Bankruptcy Prediction of Corporate Coupon Bond with Modified First Passage Time Approach

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Abstract: Most corporations considering debt liabilities issue risky coupon bonds for a finite maturity which tipically matches the expected life of the assets being financed. For valuing these coupon bond, we can consider the common stock and and coupon bonds as a compound option. The other problem is bond indenture provisions often include safety covenants that give bond investors the right to reorganize a firm if its value falls below a given barrier. This paper will shown how to value bonds with coupon based on the first passage time approach. We will construct a formula for probability of default at the maturity date by computing the historical low of firm values. Using Indonesian corporate coupon bond data, we will predict the bankruptcy of this firm.

Keywords: safety covenants, default barrier, probability of default, compound option

1. Introduction

Credit risk management is one of the most important recent developments inthe finance industry.It has been the subject of considerableresearch interest in banking and finance communities, and has recently drawn theattention of statistical researchers. Credit Risk is the risk induced from credit events such as credit rating change, restucturing, failure to pay, bankruptcy, etc. More formal denition, credit risk is the distribution of financial losses due to unexpected changes in the credit quality of a counterparty in a financial agreement (Giesecke, 2004).Central to credit risk is the default event, which occurs if the debtor is unable tomeet its legal obligation according to the debt contract.

Merton (1974) firstly builds a model based on the capital structure of the firm, which becomes the basis of the structural approach. He assumes that the firm is financed by equity and a zero coupon bond with face value K and maturity date T. In this approach, the company defaults at the bond maturity time T if its assets value falls below the face value of the bond at time T.

Black and Cox (1976) extends the definition of default event and generalize Merton's method into the First Passage Approach. In this approach, the firm defaults when the history low of the firm assets value below some barrier D. Thus the default event could take place before the maturity date T. This theory also need assumption that the corporations issues only one zero coupon bond. Reisz and Perlich (2004) point out that if the barrier is below the bond's face value, then default time definition of Black and Cox theory does not reflect economic reality anymore. In their paper, they modified the classic First Passage Time Approach, and re-defining the formula of default time.

Up to this time, most corporation tend to issue risky coupon bond. At every coupon date until the final payment, the firms have to pay the coupon. At the maturity date, the bondholder receive the face value of the bond. The bankruptcy of the firm occurs when the firm fails to pay the coupon at the coupon payment and/or the face value of the bond at the maturity date. Geske (1977) has derived formulas for valuing coupon bonds. In a later paper, Geske (1979) suggested that when company has coupon bond outstanding, the common stock and coupon bond can be viewed as a compound option.

In this paper we proposed a method for unifying some theory above. We want to produce a new theory in credit risk that fulfill assumptions in real finance industry. We will derive probability of default formula for risky coupon bond with modified first passage time approach. We construct the formula by computing the historical low of firm values.

2. Theoritical Framework

2.1 Merton's Model

Consider a firm with market value at time *t*, V_t, which is financed by equity and a zero coupon bond with face value, *K*, and maturity date, *T*. The firm's contractual obligation is to repay the amount *K* to the bondholder at time *T*. Merton and Black & Scholes (1973) indicated that most corporate liabilities may be viewed as an option. They derived a formula for valuing call option and discussed the pricing of a firm's common stock and bonds when the stock is viewed as an option on the value of the firm. Thus, valuing the equity price of the firm is identical to the equation for valuing a European call option.

The firm is assumed to default at the bond maturity date *T*, if the total assets value of the firm is not sufficient to pay its obligation to the bondholder. Thus the default time τ is a discrete random variable given by

$$\tau = \begin{cases} T & \text{jika} \quad V_T < K \\ \infty & \text{jika} \quad V_T \geq K \end{cases} \tag{1}$$

Figure 1 shows the default event graphically

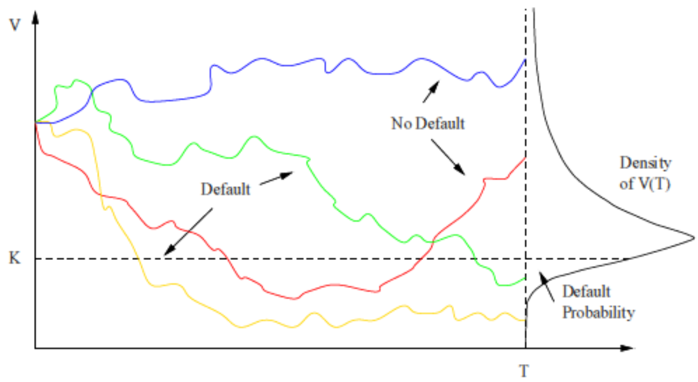

Figure 1. Default Event in the Merton's Model

To calculate the probability of default, we make assumption that the standard model for the evolution of asset prices over time is Geometric Brownian Motion:

$$dV_t = \mu V_t dt + \sigma V_t dW_t \quad \text{and} \quad V_0 > 0 \tag{2}$$

Where µ is a drift parameter, $\sigma > 0$ is a volatility parameter, and *W* is a standard Brownian Motion. Setting $m = \mu - \frac{1}{2}\sigma^2$Itto's lemma implies that

$$V_t = V_0 \exp(mt + \sigma) \tag{3}$$

Since W_t is normally distributed with mean zero and variance *T*, probability of default is given by

$$P(\tau = T) = P[V_T < K] = P[\sigma W_T < \log L - mT] = \Phi\left(\frac{\log L - mT}{\sigma\sqrt{T}}\right) \tag{4}$$

where $L = \frac{K}{V_0}$ and Φ is the cumulative standard normal distribution function.

Classic First Passage Time Model (Black & Cox's Model)

In Merton's model, the firm can only default at the maturity date T. As noted by Black & Cox (1976), bond indenture provisions often include safety covenants that give bondholder right to reorganize a firm if its value falls below a given barrier.

We still use geometric Brownian motion to model the total assets of the firm V_t. Suppose the default barrier B is a constant valued in $(0,V_0)$, then the default time τ is modified to

$$\tau = \inf\{t > 0 : V_t < B\} \tag{5}$$

This definition says a default takes place when the assets of the firm fall to some positive level B for the first time. The firm assumed to take the position of not default at time $t = 0$.
So, the probability of default is calculated as

$$P(\tau \le T) = P[M_T < B] = P\left[\min_{s\le T}(ms + \sigma W_s) < \log\left(\frac{B}{V_0}\right)\right]$$

Where M is the historical low of firm values

$$M_t = \min_{s\le T} V_s$$

Since the distribution historical low of an arithmetic Brownian Motion is inverse Gaussian, then the probability of default can be calculated explicitly by

$$P(\tau \le T) = \Phi\left(\frac{\ln\left(\frac{B}{V_0}\right) - mT}{\sigma\sqrt{T}}\right) + \left(\frac{B}{V_0}\right)^{\frac{2m}{\sigma^2}} \Phi\left(\frac{\ln\left(\frac{B}{V_0}\right) + mT}{\sigma\sqrt{T}}\right) \tag{6}$$

Figure 2 shows the default event graphically for Black & Cox's model.

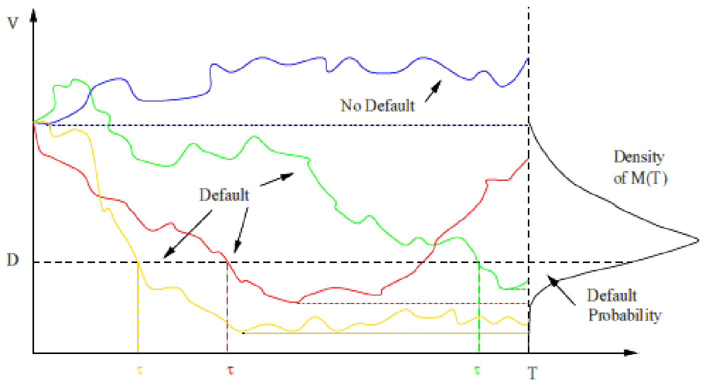

Figure 2. Default Event in the Black & Cox's Model

2.2 Coupon Bond (Geske's Model)

In practice, the most common form of debt instrument is acoupon bond.In the U.S and in many other countries, coupon bonds paycoupons every six months and face value at maturity.Suppose the firm has only common stock and coupon bond outstanding. The coupon bond has n interest payments of c dollars each. The firm is assumed to default at the coupon date, if the total assets value of the firm is not sufficient to pay the coupon payment to bondholder. And at the maturity date, the firm can default if the total assets is below the face value of the bond. For this case, if the firm defaults on a coupon payment, then all subsequent coupon payments (and payments of face value) are also default on.

Geske (1979) proposed a theory for valuing risky coupon bond. When the corporation has coupon bonds outstanding, the common stock can be considered as a compound option (Geske, 1977). A compound option is an optionon an option. In other words, the underlying asset is anotheroption(Wee, 2010). For coupon bond, valuing equity price of coupon bond is identical to valuing a European call option on call option.

At every coupon date until the final payment, the firm have the option of buying the coupon or forfeiting the firm to bondholder. The final firm option is to repurchase the claims on the firm from the bondholders by paying off the principal at maturity. The financing arrangements for making or missing the interest payments are specified in the indenture conditions of the bond. In Figure 3 we illustrate the default event of Geske's model.

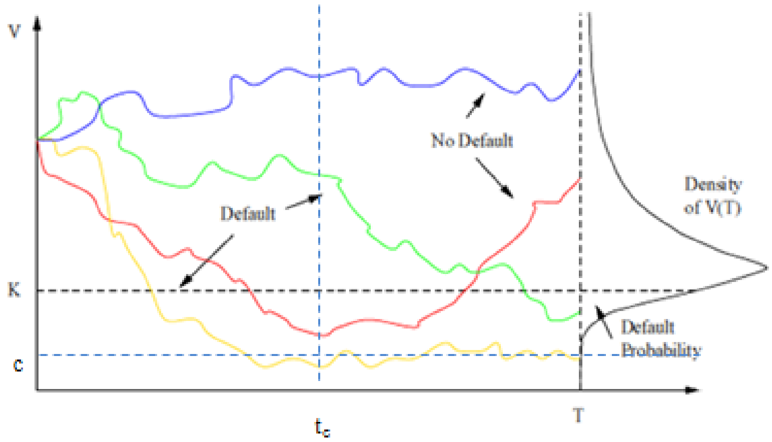

Figure 3. Default Event in the Geske's Model

3. Valuation of Coupon Bond with Modified First Passage Time Approach

3.1 Modified First Passage Time Approach (Reisz & Perlich's Model)

In their paper, Reisz & Perlich (2004) point out that if the barrier is below the face value of the bond, then our earlier definition (5) does not reflect economic reality anymore. It does not capture the situation when the firm is in default because $V_T<K$ although $M_T>B$.

Then, they proposed a redefine default as firm value falling below the barrier $B<K$ at any time before maturity or firm value falling below face value K at maturity. Formally, the default time is now given by

$$\tau = \min(\tau_1, \tau_2) \tag{7}$$

Where

τ_1 = the maturity time T if assets $V_T<K$ at T
τ_2 = the first passage time of assets to the barrier B

In other words, the default time is defined as the minimum of the first passage default time (5) and Merton's default time (1). This definition of default is consistent with the payoff to equity and bonds. Even if the firm value does not fall below the barrier, if assets are below the bond's face value at maturity he firm default. The default event for Reisz & Perlich's model is shown at Figure 4.

Assuming that the firm can neither repurchase shares nor issue new senior debt, the payoffs to the firm's liabilities at debt maturity T are summarized in Table 1 and Table 2.

Table 1. Payoffs at Maturity in The Modified First Passage Time Approach for $B \geq K$

State of the firm	Assets		Bond	Equity
No Default	$M_T>B$		K	$V_T - K$
Default	$M_T \leq B$	$B>K$	B	0
		$B = K$	K	0

Table 2. Payoffs at Maturity in The Modified First Passage Time Approach for $B<K$

State of the firm	Assets	Bond	Equity
No Default	$M_T>B$, $V_T\geq K$	K	$V_T - K$
Default	$M_T>B$, $V_T< K$	V_T	0
	$M_T\leq B$	B	0

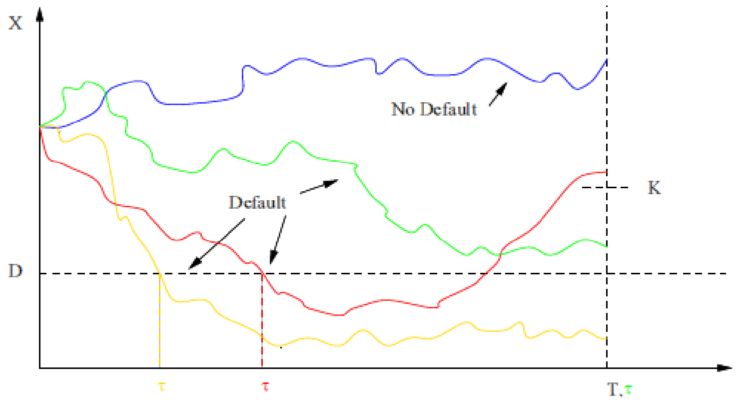

Figure 4. Default Event in the Reisz & Perlich's Model

3.1 Valuation of Coupon Bond with Modified First Passage Time Approach

In this section, we want to begin with assumption that the firm have assets value V_t, which is financed by equity and a single coupon bond with face value K and only one time coupon payment at t_c for the bond period.

Suppose the default barrier B is a constant valued in $(0,V_0)$ and $c<B<K$, then the default time τ is given by

$$\tau = \min(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5) \tag{8}$$

Where

τ_1 = the maturity time T if assets $V_T<K$ at T
τ_2 = the first passage time of assets to the barrier B at time (t_c,T)
= $\inf\{t_c < t < T : V_t < B\}$
τ_3 = the coupon payment date if assets $V_T<B$ or assets $V_T<c$ at time t_c
τ_4 = the first passage time of assets to the barrier B at time $(0,t_c)$
= $\inf\{0 < t \leq t_c : V_t < B\}$
τ_5 = ∞, otherwise

With the definition above, we can summarize the default time by

$$\tau = \min(\tau_1, {\tau_2}^*) \tag{9}$$

Where

${\tau_2}^*$ =the first passage time of assets to the barrier B at time $(0,T)$
$= \inf\{0 < t < T : V_t < B\}$

The default event is shown at Figure 5.

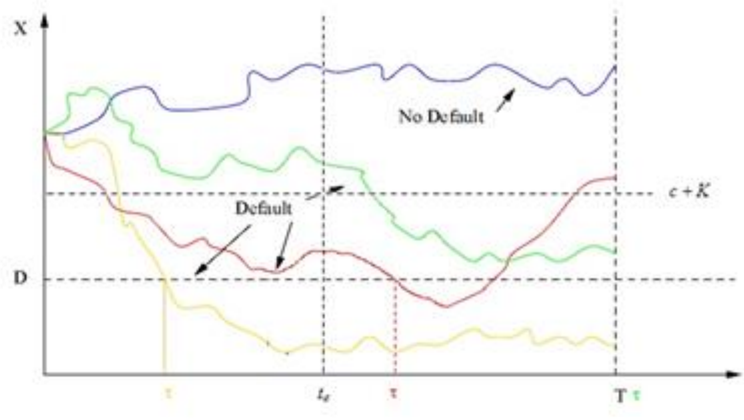

Figure 5. Default Event for Coupon Bond with Modified First Passage Time Approach

We have to check wether this default definition is consistent with the payoff to investor. We need to consider two scenarios.

1. $B \geq c + K$
   a. If the firm value never falls below the barrier B over the term of the bond ($M_T > B$), then at coupon payment the bond investor receive the coupon c, and at the maturity date receive the face value $c + K$, $K < V_0$. The equity holders receive the remaining $V_T - (c + K)$ at the maturity date.
   b. If the firm value falls below the barrier at some point during the bond's term ($M_T \leq B$), then the firm default. In this case, the firm stop operating, bond investors take over its assets B and equity investor receive nothing.
   Bond investor is fully protected: they receive at least the face value and coupon $c + K$ upon default and the bond is not subject to deafult rik anymore.
   c. If the assets value $V_T$is less then $c + K$, the ownership of the firm will be transferred to the bondholder, who lose the amount $(c + K) - V_T$. Equity is worthless because of limited liability.
2. $B < c + K$
   This anomaly does not occur if we assume $B < c + K$ so that bondholder is both exposed to some default risk and compensated for bearing that risk.
   a. If the firm value never falls below the barrier B over the term of the bond ($M_T > B$) and $V_T \geq c + K$, then at coupon payment the bond investor receive the coupon c, and at the maturity date receive the face value $c + K$, $K < V_0$. The equity holders receive the remaining $V_T - (c + K)$ at the maturity date.
   b. If $M_T > B$ but $V_T < c + K$, then the firm default, since the remaining assets are nor sufficient to pay off the debt in full. Bondholder collect the remaining assets V_T and equity become worthless.
   c. If $M_T \leq B$, then the firm default as well. Bond investor receive $B < K$ at default and equity become worthless.

To calculate probability of default for this case, first we define M as the historical low of firm values, that is

$$M_t = \min_{s \leq T} V_s$$

Then, we get the corresponding probability of default as

$$\begin{aligned} P(\tau \leq T) &= P(\min(\tau_1, {\tau_2}^*) \leq T) \\ &= 1 - P(\min(\tau_1, {\tau_2}^*) > T) \\ &= 1 - P(\tau_1 > T, {\tau_2}^* > T) \\ &= 1 - P(M_T > B, V_T < K) \end{aligned}$$

Using the joint distribution of an Arithmetic Brownian Motion and its running minimum, we get

$$P(\tau \leq T) = \Phi\left(\frac{\ln\left(\frac{B}{V_0}\right) - mT}{\sigma\sqrt{T}}\right) + \left(\frac{B}{V_0}\right)^{\frac{2m}{\sigma^2}} \Phi\left(\frac{\ln\left(\frac{B^2}{KV_0}\right) + mT}{\sigma\sqrt{T}}\right) \tag{10}$$

The probability of default for coupon bond with modified first passage time approach is higher than the corresponding probability of the classical approah, equation (6).

4. Empirical Study in Indonesian Bond Market

In this case study we use data sets from and Indonesian Bond Market Directory 2011 that is published by Indonesian Stock Exchange (IDX) and Indonesian Bond Pricing Agency (IBPA). We use bond that is issued by PT Bank Lampung (BPD Lampung), namely Oligasi II Bank Lampung Tahun 2007, with code number BLAM02 IDA000035208. The profile structure of this bond is given at Table 2. Total assets data of the firm is published by Indonesian Bank is given at Table 3.

Table 2. Profile Structure of Obligasi II Bank Lampung Tahun 2007

Outstanding	Listing Date	Maturity Date	Issue Term	Coupon Structure
300,000,000,000	Nov 12, 2007	Nov 9, 2012	5 years	Fixed 11.85 %

Tabel 3.1 Total Asset Value of PT Bank Lampung Tbk in last 2 years

Years	Month	Total Assets Value	Years	Month	Total Assets Value
2011	January	2,454,583,000,000	2012	January	3,178,145,000,000
	February	2,583,697,000,000		February	3,402,327,000,000
	March	2,854,050,000,000		March	3,611,431,000,000
	April	2,837,780,000,000		April	3,845,560,000,000
	May	2,846,133,000,000		May	4,085,869,000,000
	June	3,020,574,000,000		June	4,082,279,000,000
	July	3,129,483,000,000		July	3,978,328,000,000
	August	2,919,372,000,000		August	3,447,473,000,000
	September	3,115,696,000,000		September	4,047,746,000,000
	October	3,000,063,000,000		October	4,026,786,000,000
	November	3,043,340,000,000		November	4,295,751,000,000
	December	3,130,048,000,000		December	4,221,274,000,000

For deriving the probability of default of bond, we have to construct the formula by computing the historical low of firm values. All the computation is done by R programming. In this study, we use a fixed barrier level in 2,000,000,000,000.

Using formula (10) we have the probability of default for Obligasi II Bank Lampung Tahun 2007 is 0.00003627191. This probability of default is very small because of the outstanding of the bond is very low than the total assets value. It can be seen from Table 2 and Table 3, that the face value of the bond is 300,000,000,000,000 and the total assets value at the end of 2012 is 4,221,274,000,000. In the normal situation, the total assets value is very sufficient for paying the principal of the bond.

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