relative sliding velocity. Most prominent friction model used is a simple logarithmic velocity-dependent function, often assumed to be monotonically weakening or strengthening with increasing relative sliding velocity. The effect of a velocity-dependent friction on the contact between a soft structure against a driven rigid flat was analyzed numerically [7-10].

It should be noted in this study that the "inherent" friction coefficient between indenter and rubber surface contact is applied than "effective" friction. Although the "effective" friction was developed and measured from frictional vibration model by deriving the mean energy consumption rate, this friction coefficient strongly depended on the mechanical properties of tester equipment [15]. Based on inherent friction coefficient, in general, the formulation used for velocity-dependent kinetic friction can be described as follows:

$$F_k = \left[\mu_0 + \mu_1 \exp\left(\alpha V_{rel}\right)\right] F_n \tag{41}$$

The parameters μ_0, μ_1 correspond to the friction coefficients with a positive constant value, V_{rel} corresponds to the relative sliding velocity, and α is a constant parameter than can be a positive or negative value. Thus, based on Fig. 1(b), the relative sliding velocity along the slip phase between mass and indenter is $V_{rel} = \dot{x}$ due to the fixed position of the indenter. By inserting Eq. (41) into Eq. (4), the equation of motion along the slip phase can be expressed as,

$$m\ddot{x} + \left[c\dot{x} + \mu_1 F_n \exp\left(\alpha \dot{x}\right)\right] + kx = cV + kVt - \mu_0 F_n \tag{42}$$

It can be seen that the dynamic properties of the system as described at the left hand side are modified by the velocity-dependent friction component, meanwhile, the right hand side describes the excitation forces that act on it. There is no oscillation term on this excitation force, therefore, the sliding system oscillates on its natural frequency. If Eq. (42) is simplified as a linear vibration, the damping term due to the velocity-dependent friction can be made as $\mu_1 F_n \exp(\alpha\dot{x}) \approx c_v\dot{x}$, which c_v is assumed to be constant. Thus, a total damping factor ζ_{tot} of the sliding system is,

$$\zeta_{tot} = \frac{c + c_v}{2\sqrt{km}} \tag{43}$$

It should be noted that the mass velocity $\dot{x}$ increases with respect to the driving speed V and also the stick-slip parameter λ as described in Eqs. (15,18,29). With a positive value of α, it reflected that high driving speed V increases the value of c_v and the total damping factor ζ_{tot}. Thus, modification of Eq. (42) by velocity-dependent friction component might change the behavior of the sliding system at three possibility conditions,

1. Stick-slip conditions, occurs if the stick-slip parameter λ dominantly influences than the damping term. Referring to Eq. (39), this condition can be stated as: $\dfrac{\zeta_{tot}}{1+\delta_s\delta_e} < \dfrac{\lambda}{2}$
2. Oscillatory slip (relative to the driving velocity V), occurs if the sliding system is in under-damped condition. In this case, the damping term dominantly influences than the stick-slip parameter, however, the total damping factor is still smaller than unity. This condition can be stated as: $\dfrac{\zeta_{tot}}{1+\delta_s\delta_e} > \dfrac{\lambda}{2}$ and $\zeta_{tot} < 1$.
3. Continuously slip (relative to the driving velocity V), occurs if the sliding system is in over-damped condition. In this case, the total damping factor is higher than unity. This condition can be stated as: $\zeta_{tot} > 1$.

Regarding to the velocity dependent friction, a positive value of α might turn out the stick-slip of the system to a full slip (steady sliding) condition in high driving speed V, on the other hand, the system remains in the stick-slip condition with a negative value of α.

4. Conclusions

An analytical model to identify the stick-slip behavior of the soft material surface due to a sliding indentation is presented. A stick-slip model of a sliding system with solely tangential motion is developed. The sliding system used is an indenter with a fixed indentation that slides against a low damping viscoelastic soft material.

Using the coulomb friction law, the exact solution for the un-damped sliding system is given. Results show that the stick-slip amplitude strongly depended on the friction force, driving velocity and material stiffness. The stick-slip amplitude increases with respect to the driving velocity and the difference between static and kinetic friction force.

An approximate solution is presented for the sliding system with low viscous damping by neglecting some small terms of the exact solution. If compared to the un-damped sliding system, the damping factor increases the slipping time and reduces the sticking time, consequently, the stick intensity decreases and the sliding system tends to a full slip or steady sliding. Moreover, the damping factor decreases the stick-slip amplitude compared to undamped system. Also, the effect of velocity-dependent friction may change the sliding system behaviour from the stick-slip contact to full slip (steady sliding) or vice versa.

Nomenclature

A	displacement amplitude of un-damped sliding system (m)
A_d	displacement amplitude of damped sliding

	system (m)
c	viscous damping coefficient (Ns/m)
c_v	damping coefficient due to sliding velocity effect (Ns/m)
F_d	driving force (N)
F_e	elastic or spring force (N)
F_e^*	normalized spring force (-)
F_k	kinetic friction force (N)
F_s	static friction force (N)
F_t	tangential force of indenter (N)
k	tangential stiffness (N/m)
m	effective oscillating mass (kg)
t	time (s)
t_{dslip}	slipping time of damped system (s)
t_{dstick}	sticking time of damped system (s)
t^*_{st}	sticking degree (-)
t_{slip}	slipping time of un-damped system (s)
t_{stick}	sticking time of un-damped system (s)
t_o	initial sticking time of un-damped system (s)
t_{od}	initial sticking time of damped system (s)
V	driving speed (m/s)
V_{rel}	relative sliding velocity (m/s)
V^*	velocity ratio (-)
T	stick-slip period of un-damped system (s)
T_d	stick-slip period of damped system (s)
x	displacement of oscillating mass (m)
$\dot{x}$	velocity of oscillating mass (m/s)
$\ddot{x}$	acceleration of oscillating mass (m/s^2)
Δx	stick-slip amplitude of un-damped system (m)
Δx_d	stick-slip amplitude of damped system (m)
x_s	$= \dfrac{F_s - F_k}{k}$ (m)
α	factor of velocity-dependent friction (-)
γ	friction force ratio (-)
δ_e, δ_s, δ_t	damping parameter (-)
ξ	normalized slip displacement (-)
μ_0, μ_1	friction coefficient (-)
φ, φ_d	stick angle (rad)
ζ	damping factor (-)
ζ_{tot}	total damping factor (-)
ω	damped natural frequency (rad/s)
ω_n	un-damped natural frequency (rad/s)
λ	dimensionless (stick-slip) parameter of un-damped system (-)
λ_d	dimensionless (stick-slip) parameter of damped system (-)
τ	dimensionless time (rad)

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