# IICMA 2013

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The Organizing Committee is not responsible for any errors in the papers as these are individual author responsibility.

April 2014

# FOREWORDS

## President of the Indonesian Mathematical Society (IndoMS)

# FOREWORDS

Assalamu'alaikum Warahmatullahi Wabarakatuh

Good morning and best wishes for all of us

It is my pleasure to say that the proceedings of the Second IndoMS International Conference on Mathematics and Its Applications (IICMA) 2013 from November 6 to November 7 at Yogyakarta-Indonesia finally published. The IICMA 2013 is the second IICMA, after IICMA 2009, which is organized by the Indonesian Mathematical Society (IndoMS) in collaboration with Department of Mathematics, Faculty of Mathematics and Natural Sciences, Gadjah Mada University and funded by Directorate of Research and Community Services, the Directorate General of Higher Education, Ministry of Education and CultureRepublic of Indonesia.

IICMA 2013 is one of the activities of IndoMS period 2012-2014. Organizing an IICMA 2013 is not only a continuing academic activity for IndoMS, but it is also a good opportunity for discussion, dissemination of the research result on mathematics including: Analysis, Applied Mathematics, Algebra, Theoretical Computer Science, Mathematics Education, Mathematics of Finance, Statistics and Probability, Graph and Combinatorics, also to promote IndoMS as a non-profit organization which has a member more than 1,400 people from around Indonesia area.

We would like to express our sincere gratitude to all of the Invited Speakers from the Netherlands, Georgia, India, Germany, Singapore and also Indonesia from Universities (ITB, UPI, University of Jember) and LAPAN Bandung, all of the speakers, members and staffs of the organizing committee of IICMA 2013. Special thanks to the Secretary of International Mathematics Union (IMU), the Directorate General of Higher Education, the Dean of Faculty of Mathematics and Natural Sciences-Gadjah Mada University, the Head of Department of Mathematics together with all staffs and students, also for supporting of lecturers and staffs as an organizing committee from Indonesian University, Padjadjaran University, University North of Sumatera, Sriwijaya University and Bina Nusantara University. Finally, we also would like to to give a big thanks for all reviewers who help us to review all papers which are submitted after IICMA.

With warmest regards,

Budi Nurani Ruchjana
President IndoMS 2012-2014

## Chair of the Committee IICMA 2013

On behalf of the Organizing Committee of IndoMS International Conference on Mathematics and its Applications (IICMA) 2013, I would like to thanks all participants of the conference. This conference was organized by Indonesia Mathematical Society (IndoMS) and hosted by Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, Yogyakarta, Indonesia, during 6-7 November 2013.

In IICMA 2013, there will be 122 talks which consists of 10 invited and 112 contributed talks coming from diverse aspects of mathematics ranging from Analysis, Applied Mathematics, Algebra, Theoretical Computer Science, Mathematics Education, Mathematics of Finance, Statistics and Probability, Graph and Combinatorics. However, the number of paper which were sent and accepted in this proceedings is 33 papers. We would also like to give our

- Prof. Dr. S. Arumugam (Combinatorics, Kalasalingam University, India)
- Prof. Dr. Bas Edixhoven (Algebra, Universiteit Leiden-the Netherlands)
- Prof. Dr. Dr. h.c. mult. Martin Grotschel (Applied Math, Technische Universitat Berlin, Germany and International Mathematics Union)
- Prof. Dr. Kartlos Joseph Kachiashvili (Statistics, Tbilisi State University-Georgia)
- Prof. Dr. Berinderjeet Kaur (Mathematics Education, National Institute of Education, Singapore)
- Prof. Hendra Gunawan, Ph.D (Analysis, ITB-Bandung, Indonesia)
- Prof. Dr. Edy Hermawan (Atmospheric Modeling, LAPAN Bandung)
- Prof. H. Yaya S. Kusumah, M.Sc., Ph.D (Mathematics Education, UPI-Bandung, Indonesia)
- Prof. Dr. Slamin (Combinatorics, Universitas Jember, Indonesia)
- Dr. Aleams Barra (Algebra, ITB-Bandung, Indonesia)

We thank all who sent the papers or proceedings of IICMA 2013. We also would like to give our gratitude for all reviewers who worked hard for making this proceedings done.

IndoMS conveys high appreciation for the Directorate General of Higher Education (DGHE) for the most valuable support in organizing the conference. We also would like to give our gratitude to Universitas Gadjah Mada, especially to Department of Mathematics, Faculty of Mathematics and Natural Sciences for providing the places and staffs for this conference.

It remains to thank all members of Organizing Committee spread across 3 cities, Depok, Bandung and Yogyakarta who have worked very hard to make this conference happens.

Yogyakarta, January 5th, 2014
On behalf of the Committee
Dr. Kiki Ariyanti Sugeng - Chair.

# ACKNOWLEDGEMENT

The organizing Committee of the IICMA 2013 and Indonesian Mathematical Society (IndoMS) wish to express their gratitude and appreciation to all Sponsors and Donors for their help and support for the Program, either in form of financial support, facilities, or in other form. The Committee addresses great thank especially to:

a. Directorate General of Higher Education (DGHE)
b. The Rector of the Gadjah Mada University
c. The Dean of Faculty ofMathematics and Natural Sciences, Gadjah Mada University.
d. The Head of the Department of Mathematics, Gadjah Mada University.
e. All sponsors of the conference

The Committee extends its gratitude to all invited speakers, parallel session speakers, and guests for having kindly and cordially accepted the invitation and to all participants for their enthusiastic response.

Finally the Committee also would like to acknowledge and appreciate for the support and help of all IndoMS members in the preparation for and the running of the program

# CONTENTS

# HYBRID MODEL OF IRRIGATION CANAL AND ITS CONTROLLER USING MODEL PREDICTIVE CONTROL

SUTRISNO

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**Abstract.** In this paper, we formulate a hybrid model of irrigation canal that contains four reaches where each of reach has two state events. These state events of this hybrid model are triggered by the height of the water level which has two different surfaces. We formulate this hybrid model in piecewise affine (PWA) form, transform it into mixed logical dynamic (MLD) form using hybrid system description language (HYSDEL) that was embedded in hybrid toolbox for MATLAB and control this MLD using model predictive control (MPC). We control this irrigation canal so that the water level on each reach will be located at the desired level. Finally, we simulate this system and its controller to desire given desired level or set point. From the simulation results, the water level of all reaches are located at the desired level.

Key words and Phrases: Hybrid system, piecewise-affine (PWA) system, mixed logical dynamic (MLD) system, Model predictive control, irrigation canal

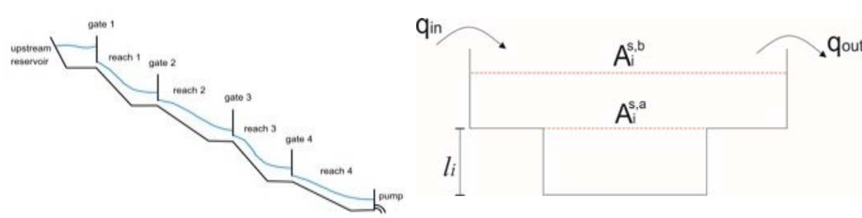
## 1. INTRODUCTION

Following [4], hybrid system is a mathematical model that consists of dynamics of real valued variables, discrete variables and their interaction. A particular case of discrete hybrid systems is piecewise-affine (PWA) systems whose mode depends on the current location (or event) of the state vector. Hybrid system in PWA form can be transformed into equivalent mixed logical dynamical (MLD) form [2] and it can be done using HYSDEL [14] that was embedded in hybrid control toolbox for MATLAB [2]. This MLD form is more suitable for solving optimization problem corresponding to the optimal control problem [4]. The state of an MLD model contains state and input of the PWA model and some auxiliary variables i.e. binary variable and real variable. Furthermore, besides the original constraint of PWA, MLD has auxiliary constraint in the form of matrix inequality. The controllability and observability of this hybrid model was given by [3] in order to be controllable and observable by a control design.

Hybrid predictive control is a famous control method that was applied in some problems like optimal semi-active suspension [5], direct injection stratified charge engines [6] and the stability of MPC for hybrid system was guaranteed by [7]. Predictive control can be applied to control an MLD system with some modifications such as the corresponding optimization problem. Predictive control for MLD system for finite time horizon can be done by forming the vector prediction of all state variables of MLD over time horizon, substituting them into objective function and solving the corresponding optimization problem. In case where the objective function has quadratic form, the corresponding optimization can be solved using mixed integer quadratic programming (MIQP) [2].

Irrigation canal is needed to be controlled to ensure that the water level of each reach is located at the desired level. To control this system, we need the mathematical model of the system. For flat irrigation canal (there is no different surface for any water level), the mathematical model was given by [13] in the linear discrete dynamics (non-hybrid model) and the control designs for this system were given by some researches like coordinated DMPC [11], distributed model predictive control [10, 12] and its performance analysis was given by[8]. For non-flat irrigation canal (there is different surfaces for some water levels), we need a new mathematical model formulation, that is a new hybrid model in order to control this system.

In this paper, we formulate the hybrid model of irrigation canal that have two different surfaces for each reach. This hybrid model will be presented as PWA form and it will be transformed equivalently into MLD form in order to be more suitable for designing its controller. We will apply MPC to control this MLD model. MPC for MLD contains a mixed integer quadratic optimization and we will solve it using MIQP. To observe how this hybrid model and its controller are working, we simulate this system so that the water level of each reach will be located at the desired level and show the output that is the water level of each reach.




(a) Irrigation canal with their surface areas four reaches
(b) A reach with two state events triggered by

Figure 1. Irrigation canal and a reach

## 2. MODELING OF HYBRID SYSTEM OF IRRIGATION CANAL

A hybrid model presents the dynamic of a system that constituted by parts described by logical parameter such as on/off switches, event states, if-then-else rules, etc [4]. Firstly, we will formulate the hybrid model of irrigation canal in the PWA form. Consider an irrigation canal system illustrated by Figure (1a) that has four reaches. Let $x_i(k)$ be the water level in meter unit (m) for reach-$i$, $A_i^s$ be the water surface area ($m^2$) for reach $i$, $q_i^{in}$-and $q_i^{out}$ be the in-flow ($m^3/s$) and out-flow ($m^3/s$) for reach-$i$ which are measured on upstream and downstream respectively and $p_4$ be the flow that passing the pump on reach-4. Let $T_s$ be the sampling time, then the dynamic for reach-$i$ can be written as [13].

$$x_i(k+1) = x_i(k) + \frac{T_s}{A_i^s} q_i^{in}(k) - \frac{T_s}{A_i^s} q_i^{out}(k) \quad (1)$$

For reach-$i$, let $A_i^{s,b}$ be the water surface area if $x_i > l_i$ (mode-0) and $A_i^{s,a}$ be the water surface area if $x_i \le l_i$ (mode-1) as illustrated by Figure (1b). Then the dynamic of reach $i$ can be written as the following PWA model

$$x_i(k+1) = \begin{cases} x_i(k) + \frac{T_s}{A_i^{s,b}} q_i^{in}(k) - \frac{T_s}{A_i^{s,b}} q_i^{out}(k) & \text{if } x_i > l_i, \text{(mode-0)} \\ x_i(k) + \frac{T_s}{A_i^{s,a}} q_i^{in}(k) - \frac{T_s}{A_i^{s,a}} q_i^{out}(k) & \text{if } x_i \le l_i, \text{(mode-1)} \end{cases}$$

$y_i(k) = x_i(k)$,

for $i = 1,2,3$ and especially for $i = 4$, $q_4^{out}$ is replaced by $p_4$.

## 3. MODEL PREDICTIVE CONTROL FOR MLD SYSTEM

Model predictive control is one of a modern control design that has been applied by many researches to solve an optimal control problem including optimal control problem for a hybrid system. Let $s$ be the number of the model parts or events and $k$ denote the time step, then PWA model for discrete time in general form can be described as follows [4].

$$x_c(k+1) = \begin{cases} A_1 x_c(k) + B_1 u_c(k) & \text{if } \delta_1 = 1, \\ A_2 x_c(k) + B_2 u_c(k) & \text{if } \delta_2 = 1, \\ \vdots \\ A_s x_c(k) + B_s u_c(k) & \text{if } \delta_s = 1. \end{cases}$$

$$y_c(k) = Cx_c(k) + Du_c(k) \quad (2)$$

where $x_c(k) \in \mathbb{R}^{nc}$, $u_c(k) \in \mathbb{R}^{mc}$ and $y(k) \in \mathbb{R}^{pc}$ denote the state, input and output vectors respectively at time step k, $i = 1,2,...,s$, $\delta_i \in \{0,1\}$, and matrices $A_i, B_i, C$ and $D$ are real matrices with appropriate dimensions. This PWA model can be transformed into equivalent MLD and vice versa as follows [1]. Let $\delta \in \{0,1\}$ be the binary variable, $n_c > 0$ be the number of the state of the system then the MLD model can be described as follows.

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) \quad (3)$$

$$y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) \quad (4)$$

$$E_2\delta(k) + E_3z(k) \le E_1u(k) + E_4x(k) + E_5 \quad (5)$$

where $x(k) = \begin{bmatrix} x_c(k) \\ x_l(k) \end{bmatrix}$ is the new state vector for MLD model, $x_c(k) \in \mathbb{R}^{n_c}$, $x_l(k) \in \{0,1\}^{n_l}$, $y(k) = \begin{bmatrix} y_c(k) \\ y_l(k) \end{bmatrix} \in \mathbb{R}^{p_c} \times \{0,1\}^{p_l}$ is the new output vector for MLD model, $u(k) = \begin{bmatrix} u_c(k) \\ u_l(k) \end{bmatrix} \in \mathbb{R}^{m_c} \times \{0,1\}^{m_l}$ is the new input vector for MLD model, $z(k) \in R^{r_c}$ and $\delta(k) \in \{0,1\}^{r_l}$ are auxiliary variables. $A, B_i, C, D_i$ and $E_i$ are real constant matrices and $E_5$ is a real vector.

To control this MLD model, that is determining some input values such that the output track some desired set point or reference trajectories, we will use MPC controller by assuming that this MLD model is controllable and reachable. The objectives of the MPC can be described as the state gain to the set point or references. MPC minimizes this objective function that can be described as the following optimization [4].

$$\min_{[u,\delta,z]^T} J = \sum_{t=0} \|u(k) - u_r\|^2_{Q_2} + \|\delta(k) - \delta_r\|^2_{Q_3} + \|z(k) - z_r\|^2_{Q_4} \quad (6)$$

$$+ \|x(k) - x_r\|^2_{Q_1} + \|y(k) - y_r\|^2_{Q_5}\Big] \quad (7)$$

subject to :

$$x(k + 1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) \quad (8)$$

$$- E_4x(k) - E_1u(k) + E_2\delta(k) + E_3z(k) \leq E_5 \quad (9)$$

where $T$ be the prediction horizon length, $Q_1$, $Q_2$, $Q_3$, $Q_4$, and $Q_5$ are the symmetric and positive definite weighting matrices, $x_r$, $u_r$, $\delta_r$, and $z_r$ are the references and $\|v\|^2_Q = v^TQv$.

The optimization (6) can be transformed into mixed integer quadratic (MIQ) optimization by forming the vector predictions of state, input $u$, $\delta$, and $z$ over horizon prediction $T$ and substituting them into the objective function. This transformation gives the following MIQ optimization as follows.

$$\mathbf{R}^* = \arg\min_{\mathbf{R}} \mathbf{R}^0 S_1 \mathbf{R} + 2(S_2 + x^0_0 S_3)\mathbf{R} \quad (11)$$

subject to :

$$F_1\mathbf{R} \leq F_2 + F_3x_0 \quad (12)$$

$$\mathbf{ABR} = x_f - A^Tx_0, \quad (13)$$

$$\mathbf{R} = [u(0),\ldots,u(T-1),\ldots,\delta(0),\ldots, \quad (14)$$

$$\delta(T-1),z(0),\ldots,z(T-1)]^T \quad (15)$$

where $S_1$, $S_2$, and $S_3$ are the real constant matrices with appropriate dimension, $\mathbf{A} = [A^{T-1}, A^{T-2}, \ldots, A^2, A, I]$, $\mathbf{B} = [\mathbf{B_1}\ \mathbf{B_2}\ \mathbf{B_3}]$, $F_1 = \mathbf{E} - \mathbf{E_4}G_4$, $F_2 = \mathbf{E_5}$, $F_3 = \mathbf{E_4}G_0$, $\mathbf{E} = [-\mathbf{E_1}\ \mathbf{E_2}\ \mathbf{E_3}]$, $G_0 = [I, A, A^2, \ldots, A^{T-1}]'$, $E_5 = [E_5, E_5, \ldots, E_5]'$,

$E_i = \begin{bmatrix} E_i & & \\ & E_i & \\ & & \ddots \\ & & & E_i \end{bmatrix}$, $i = 1,2,3,4$, and $\mathbf{B}_i = \begin{bmatrix} B_i & \ldots \\ & B_i \end{bmatrix}$, $i = 1,2,3$.

Optimization (11) can be solved using MIQP that embedded in hybrid toolbox [2]. The optimal value $\mathbf{R}^{*} = [u^{*},\delta^{*},z^{*}]$ obtained from (11) contains the optimal value for $u^{*}$, $\delta^{*}$, and $z^{*}$ over time horizon. The control action that will be applied to the system is $u^{*}$ at current time step.

## 4. SIMULATION RESULTS

We simulate the hybrid model of irrigation canal that contains four reaches associates to (3)-(4) by applying MPC controller that the optimal input values are obtained by optimization (11) with two initial state values. For all $i = 1,2,3,4$, the parameter values for this system are $l_i = 2$, $A^{s,a}{}_i = 600$, $A^{s,b}{}_i = 800$, and $0 \leq u_i(k) \leq 4$. The first simulation is using initial state or initial water level $x(0) = [1.3,2,2.5,1.8]^T$. The simulation results including desired water levels (set points) for all reaches are appeared on Figure (3-4).

Figure (2) shows the optimal control actions for all reaches generated by MPC. These control actions was determined by MIQP programming (11). At time step $k$(second), the value $u_1(k)$ presents the inflow ($m^3/s$) for reach-1 that passed through the gate-1 and analogously for $u_i(k)$ for $i = 2,3,4,5$. These control actions are applied to the system and give the outputs (water level for all reaches) that given by Figure (3).

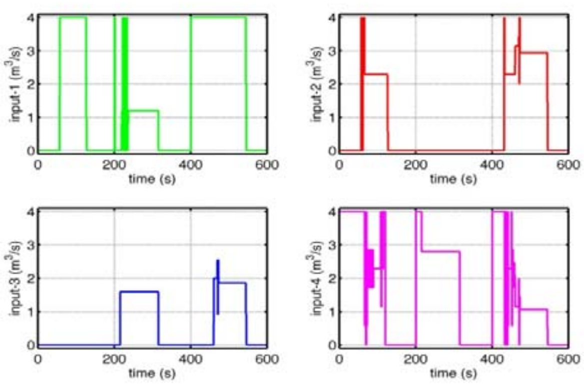



Figure 2. Optimal control inputs where $x(0) = [1.3,2,2.5,1.7]$

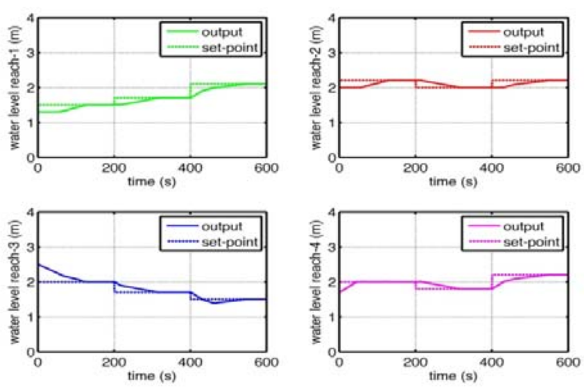



Figure 3. Water levels for all reaches where $x(0) = [1.3,2,2.5,1.7]$

Figure (3) shows the water level for all reaches. For reach-1, the initial water level is 1.3 m. From this figure, it can be seen that the water level is located at the desired level after 100 s and analogously for reach-*i* for $i = 2,3,4$. From this figure, it can be conclude that the controller brings the water levels of all reaches to the desired levels very well.

Figure (4) shows the modes for all reaches along simulation. For reach-1, the initial water level is 1.3 m, lower than $l_1 = 2m$, it means that this state is located at mode-1. At time step 600 s, the water level of reach-1 is 2.1 m, upper than

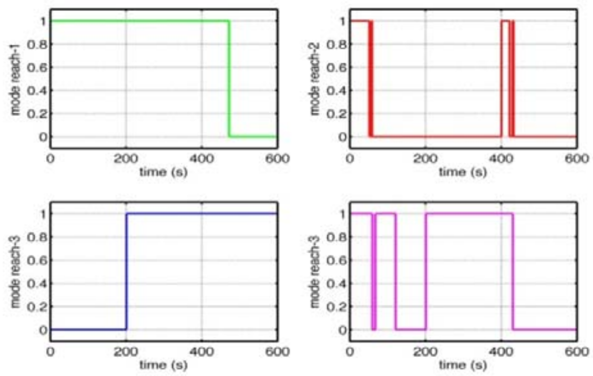



Figure 4. Modes for all reaches where $x(0) = [1.3, 2, 2.5, 1.7]$

$l_1 = 2m$, it means that this level is located at mode-0. We can observe analogously for other reaches. These mode values are dependent to the water level showed by Figure (3).

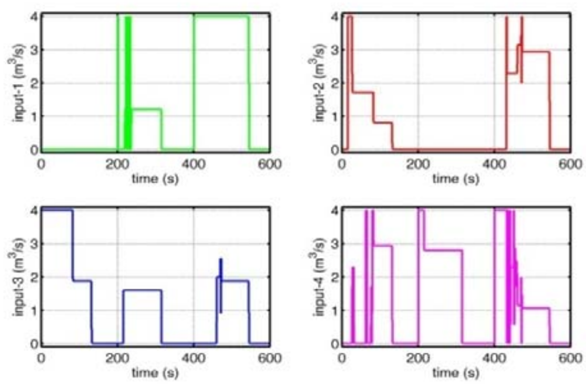



Figure 5. Optimal control inputs where $x(0) = [1.8, 2.5, 1.6, 2.2]$

The second simulation is using initial state $x(0) = [1.8, 2.5, 1.6, 2.2]$. Analogously to the first simulation, Figure (5) shows the optimal input values for all reaches, Figure (6) shows the water level for all reaches and Figure (7) shows the modes for all reaches. From Figure (6), it can be observed that the output of this system i.e. the water level for each reach followed the given desired level or set point. The mode for each reach on Figure (7) is following the output values on

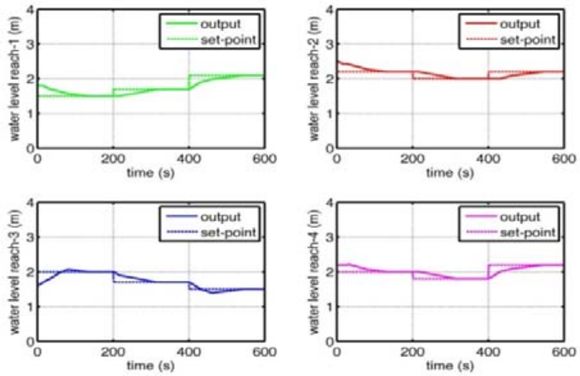



Figure 6. Water levels for all reaches where $x(0) = [1.8, 2.5, 1.6, 2.2]$

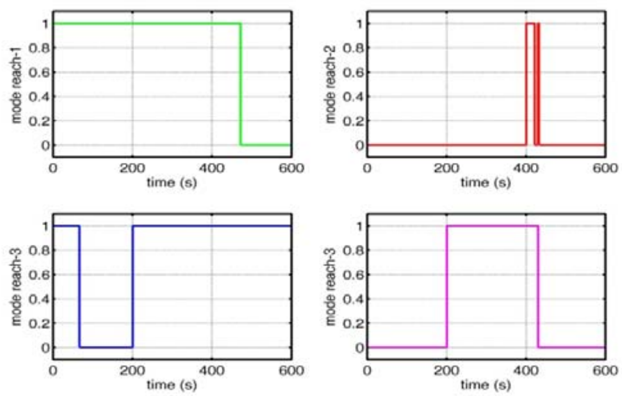



Figure 7. Modes for all reaches where $x(0) = [1.8, 2.5, 1.6, 2.2]$

Figure (6), that is if the water level is lower or equal to 2 m, then the mode is 1 and if the water level is upper than 2 m, then the mode is 0. These mode values are dependent to the water level showed by Figure (6).

## 5. CONCLUSIONS

In this work, the modeling and control of irrigation canal with two state events were considered. The mathematical model of this system was presented as hybrid model in the PWA form and it can be transformed equivalently into MLD form using HYSDEL and *mld* function on hybrid toolbox for MATLAB. The controller for this MLD model was designed using MPC for MLD. From the simulation results, the water level for all reaches were followed the given desired levels very well.

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