BAB III

MODEL DISTRIBUSI KECELAKAAN

3.1. Distribusi Poisson

Banyak cara penurunan distribusi Poisson. Dalam kecelakaan lalulintas jalan raya distribusi Poisson diturunkan dengan hipotesa bahwa rata-rata (mean) banyaknya kecelakaan per satuan waktu (λ) sama.

Kita mengasumsikan sebagai berikut :

(a) P(1 keecelakaan dalam waktu $(t, t+\delta t)$) = $\lambda\delta t+o(\delta t)$

(b) P(0 kecelakaan dalam waktu $(t,t+\delta t)$) = $1-\lambda\delta t-o(\delta t)$

(c) P(2 kecelakaan atau lebih dalam waktu $(t,t+\delta t)$) = $o(\delta t)$

(d) Kecelakaan dalam selang waktu yang berbeda bebas satu dengan yang lainnya.

Dengan λ = rata-rata banyaknya kecelakaan per satuan waktu

$o(\delta t)$ = suatu harga yang bernilai kecil yang bisa diabaikan.

Misalkan $P_k(t)$ = P(k kecelakaan dalam waktu (0,t))

maka, $P_k(t+\delta t)$ = P(k kecelakaan dalam waktu $(0,t+\delta t)$)

= P(k kecelakaan dalam waktu (0,t) dan tidak ada kecelakaan dalam waktu $(t,t+\delta t)$) + P((k-1) kecelakaan dalam waktu (0,t) dan 1 kecelakaan dalam waktu $(t,t+\delta)$) + P((k-i) kecelakaan dalam waktu (0,t) dan i kecelakaan dalam waktu $(t, t+\delta t)$).

dengan $i \geq 2$.

$$P_k(t+\delta t) = P_k(t)(1-\lambda\delta t-o(\delta t) + P_{k-1}(t)(\lambda\delta t+0(\delta t)) + P_{k-i}(t)o(\delta t)$$

$$P_k(t+\delta t) = P_k(t)(1-\lambda\delta t) + P_{k-i}(t)\lambda\delta t \qquad (3.1.1)$$

Kedua sisi persamaan (3.1.1) dikurangi P(t) kemudian dibagi δt, seterusnya ambil δt menuju 0 :

$$P_k(t+\delta t) - P_k(t) = P_k(t)(1-\lambda\delta t) - P_k(t) + P_{k-1}(t)\lambda\delta t$$

$$\frac{P_k(t+\delta t) - P_k(t)}{\delta t} = -\lambda P_k(t) + \lambda P_{k-1}(t)$$

$$\frac{dP_k(t)}{dt} = -\lambda P_k(t) + \lambda P_{k-1}(t) \; ; \; k = 0,1,2, \ldots (3.1.2)$$

Penyelesaian persamaan (3.1.2) diperoleh dengan memakai metoda fungsi pembangkit.

Misalkan fungsi pembangkit yang berhubungan dengan $P_k(t)$ adalah :

$$G_k(s) = \sum_{k=o}^{\sim} s^k P_k(t) \qquad (3.2.3)$$

Persamaan (3.1.2) dikalikan dengan s^k, kemudian dijumlahkan atas k, diperoleh :

$$\sum_{k=o}^{\sim} \frac{s^k \, dP_k(t)}{dt} = \sum_{k=o}^{\sim} s^k(-\lambda P_k(t) + \lambda P_{k-1}(t)) \qquad (3.1.4)$$

Ternyata persamaan (3.1.4) merupakan turunan pertama fungsi pembangkit (3.1.3)

$$\frac{d\,G_k(s)}{dt} = \sum_{k=o}^{\sim} s^k \frac{d\,P_k(t)}{dt}$$

$$= \sum_{k=o}^{\sim} \lambda\, s^k\,(-P_k(t) + P_{k-1}(t)$$

$$= -\lambda \sum_{k=o}^{\sim} s^k\, P_k(t) + \lambda \sum_{k=1}^{\sim} s^k\, P_{k-1}(t)$$

$$= -\lambda \sum_{k=o}^{\sim} s^k\, P_k(t) + \lambda s \sum_{k=1}^{\sim} s^{k-1}\, P_{k-1}(t)$$

$$= -\lambda \sum_{k=o}^{\sim} s^k\, P_k(t) + \lambda s \sum_{k=o}^{\sim} s^k\, P_k(t)$$

$$= \lambda(s-1) \sum_{k=o}^{\sim} s^k\, P_k(t)$$

$$= \lambda(s-1)\; G_k\;(s) \qquad (3.1.5)$$

$$\frac{d\,G_k(s)}{G_k(s)} = \lambda(s-1)\;dt$$

$$\ln\,G_k(s) = \lambda t(s-1)$$

Dari persamaan (3.1.5) diperoleh $G_k(s) = c\;e^{\lambda t(s-1)}$ (3.1.6)

Untuk $t = 0$ maka $G_k(s) = 1$ diperoleh $c = 1$

dan persamaan (3.1.6) menjadi $G_k(s) = e^{\lambda t(s-1)}$ (3.1.7)

Distribusi peluang $P_k(t)$ merupakan koefisien dari s^k dalam $G_k(s)$ persamaan (3.1.8) diperoleh

$$G_k(s) = e^{\lambda t(s-1)}$$

$$= e^{-\lambda t} \sum_{k=o}^{\infty} \frac{(\lambda ts)^k}{k\,!}$$

$$= e^{-\lambda t} \left[1 + \lambda ts + \frac{(\lambda ts)^2}{2\,!} + \ldots + \frac{(\lambda + s)^n}{n\,!} + \ldots \right]$$

$$= \sum_{k=o}^{\infty} \frac{e^{-\lambda t}(\lambda ts)^k}{k\,!}$$

$$= \sum_{k=o}^{\infty} \frac{e^{-\lambda t}\,(\lambda t)^k\, s^k}{k\,!}$$

Jadi $P_k(t) = \dfrac{e^{-\lambda t}\,(\lambda t)^k}{k\,!}$, $k = 0, 1, 2$ (3.1.8)

merupakan peluang mendapat k kecelakaan dalam waktu t, dan peluang mendapat k kecelakaan per satuan waktu adalah :

$$P_k(t) = \frac{e^{-\lambda}\,\lambda^k}{k\,!} \quad ; k = 0, 1, 2 \qquad (3.1.9)$$

3.2. Distribusi Binomial Negatif

Banyak cara penurunan distribusi Binomial Negatif. Dalam kecelakaan lalu lintas jalan raya ada dua hipotesa yang dapat digunakan untuk menurunkan distribusi di atas yaitu :

1. Hipotesa proneness
2. Hipotesa liability

Kedua hipotesa tersebut terpakai secara luas meski tak pernah terdefinisi secara memuaskan.

Proneness berhubungan dengan faktor pengemudi, sedangkan liability berhubungan dengan semua faktor penyebab kecelakaan. Jadi proneness merupakan hal yang khusus dari liability.

1. Distribusi Binomial Negatif Proneness

Distribusi ini diturunkan berdasarkan pernyataan yang menyatakan bahwa mean banyaknya kecelakaan per satuan waktu (λ) berubah-ubah.

Distribusi Poisson bersyarat

$$P(k/\lambda) = \frac{e^{-\lambda} \lambda^{k}}{k\,!} \quad ; k = 0, 1, 2 \qquad (3.2.1)$$

Dan distribusi λ, merupakan distribusi Gamma.

$$dF = \frac{c^{r}}{\Gamma\,(r)} e^{-c\lambda} \lambda^{r-1}\, d\lambda \; ; \qquad (3.2.2)$$

$$0 \leq \lambda < \infty \,, \; c > 0, \; r > 0$$

Dari persamaan (3.2.1) dan (3.2.2) diperoleh

$P(k) = P\,(k/\lambda)\; P(\lambda)$

$$= \int_{o}^{\infty} \frac{e^{-\lambda}\lambda^{k}}{k\,!} \frac{c^{r}}{\Gamma\,(r)} e^{-c\lambda} \lambda^{r-1}\, d\lambda \; ; k = 0, 1, 2, \ldots$$

$$= \frac{c^r}{\Gamma(r)k!} \int_0^\infty e^{-\lambda(c+1)} \lambda^{k+r-1} \, d\lambda \; ; \quad k = 0, 1, 2, \ldots$$

Misalkan $\lambda(c+1) = \mu$, maka $d\mu = (c+1)\, d\lambda$

$$P(k) = \frac{c^r}{\Gamma(r)k!} \int_0^\infty e^{-\mu} \left(\frac{\mu}{c+1}\right)^{k+r-1} \left(\frac{d\mu}{c+1}\right)$$

$$= \frac{c^r}{\Gamma(r)k!} \frac{1}{(c+1)^{k+r}} \int_0^\infty e^{-\mu} \mu^{k+r-1} \, d\mu$$

$$= \frac{\Gamma(k+r)}{\Gamma(r)\, k!} \left(\frac{c}{c+1}\right)^r \left(\frac{1}{c+1}\right)^k$$

$$= \frac{(k+r-1)!}{(r-1)!\, k!} \left(\frac{c}{c+1}\right)^r \left(\frac{1}{c+1}\right)^k$$

$$= \binom{k+r-1}{k} \left(\frac{c}{c+1}\right)^r \left(\frac{1}{c+1}\right)^k \; ; \; k = 0, 1, 2$$

$$= \frac{r(r+1) \ldots (r+k-1)}{k\,!} \left(\frac{c}{c+1}\right)^r \left(\frac{1}{c+1}\right)^k$$

$$= (-1)^k \frac{-r(-r-1)(-r-2) \ldots (-r-k+1)}{k\,!} \left(\frac{c}{c+1}\right)^r \left(\frac{1}{c+1}\right)^k \qquad (3.2.3)$$

$$= (-1)^k \binom{-r}{k} \left(\frac{c}{c+1}\right)^r \left(\frac{1}{c+1}\right)^k$$

$$= \binom{-r}{k} \left(\frac{c}{c+1}\right)^r \left(-\frac{1}{c+1}\right)^k$$

Persamaan (3.2.3) dikenal sebagai distribusi binomial negatif yang sebelumnya dikenal sebagai distribusi pertanggungan tak sama

Mean dan variansi dapat diperoleh dari penurunan fungsi pembangkit dengan mengambil s = 1. Fungsi pembangkit dari distribusi Binomial negatif adalah :

$$G(s) = \sum_{k=o}^{\sim} s^k \; P(k)$$

$$= \sum_{k=o}^{\sim} s^k \binom{-r}{k} \left(\frac{c}{c+1}\right)^r \left(-\frac{1}{c+1}\right)^k$$

$$= \left(\frac{c}{c+1}\right)^r \sum_{k=o}^{\sim} \binom{-r}{k} \left(-\frac{s}{c+1}\right)^k$$

$$= \left(\frac{c}{c+1}\right)^r \left(1 - \frac{s}{c+1}\right)^{-r} \qquad (3.2.5)$$

$$\text{Mean} = \mu_1'(k) = E(X) = \left.\frac{d\,G(s)}{ds}\right|_{s=1} = \frac{r}{c} \qquad (3.2.6)$$

$$u_2'(k) = E(X(X-1)) = \left.\frac{d^2\,G(s)}{ds^2}\right|_{s=1} = \frac{r}{c}\left(\frac{r+1}{c}\right) \qquad (3.2.7)$$

$$\text{Variansi} = E(X(X-1)) + E(X) - (E(X))^2$$

$$= \frac{r\,(c+1)}{c^2} \qquad (3.2.8)$$

2. Distribusi Binomial Negatif Liability

Distribusi ini diturunkan berdasarkan liability yang menyatakan bahwa peluang mendapat kecelakaan mula-mula sama dan berubah setelah k kecelakaan, peluang untuk mendapatkan satu kecelakaan lagi setelah k kecelakaan adalah f(k,t), f(k,t) merupakan fungsi pertanggungan monoton naik dari k.

Kita menggunakan ;

(a) $P(1 \text{ kecelakaan dalam waktu } (t,t+\delta t)) = f(k,t)\delta t$

(b) $P(0 \text{ kecelakaan dalam waktu } (t,t+\delta t)) = 1 - f(k,t)\delta t$

(c) $P(2 \text{ kecelakaan atau lebuh dalam waktu } (t,t+\delta t)) = o(\delta t)$

(d) Kecelakaan dalam selang waktu yang berbeda bebas satu dengan yang lainnya.

dengan

f(k,t) = fungsi pertanggungan monoton linier naik

$o(\delta t)$ = suatu harga yang bernilai kecil yang bisa diabaikan

$P(k,t+\delta t)$ = P(k kecelakaan dalam waktu $(0,t+\delta t)$)
= P(k kecelakaan dalam waktu (0,t) dan tak ada kecelakaan dalam waktu $(t,t+\delta t)$) +
P((k-1) kecelakaan dalam waktu $(t,t+\delta t)$) +
P((k-i) kecelakaan dalam waktu (0,t) dan i kecelakaan dalam waktu $(t,t+\delta t)$) ; $i \geq 2$

dari asumsi di atas diperoleh :

$$p(k,t+\delta t) = p(k,t)(1-f(k,t)\delta t) + p(k-1,t)f(k-1,t)\delta t + p(k-i,t)o(\delta t)$$

$$= p(k,t)(1-f(k,t)\delta t) + p(k-1,t)f(k-1,t)\delta t \quad (3.2.9)$$

$$p(0,t+\delta t) = p(0,t)\,[1 - f(0,t)\delta t] + o\delta t$$

Kedua sisi persamaan (3.2.9) dikurangi $p(k,t)$, kemudian dibagi δt dan ambil δt menuju 0 diperoleh :

$$\begin{cases} p(k,t+\delta t) - p(k,t) = p(k,t)\delta t - f(k,t)\delta t\, p(k,t) \\ \qquad\qquad + p(k-1,t)f(k-1,t)\delta t - p(k,t) \\ \\ p(o,t+\delta t - p(o,t) = p(o,t)\delta t - f(o,t)\delta t\, p(o,t) - p(k,t) \end{cases}$$

$$\frac{p(k,t+\delta t) - p(k,t)}{\delta t} = -p(k,t)f(k,t)+p(k-1,t)f(k-1,t)\delta t$$

$$\frac{p(0,t+\delta t) - p(0,t)}{\delta t} = -p(0,t)f(0,t)\delta t$$

$$\left.\begin{aligned} \frac{\partial p\,(k,t)}{\partial t} &= -p(k,t)f(k,t)+p(k-1,t)f(k-1,t)\ ; \\ & k = 1,2,\ldots. \\ \frac{\partial p\,(0,t)}{\partial t} &= -p(0,t)f(0,t) \end{aligned}\right\} \quad (3.2.10)$$

Dalam hal jika $f(k,t) = \beta + \gamma k$; $\beta>0$, $\gamma>0$ persamaan (3.2.10) menjadi :

$$\left.\begin{aligned} \frac{\partial p\,(k,t)}{\partial t} &= -p(k,t)f(\beta+\gamma k)+p(k-1,t)(\beta+\gamma(k-1)) \\ \frac{\partial p\,(0,t)}{\partial t} &= -\beta p(0,t) \end{aligned}\right\} (3.2.11)$$

Penyelesaian persamaan (3.2.11) diperoleh dengan memakai metode fungsi pembangkit.

Misalkan fungsi pembangkit yang berhubungan dengan p(k,t) adalah :

$$G(k,s) = \sum_{k=o}^{\infty} s^k \ (\beta+\gamma k)p(k,t) \qquad (3.2.12)$$

Persamaan (3.2.11) dikalikan dengan s^k, kemudian dijumlahkan atas k maka diperoleh

$$\left.\begin{aligned} \sum_{k=o}^{\infty} s^k \frac{\partial p(k,t)}{\partial t} &= \sum_{k=o}^{\infty} -s^k \ (\beta+\gamma k)p(k,t) \ + \\ &\quad \sum_{k=o}^{\infty} s^k \ (\beta + \gamma(k-1))p(k-1,t) \\ \frac{\partial p(0,t)}{\partial t} &= - \beta p(0,t) \end{aligned}\right\} (3.1.13)$$

Persamaan (3.2.13) dijumlahkan diperoleh :

$$\begin{aligned} \frac{\partial p(0,t)}{\partial t} + \sum_{k=o}^{\infty} s^k \frac{\partial p(k,t)}{\partial t} &= -\beta p(0.t) \\ &\quad -\sum_{k=o}^{\infty} s^k \ (\beta+\gamma k)p(k,t) \ + \\ &\quad s \sum_{k=1}^{\infty} s^{k-1}(\beta+\gamma(k-1))p(k-1,t) \end{aligned}$$

$$\sum_{k=o}^{\infty} s^k \frac{\partial p(k,t)}{\partial t} = -\sum_{k=o}^{\infty} s^k (\beta+\gamma k)p(k,t) +$$

$$s\sum_{k=o}^{\infty} s^k (\beta + \gamma k)p(k,t)$$

$$= (s-1)\sum_{k=o}^{\infty} s^k (\beta+\gamma k)p(k,t) \qquad (3.2.14)$$

Persamaan (3.2.14) dikalikan $(\beta+\gamma k)$ diperoleh :

$$\sum_{k=o}^{\infty} s^k \frac{\partial p(k,t)}{\partial t}(\beta+\gamma k) = (s-1)\sum_{k=o}^{\infty} s^k (\beta+\gamma k)^2 p(k,t) \qquad (3.2.15)$$

Ternyata persamaan (3.2.15) merupakan turunan pertama fungsi pembangkit (3.2.12),

$$\frac{\partial G(k,s)}{\partial t} = \sum_{k=o}^{\infty} s^k(\beta+\gamma k) \frac{\partial p(k,t)}{\partial t}$$

$$= (s-1)\sum_{k=o}^{\infty} s^k (\beta+\gamma k)p(k,t)(\beta+\gamma k)$$

$$= (s-1)\left[\sum_{k=o}^{\infty} \beta s^k (\beta+\gamma k)p(k,t) + \sum_{k=o}^{\infty} \gamma k s^k (\beta+\gamma k)p(k,t)\right]$$

$$= (s-1)\left[\sum_{k=o}^{\infty} \beta s^k \ (\beta+\gamma k)p(k,t) + \right.$$

$$\left. \gamma s \sum_{k=1}^{\infty} ks^{k-1} \ (\beta+\gamma k)p(k,t) \right]$$

$$\frac{\partial G(k,s)}{\partial t} = (s-1)\left[\beta G(k,s) + \gamma s \frac{\partial G(k,s)}{\partial s} \right] \quad (3.2.16)$$

Persamaan (3.2.16) merupakan differensial parsial linier berbentuk $P_p + Q_q = R$

dengan,

$$p = \frac{\partial G(k,s)}{\partial t} \ , \ P = 1 \ , \ q = \frac{\partial G(k,s)}{\partial s} \ ,$$

$$Q = -\gamma s(s-1) \ , \ R = (s-1)\beta G(k,s)$$

Dapat diselesaikan dengan sistem pembantu (sistem Lagrange).

$$\frac{dt}{p} = \frac{ds}{Q} = \frac{dG(k,s)}{R}$$

$$\frac{dt}{1} = \frac{ds}{-\gamma s(s-1)} = \frac{dG(k,s)}{(s-1)\beta G(k,s)} \quad (3.2.17)$$

Dari persamaan (3.2.17) diperoleh :

$$\frac{dt}{1} = \frac{ds}{-\gamma s(s-1)}$$

$$\gamma dt = \frac{ds}{s} - \frac{ds}{s-1}$$

Diperoleh

$$C_1 = e^{\gamma t}\left(\frac{s-1}{s}\right) \qquad (3.2.18)$$

Dari persamaan (3.2.17) juga diperoleh :

$$\frac{ds}{-\gamma s(s-1)} = \frac{dG(k,s)}{(s-1)\beta G(k,s)}$$

$$\frac{dG(k,s)}{G(k,s)} = -\frac{\beta}{\gamma}\frac{ds}{s}$$

Diperoleh $G(k,s) = c_2 s^{-\beta/\gamma}$ (3.2.19)

C_2 merupakan fungsi dari C_1 maka persamaan (3.2.19) menjadi

$$G(k,s) = s^{-\beta/\gamma}\,\emptyset\left(e^{\gamma t}\left(\frac{s-1}{s}\right)\right) \qquad (3.2.20)$$

Pada saat t = 0 maka $G(k,s) = 1 = s^{-\beta/\gamma}\,\emptyset\left(e^{\gamma t}\left(\frac{s-1}{s}\right)\right)$

Jadi:

$$\emptyset\left(\frac{s-1}{s}\right) = s^{\beta/\gamma} \qquad (3.2.21)$$

Misalkan $\frac{s-1}{s} = \theta$, maka $s = \frac{1}{1-\theta}$ substitusi ke persamaan (3.2.21) diperoleh $\emptyset(\theta) = \left(\frac{1}{1-\theta}\right)^{\beta/\gamma}$

kemudian ganti θ dengan $e^{\gamma t}\left(\frac{s-1}{s}\right)$ diperoleh

$$\phi\left[e^{\gamma t}\left(\frac{s-1}{1}\right)\right] = \left(\frac{1}{1-e^{\gamma\beta}\left(\frac{s-1}{s}\right)}\right)^{\beta/\gamma}$$

Substitusi kembali pada persamaan (3.2.20) diperoleh

$$G(k,s) = s^{-\beta/\gamma}\left(\frac{1}{1-e^{\gamma\beta}\left(\frac{s-1}{s}\right)}\right)^{\beta/\gamma}$$

$$= s^{-\beta/\gamma}\left[1-e^{\gamma t}\left[\left(\frac{s-1}{1}\right)\right]\right]^{-\beta/\gamma}$$

$$= (s-e^{\gamma t}(s-1))^{-\beta/\gamma}$$

$$= (s-e^{\gamma t}\ s+e^{\gamma t})^{-\beta/\gamma}$$

$$= (e^{\gamma t}-s(e^{\gamma t}-1))^{-\beta/\gamma} \qquad (3.2.22)$$

Distribusi peluang p(k,t) merupakan koefisien dari s^k dalam fungsi pembangkit G(k,s) persamaan (3.2.22)

$$G(k,s) = (e^{\gamma t} - s(e^{\gamma t}-1))^{-\beta/\gamma}$$

$$= \left[1-s\frac{e^{\gamma t}-1}{e^{\gamma t}}\right]^{-\beta/\gamma}(e^{\gamma t})^{-\beta/\gamma}$$

$$= \sum_{k=o}^{\infty}\binom{-\beta/\gamma}{k}\left[-\frac{s(e^{\gamma t}-1)}{e^{\gamma t}}\right]^{k} e^{-\beta t}$$

$$= \sum_{k=o}^{\infty}\binom{-\beta/\gamma+k-1}{k}(1-e^{\gamma t})^{k}\, e^{-\beta t}\, s^{k}$$

maka

$$p(k,t) = \begin{bmatrix} \beta/\gamma+k-1 \\ k \end{bmatrix} (1-e^{\gamma t})^{k}\ e^{-\beta t} \qquad (3.2.23)$$

Persamaan (3.2.23) merupakan distribusi binomial negatif seperti persamaan (3.2.3) dengan $r = \beta/\gamma$ dan $c = (e^{\gamma t}-1)$ ternyata distribusi Binomial negatif baik yang diturunkan oleh hipotesa proneness maupun liability tidak ada bedanya dalam satu peubah.

3.3. Distribusi Binomial Negatif Dua Peubah

Pada distribusi binomial negatif satu peubah kita tidak dapat membedakan antara hipotesa proneness dengan liability.

Untuk itu kita membagi periode waktu dalam dua bagian. Misal selang waktu $(0,t) = \delta_1$ dan $(t,T) = \delta_2$, kemudian dimisalkan banyaknya kecelakaan dalam masing-masing selang waktu δ , δ_1, dan δ_3 adalah k, k_1, k_2 dan $k = k_1 + k_2$

1. Distribusi Binomial negatif dua peubah proneness

Penurunan distribusi ini sama seperti penurunan distribusi Binomial negatif satu peubah.

Distribusi Poisson bersyarat

$$P(k_i/\lambda) = \frac{e^{-\lambda\delta_i}\ (\lambda\ \delta_i)^{k_i}}{k_i\ !},\ i = 1,\ 2 \qquad (3.3.1)$$

Dan distribusi λ berdistribusi gamma

$$dF = \frac{c^r}{\Gamma(r)} e^{-c\lambda} \lambda^{r-1} d\lambda \qquad (3.3.2)$$

$0 \leq \lambda (\infty , c . 0, r . 0$

Dari persamaan(3.3.1) dan ((3.3.2) diperoleh

$$P(k_i) = P(k_i/\lambda) P(\lambda)$$

$$= \int_0^\infty \frac{e^{-\lambda \delta_i} (\lambda \delta_i)}{k_i !} \frac{c^r}{\Gamma(r)} e^{-c\lambda} \lambda^{r-1} d\lambda$$

$$= \frac{c^r \delta_i^{k_i}}{\Gamma(r) k_i !} \int_0^\infty e^{-\lambda (c + \delta_i)} \lambda^{k_i + r-1} d\lambda$$

Misalkan $\mu = \lambda (c + \delta_i)$, maka $d\mu = (c + \delta_i) d\lambda$

$$P(k_i) = \frac{c^r \delta_i^{k_i}}{\Gamma(r) k_i !} \int_0^\infty e^{-\mu} \left(\frac{\mu}{c + \delta_i} \right)^{k_i + r-1} \left(\frac{d\mu}{c + \delta_i} \right)$$

$$P(k_i) = \frac{c^r \delta_i^{k_i}}{\Gamma(r) k_i !} \left(\frac{1}{c + \delta_i} \right)^{k_i + r-1} \int_0^\infty e^{-\mu} \mu^{k_i + r-1} d\mu$$

$$P(k_i) = \frac{\Gamma(k_i + r)}{\Gamma(r) k_i !} \left(\frac{c}{c + \delta_i} \right)^r \left(\frac{\delta_i}{c + \delta_i} \right)^{k_i} ; i = 1, 2$$

$$P(k_i) = \binom{k_i + r-1}{k_i} \left(\frac{c}{c + \delta_i} \right)^r \left(\frac{\delta_i}{c + \delta_i} \right)^{k_i} ; i = 1, 2$$

(3.3.3.)

$$P(k_1) = \binom{k_1 + r - 1}{k_1} \left[\frac{c}{c + \delta_1}\right]^r \left[\frac{\delta_1}{c + \delta_1}\right]^{k_1}$$

$$P(k_2) = \binom{k_2 + r - 1}{k_2} \left[\frac{c}{c + \delta_2}\right]^r \left[\frac{\delta_2}{c + \delta_2}\right]^{k_2}$$

$$P(k) = \binom{k + r - 1}{k} \left[\frac{c}{c + \delta}\right]^r \left[\frac{\delta}{c + \delta}\right]^{k}$$

$P(k_1)$ dan $P(k_2)$ merupakan distribusi marginal Binomial negatif.

Distribusi Binomial negatif dua peubah dapat diturunkan dari distribusi Poisson bersyarat

$$P(k_1, k_2/\lambda) = \frac{e^{-\lambda\delta_1} (\lambda\, \delta_1)^{k_1}}{k_1\, !} \; \frac{e^{-\lambda\delta_2} (\lambda\, \delta_2)^{k_2}}{k_2\, !} \qquad (3.3.4)$$

dengan distribusi λ yang berdistribusi gamma

$$P(\lambda) = \frac{c^r}{P(r)} e^{-c\lambda} \lambda^{r-1}\, d\lambda \qquad (3.3.5)$$

$$0 \le \lambda < \infty \,,\; c > 0 ,\; r > 0$$

diperoleh

$$P(k_1, k_2) = \int_0^\infty \frac{e^{-\lambda\delta_1}(\lambda\, \delta_1)^{k_1}}{k_1\, !} \; \frac{e^{-\lambda\delta_2}(\lambda\, \delta_2)^{k_2}}{k_2\, !} \; \frac{c^r}{\Gamma(r)} e^{-c\lambda} \lambda^{r-1}\, d\lambda$$

$$= \frac{c^r\, \delta_1^{k_1}\, \delta_2^{k_2}}{\Gamma(r)\, k_1\, !\, k_2\, !} \int_0^\infty e^{-\lambda(c+\delta_1+\delta_2)} \lambda^{k_1+k_2+r-1}\, d\lambda$$

Misalkan $\mu = \lambda\,(c + \delta_1 + \delta_2)$, maka $d\mu = (c + \delta_1 + \delta_2)\,d\lambda$

$$P(k_1, k_2) = \frac{c^r \delta_1^{k_1} \delta_2^{k_2}}{\Gamma(r)\, k_1!\, k_2!} \int_0^{\infty} e^{-\mu} \left[\frac{\mu}{c+\delta_1+\delta_2}\right]^{k_1+k_2+r-1} \left[\frac{d\mu}{c+\delta_1+\delta_2}\right]$$

$$= \frac{c^r \delta_1^{k_1} \delta_2^{k_2}}{\Gamma(r)\, k_1!\, k_2!} \left[\frac{1}{c+\delta_1+\delta_2}\right]^{k_1+k_2+r} \int_0^{\infty} e^{-\mu} \mu^{k_1+k_2+r-1}\, d\mu$$

$$= \frac{\Gamma(k_1+k_2+r)\, \delta_1^{k_1} \delta_2^{k_2}}{\Gamma(r)\, k_1!\, k_2!}\, c^r \left[\frac{1}{c+\delta_1+\delta_2}\right]^{k_1+k_2+r}$$

karena $k_1 + k_2 = k$ dan $\delta_1 + \delta_2 = \delta$, maka

$$P(k_1, k_2) = \frac{\Gamma(r+k)\, \delta_1^{k_1} \delta_2^{k_2}}{\Gamma(r)\, k_1!\, k_2!} \left[\frac{c}{c+\delta}\right]^r \left[\frac{1}{c+\delta}\right]^k \tag{3.3.6}$$

Fungsi pembangkit dari distribusi Binomial negatif dua peubah :

$$G(s_1, s_2) = \sum_{k=0}^{\infty} \sum_{k_1=0}^{k} s_1^{k_1} s_2^{k_2} \frac{\Gamma(r+k)\, \delta_1^{k_1} \delta_2^{k_2}}{\Gamma(r)\, k_1!\, k_2!} \frac{c}{(c+\delta)^{r+k}}$$

$$= \left[\frac{c}{c+\delta}\right]^r \sum_{k=0}^{\infty} \sum_{k_1=0}^{k} \binom{r+k-1}{k} \binom{k}{k_1} \frac{(\delta_1 s_1)^{k_1} (\delta_2 s_2)^{k_2}}{(c+\delta)^k}$$

$$= \left(\frac{c}{c+\delta}\right)^{r} \sum_{k=o}^{\infty} \binom{r+k-1}{k} \frac{1}{(c+\delta)^{k}}$$

$$\sum_{k_1=o}^{k} \binom{k}{k_1} (\delta_1 s_1)^{k_1} (\delta_2 s_2)^{k-k_1}$$

$$= \left(\frac{c}{c+\delta}\right)^{r} \sum_{k=}^{\infty} \binom{-r}{k} \left(-\frac{1}{(c+\delta)}\right)^{k} (\delta_1 s_1 + \delta_2 s_2)^{k}$$

$$= \left(\frac{c}{c+\delta}\right)^{r} \sum_{k=o}^{\infty} \binom{-r}{k} \left(-\frac{\delta_1 s_1 + \delta_2 s_2}{c+\delta}\right)^{k}$$

$$= \left(\frac{c}{c+\delta}\right)^{r} \left(1 - \frac{\delta_1 s_1 + \delta_2 s_2}{c+\delta}\right)^{-r} \qquad (3.3.7)$$

Dari fungsi pembangkit (3.3.7) diperoleh momen-momennya

$$\text{Mean } (k_1) = \mu_1' \; (k_1) = \frac{\partial\, G\, (s_1, s_2)}{\partial s_1} \Bigg|_{s_1 = 1,\; s_2 = 1}$$

$$= \frac{r\,\delta_2}{c} \qquad (3.3.7)$$

$$\text{Mean } (k_2) = \mu_1' \; (k_2) = \frac{\partial\, G\, (s_1, s_2)}{\partial s_2} \Bigg|_{s_1 = 1,\; s_2 = 1}$$

$$= \frac{r\,\delta_2}{c} \qquad (3.3.8)$$

$$E\, (X_1(X_1 - 1)) = \mu_2' \; (k_1) = \frac{\partial^2\, G\, (s_1, s_2)}{\partial s_1^{2}} \Bigg|_{s_1 = 1,\; s_2 = 1}$$

$$= \frac{r\ (r + 1)\ \delta_1^{\ 2}}{c^2}$$

$$E\ (X_2(X_2 - 1)) = \mu_2{}'\ (k_2) = \left.\frac{\partial^2\ G\ (s_1, s_2)}{\partial s_2^{\ 2}}\right|_{s_1 = 1,\ s_2 = 1}$$

$$= \frac{r\ (r + 1)\ \delta_2^{\ 2}}{c^2}$$

$$\text{Variansi}\ (\ k_1) = E\ (X_1(X_1 - 1)) + E\ (X_1) - (E\ (X_1))^2$$

$$= \frac{r\ \delta_1}{c}\left[1 + \frac{\delta_1}{c}\right] \qquad (3.3.10)$$

$$\text{Variansi}\ (\ k_2) = E\ (X_2(X_2 - 1)) + E\ (X_2) - (E(X_2))^2$$

$$= \frac{r\ \delta_2}{c}\left[1 + \frac{\delta_2}{c}\right] \qquad (3.3.11)$$

$$\text{Momen campuran}\ (k_1, k_2) = \mu_1{}'\ (k_1, k_2)$$

$$= \left.\frac{\delta^2\ G\ (s_1, s_2)}{\delta\ s_1\ \partial s_2}\right|_{s_1 = 1,\ s_2 = 1}$$

$$= \frac{r\ (r + 1)\ \delta_1 \delta_2}{c^2} \qquad (3.3.12)$$

$$\text{Korelasi} = \frac{\text{kovariansi}}{\sqrt{\text{perkalian standar deviasi}}}$$

$$\rho\ (k_1, k_2) = \frac{\mu_1{}'\ (k_1, k_2) - \mu_1{}'(k_1)\ \mu_1{}'(k_2)}{\sqrt{\mu_2\ (k_1)\ \mu_2\ (k_2)}}$$

$$= \frac{1}{\sqrt{\left[1 + \frac{c}{\delta_1}\right]\left[1 + \frac{c}{\delta_2}\right]}} \qquad (3.3.13)$$

2. **Distribusi Binominal Negatif dua Peubah Liability.** Penurunan distribusi ini sama dengan penurunan distribusi Binominal negatif satu peubah liability.

Analog dengan persamaan (3.2.23) diperoleh :

$$P(k) = P(k_1 + k_2) = \binom{\beta/\gamma+k-1}{k} e^{-\beta\delta} (1 - e^{-\gamma\delta})^k \qquad (3.3.14)$$

$$P(k_1) = \binom{\beta/\gamma+k_1-1}{k_1} e^{-\beta\delta_1} (1 - e^{-\gamma\delta_1})^{k_1}$$

$$P(k_2) = \binom{\beta/\gamma+k_2-1}{k_2} e^{-\beta\delta_2} (1 - e^{-\gamma\delta_2})^{k_2}$$

$P(k_1)$ dan $P(k_2)$ merupakan distribusi marginal Binomial negatif.

Distribusi Binomial negatif dua peubah hipotesa liability adalah :

$$P(k_1,k_2) = \frac{\Gamma(\beta/\gamma + k)}{\Gamma(\beta/\gamma)k_1!\ k_2!} e^{-\beta\delta} (e^{-\gamma\delta_2} - e^{-\gamma\delta})^{k_1} (1-e^{-\gamma\delta_2})^{k_2}$$

(3.3.15)

Fungsi pembangkit dari distribusi dua peubah $P(k_1,k_2)$ adalah :

$$G(s_1,s_2) = \sum_{k=0}^{\infty} \sum_{k_1=0}^{k} s_1^{k_1} s_2^{k_2} \frac{\Gamma(\beta/\gamma+k)}{\Gamma(\beta/\gamma)k_1!\ k_2!} e^{-\beta\delta} (e^{-\gamma\delta_2} - e^{-\gamma\delta})^{k_1}$$

$$X (1 - e^{-\gamma\delta_2})^{k_2}$$

$$= e^{-\beta\delta} \sum_{k=o}^{\infty} \begin{bmatrix} \beta/\gamma+k-1 \\ k \end{bmatrix} \sum_{k=o}^{\infty} \begin{bmatrix} k \\ k_1 \end{bmatrix} (s_1 e^{-\gamma\delta_2} - s_1 e^{-\gamma\delta})^{k_1}$$

$$X\ (s_2 - s_2 e^{-\gamma\delta_2})^{k - k_1}$$

$$= e^{-\beta\delta} \sum_{k=o}^{\infty} \begin{bmatrix} \beta/\gamma+k-1 \\ k \end{bmatrix} \left[(s_1 e^{-\gamma\delta_2} - s_1 e^{-\gamma\delta}) + (s_2 - s_2 e^{-\gamma\delta_2}) \right]^k$$

$$= e^{-\beta\delta} \sum_{k=o}^{\infty} \begin{bmatrix} -\beta/\gamma \\ k \end{bmatrix} \left[- \left(s_1 (e^{-\gamma\delta_2} - e^{-\gamma\delta}) + s_2 (1 - e^{-\gamma\delta_2}) \right) \right]^k$$

$$= e^{-\beta\delta} \left[1 - s_1 (e^{-\gamma\delta_2} - e^{-\gamma\delta}) - s_2 (1 - e^{-\gamma\delta_2}) \right]^{-\beta/\gamma}$$

$$= \left[e^{\gamma\delta} - s_1 (e^{\gamma\delta}\ e^{-\gamma\delta_2} - 1) - (s_2\ (e^{\gamma\delta} - e^{\gamma\delta_1}) \right]^{-\beta/\gamma}$$

$$= \left[e^{-\gamma\delta} - s_1 (e^{\gamma\delta_1} - 1) - s_2 (e^{\gamma\delta} - e^{\gamma\delta_1}) \right]^{-\beta/\gamma} \quad (3.3.16)$$

Dari fungsi pembangkit (3.3.16) diperoleh momen-momennya

$$\text{Mean } (k_1) = \mu_1{}' (k_1) = \left. \frac{\partial\ G\ (s_1, s_2)}{\partial s_1} \right|_{s_1 = 1,\ s_2 = 1} = \frac{\beta}{\gamma} (e^{\gamma\delta_1} - 1)$$

$$(3.3.17)$$

$$\text{Mean } (k_2) = \mu_1{}' (k_2) = \left. \frac{\partial\ G\ (s_1, s_2)}{\partial s_2} \right|_{s_1 = 1,\ s_2 = 1} = \frac{\beta}{\gamma} (e^{\gamma\delta} - e^{-\gamma\delta_1})$$

$$(3.3.18)$$

$$E\,(X_1(X_1-1)) = \mu_2'(k_1) = \left.\frac{\partial^2 G(s_1,s_2)}{\partial s_1^{\,2}}\right|_{s_1=1,\; s_2=1}$$

$$= \frac{\beta}{\gamma}\left(\frac{\beta}{\gamma}+1\right)(e^{\gamma\delta_1}-1)^2$$

$$E\,(X_2(X_2-1)) = \mu_2'\,(k_2) = \left.\frac{\partial^2 G\,(s_1,s_2)}{\partial s_2^{\,2}}\right|_{s_1=1,\; s_2=1}$$

$$= \frac{\beta}{\gamma}\left(\frac{\beta}{\gamma}+1\right)(e^{\gamma\delta} - e^{-\gamma\delta_1})^2$$

$$\text{Variansi}\,(\,k_1) = \mu_2\,(k_1) = \mu_2'(k_1) + \mu_1'(k_1) - (\,\mu_1'(k_1))^2$$

$$= \frac{\beta}{\gamma}\left(e^{\gamma\delta_1}-1\right)e^{\gamma\delta_1} \qquad (3.3.19)$$

$$\text{Variansi}\,(\,k_2) = \mu_2\,(k_2) = \mu_2'(k_2) + \mu_1'(k_2) - (\,\mu_1'(k_2))^2$$

$$= \frac{\beta}{\gamma}\left(e^{\gamma\delta} - e^{\gamma\delta_1}\right)\left(\,e^{\gamma\delta} - e^{\gamma\delta_1}+1\right)$$

(3.3.20)

$$\text{Momen campuran}\,(k_1,k_2) = \mu_1'\,(k_1,k_2)$$

$$= \left.\frac{\delta^2\,G\,(s_1,s_2)}{\delta\,s_1\,\partial s_2}\right|_{s_1=1,\; s_2=1}$$

$$= \frac{\beta}{\gamma}\left(\frac{\beta}{\gamma}+1\right)\left(e^{\gamma\delta} - e^{\gamma\delta_1}\right)\left(e^{\gamma\delta_1}-1\right)$$

(3.3.21)

$$\text{Korelasi} = \rho\ (k_1,k_2) = \frac{\mu_1{}'(k_1,k_2) - \mu_1{}'(k_1)\ \mu_1{}'(k_2)}{\sqrt{\mu_2\ (k_1)\ \mu_2\ (k_2)}}$$

$$\rho\ (k_1,k_2) = \frac{1}{\sqrt{\left(1 + \frac{1}{e^{\gamma\delta_1} - 1}\right)\left(1 + \frac{1}{e^{\gamma\delta_1}(e^{\gamma\delta_2} - 1)}\right)}}$$

(3.3.22)

Ternyata dua peubah, distribusi Binomial negatif berdasarkan hipotesa proneness dengan distribusi Binomial negatif berdasarkan hipotesa liability ada bedanya.

Parameter r dan c atau β dan γ dapat diperoleh dari mean dan variansi pengamatan.

3.4. Distribusi Panjang (Long Distribution) dan Distribusi Pendek (Short Distribution)

1. Distribusi Panjang

Distribusi ini diturunkan berdasarkan konsep bahwa pengemudi sebagai mahluk yang efisiensinya berubah-ubah, pada saat-saat dimana efisiensi dari pengemudi itu rendah kita namakan Spells. Spells dapat disebabkan oleh faktor-faktor yang sifatnya sementara seperti kesehatan terganggu.

Pandang bahwa pengemudi merupakan subyek Spells dan tak ada kecelakaan terjadi di luar Spells. Banyak Spells dalam perioda yang berbeda saling bebas. Kita juga memandang bahwa semua pengemudi berpeluang sama mendapat Spells.

Misalkan λ rata-rata (mean) banyaknya Spells per pengemudi dalam waktu tertentu dan misal θ rata-rata (mean) banyaknya kecelakaan per Spells.

Fungsi pembangkit untuk banyaknya kecelakaan per Spells berdistribusi Poisson dengan parameter θ adalah $e^{\theta(s-1)}$ dan fungsi pembangkit untuk banyaknya kecelakaan dalam k Spells adalah $e^{k\theta(s-1)}$.

Dari sini diperoleh fungsi pembangkit untuk distribusi kecelakaan yang distribusinya berbentuk Compound Poisson.

Fungsi pembangkit untuk distribusi kecelakaan adalah :

$$G(s) = \sum_{k=o}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} e^{k\theta(s-1)} \qquad (3.4.1)$$

$$= \sum_{k=o}^{\infty} e^{-\lambda} \frac{(\lambda e^{\theta(s-1)})^k}{k!}$$

$$= e^{-\lambda} e^{\lambda e^{\theta(s-1)}}$$

$$= e^{\lambda (e^{\theta(s-1)} - 1)} \qquad (3.4.2)$$

$$= e^{\lambda (e^{-\theta} - 1)} e^{\lambda e^{-\theta} (e^{\theta s} - 1)}$$

$$= e^{\lambda (e^{-\theta} - 1)} \sum_{k=o}^{\infty} \frac{(\lambda e^{-\theta}(e^{\theta s} - 1))^k}{k!}$$

$$= e^{\lambda (e^{-\theta} - 1)} \sum_{k=o}^{\infty} \frac{(\lambda e^{-\theta})^k}{k!} (e^{\theta s} - 1)^k$$

$$= e^{\lambda (e^{-\theta} - 1)} \sum_{k=o}^{\infty} \frac{(\lambda e^{-\theta})^k}{k!} \sum_{m=o}^{\infty} \binom{k}{m} (e^{\theta s})^m (-1)^{k-m}$$

$$= e^{\lambda(e^{-\theta}-1)} \sum_{k=o}^{\infty} \frac{(\lambda e^{-\theta})^k}{k!} \sum_{m=o}^{\infty} \begin{bmatrix} k \\ m \end{bmatrix} (-1)^{k-m} \sum_{r=o}^{\infty} \frac{(\theta s m)^r}{r!}$$

$$= e^{\lambda(e^{-\theta}-1)} \sum_{k=o}^{\infty} \frac{(\lambda e^{-\theta})^k}{k!} \sum_{r=o}^{\infty} \frac{\theta^r s^r}{r!} \sum_{m=o}^{\infty} \begin{bmatrix} k \\ m \end{bmatrix} (-1)^{k-m} m^r$$

$$= e^{\lambda(e^{-\theta}-1)} \sum_{r=o}^{\infty} \frac{\theta^r s^r}{r!} \sum_{k=o}^{\infty} \frac{\Delta^k (0^r)}{k!} (\lambda e^{-\theta})^k \qquad (3.4.3)$$

dengan

$$\Delta^k(0^r) = \sum_{m=o}^{k} \begin{bmatrix} k \\ m \end{bmatrix} (-1)^{k-m} m^r$$

Distribusi peluang P(k) merupakan koefisien dari s^r dalam G(s) persamaan (3.4.3) diperoleh

$$P(r) = e^{\lambda(e^{-\theta}-1)} \sum_{k=o}^{\infty} \frac{\Delta^k (0^r)\, \theta^r}{k!\, r!} (\lambda e^{-\theta})^k \qquad (3.4.4)$$

Distribusi ini oleh Creswell dan Froggat dinamakan Long Distribution (Distribusi Panjang).

$$P(0) = e^{\lambda(e^{-\theta}-1)} \underbrace{\sum_{k=o}^{o} \frac{\Delta^k (0^0)\, \theta^0}{k!\ 0!} (\lambda e^{-\theta})^k}_{A}$$

$$A = \frac{\Delta^0 (0^0)\, \theta^0}{0!\ 0!} (\lambda e^{-\theta})^0$$

$$\Delta^0 (0^0) = \sum_{m=o}^{o} \begin{bmatrix} 0 \\ m \end{bmatrix} (-1)^{0-m} m^0$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} (-1)^{0-0}\, 0^0 = 1$$

$$A = 1$$

$$P(0) = e^{\lambda(e^{-\theta}-1)}$$

$$P(1) = e^{\lambda(e^{-\theta}-1)} \underbrace{\sum_{k=0}^{1} \frac{\Delta^k (0^1)\, \theta^1}{k!\ 1!} (\lambda e^{-\theta})^k}_{A}$$

$$A = \frac{\Delta^0 (0^1)\theta^1}{0!\ 1!} (\lambda e^{-\theta})^0 + \frac{\Delta^1 (0^1)\theta^1}{1!\ 1!} (\lambda e^{-\theta})^1$$

$$\Delta^0 (0^1) = \sum_{m=0}^{0} \binom{0}{m} (-1)^{0-m} m^1$$

$$= \binom{0}{0} (-1)^{0-0} 0^1 = 0$$

$$\Delta^1 (0^1) = \sum_{m=0}^{1} \binom{1}{m} (-1)^{1-m} m^1$$

$$= \binom{1}{0} (-1)^{1-0} 0^1 + \binom{1}{1} (-1)^{1-1} 1^1$$

$$= 0 + 1 = 1$$

$$A = \theta\ \lambda e^{-\theta}$$

$$P(1) = P(0)\ \theta\ \lambda e^{-\theta}$$

$$P(2) = e^{\lambda(e^{-\theta}-1)} \underbrace{\sum_{k=0}^{2} \frac{\Delta^k (0^2)\, \theta^2}{k!\ 2!} (\lambda e^{-\theta})^k}_{A}$$

$$A = \frac{\Delta^0 (0^2)\theta^2}{0!\ 2!} (\lambda e^{-\theta})^0 + \frac{\Delta^1 (0^2)\theta^2}{1!\ 2!} (\lambda e^{-\theta})^1 + \frac{\Delta^2 (0^2)\theta^2}{2!\ 2!} (\lambda e^{-\theta})^2$$

$$\Delta^0 (0^2) = \sum_{m=0}^{0} \binom{0}{m} (-1)^{0-m} m^2$$

$$= \binom{0}{0} (-1)^0 0^2$$

$$= 0$$

$$\Delta^1 (0^2) = \sum_{m=0}^{1} \binom{1}{m} (-1)^{1-m} m^2$$

$$= \binom{1}{0} (-1)^1 0^2 + \binom{1}{1} (-1)^0 1^2$$

$$= 0 + 1 = 1$$

$$\Delta^2(0^2) = \sum_{m=0}^{2} \binom{2}{m} (-1)^{2-m} m^2$$

$$= \binom{2}{0}(-1)^2 0^2 + \binom{2}{1}(-1)^1 1^2 + \binom{2}{2}(-1)^0 2^2$$

$$= 0 - 2 + 4 = 2$$

$$A = \frac{\theta^2 \lambda e^{-\theta}}{2!} + \frac{2\theta^2 (\lambda e^{-\theta})^2}{2!\ 2!}$$

$$= \frac{\theta^2 \lambda e^{-\theta}}{2!} (1 + \lambda e^{-\theta})$$

$$P(2) = P(0) \frac{\theta^2 \lambda e^{-\theta}}{2!} (1 + \lambda e^{-\theta})$$

$$P(3) = e^{\lambda(e^{-\theta}-1)} \underbrace{\sum_{k=0}^{3} \frac{\Delta^k (0^3)\ \theta^3}{k!\ 3!} (\lambda e^{-\theta})^k}_{A}$$

$$A = \frac{\Delta^0(0^3)\theta^3}{0!\ 3!} (\lambda e^{-\theta})^0 + \frac{\Delta^1(0^3)\theta^3}{1!\ 3!} (\lambda e^{-\theta})^1$$

$$+ \frac{\Delta^2(0^3)\theta^3}{2!\ 3!} (\lambda e^{-\theta})^2 + \frac{\Delta^3(0^3)\theta^3}{3!\ 3!} (\lambda e^{-\theta})^3$$

$$\Delta^0(0^3) = \sum_{m=0}^{0} \binom{0}{m} (-1)^{0-m} m^3$$

$$= \binom{0}{0} (-1)^0\ 0^3$$

$$= 0$$

$$\Delta^1(0^3) = \sum_{m=0}^{1} \binom{1}{m} (-1)^{1-m} m^3$$

$$= \binom{1}{0} (-1)^1\ 0^3 + \binom{1}{1} (-1)^0\ 1^3$$

$$= 0 + 1 = 1$$

$$\Delta^2(0^3) = \sum_{m=0}^{2} \binom{2}{m} (-1)^{2-m} m^3$$

$$= \binom{2}{0}(-1)^2 0^3 + \binom{2}{1}(-1)^1 1^3 + \binom{2}{2}(-1)^0 2^3$$

$$= 0 - 2 + 8 = 6$$

$$\Delta^3(0^3) = \sum_{m=0}^{3} \binom{3}{m} (-1)^{3-m} m^3$$

$$= \binom{3}{0} (-1)^3 0^3 + \binom{3}{1} (-1)^2 1^3$$

$$+ \binom{3}{2} (-1)^1 2^3 + \binom{3}{3} (-1)^0 3^3$$

$$= 0 + 3 - 24 + 27 = 6$$

$$A = \frac{\theta^3 \lambda e^{-\theta}}{3!} + \frac{6\theta^3(\lambda e^{-\theta})^2}{2!\ 3!} + \frac{6\theta^3(\lambda e^{-\theta})^3}{3!\ 3!}$$

$$= \frac{\theta^3 \lambda e^{-\theta}}{3!} [\ 1 + 3\ \lambda e^{-\theta} + (\lambda e^{-\theta})\ ]$$

$$P\ (3) = P\ (0) \quad \frac{\theta^3 \lambda e^{-\theta}}{3!} [\ 1 + 3\ \lambda e^{-\theta} + (\lambda e^{-\theta})\ ]$$

$$P\ (4) = e^{\lambda(e^{-\theta}-1)} \underbrace{\sum_{k=0}^{4} \frac{\Delta^k\ (0^4)\ \theta^4}{k!\ 4!} \quad (\lambda\ e^{-\theta}\ )^k}_{A}\ {}_2$$

$$A = \frac{\Delta^0(0^4)\theta^4}{0!\ 4!} (\lambda e^{-\theta})^0 + \frac{\Delta^1(0^4)\theta^4}{1!\ 4!} (\lambda e^{-\theta})^1$$

$$+ \frac{\Delta^2(0^4)\theta^4}{2!\ 4!} (\lambda e^{-\theta})^2 + \frac{\Delta^3(0^4)\theta^4}{3!\ 4!} (\lambda e^{-\theta})^3$$

$$+ \frac{\Delta^4(0^4)\theta^4}{4!\ 4!} (\lambda e^{-\theta})^4$$

$$\Delta^0(0^4) = \sum_{m=0}^{0} \binom{0}{m} (-1)^{0-m} m^4$$

$$= \binom{0}{0} (-1)^0\ 0^4$$

$$= 0$$

$$\Delta^1(0^4) = \sum_{m=0}^{1} \binom{1}{m} (-1)^{1-m} m^4$$

$$= \binom{1}{0} (-1)^1\ 0^4 + \binom{1}{1} (-1)^0\ 1^4$$

$$= 0 + 1 = 1$$

$$\Delta^2(0^4) = \sum_{m=0}^{2} \binom{2}{m} (-1)^{2-m}\ m^4$$

$$= \binom{2}{0}(-1)^2 0^4 + \binom{2}{1}(-1)^1 1^4 + \binom{2}{2}(-1)^0 2^4$$

$$= 0 - 2 + 16 = 6$$

$$\Delta^3(0^4) = \sum_{m=0}^{3} \binom{3}{m} (-1)^{3-m}\ m^4$$

$$= \binom{3}{0} (-1)^3\ 0^4 + \binom{3}{1} (-1)^2\ 1^4$$

$$+ \binom{3}{2} (-1)^1\ 2^4 + \binom{3}{3} (-1)^0\ 3^4$$

$$= 0 + 3 - 48 + 81 = 36$$

$$\Delta^4(0^4) = \sum_{m=0}^{4} \binom{4}{m} (-1)^{4-m}\ m^4$$

$$= \binom{4}{0} (-1)^4\ 0^4 + \binom{4}{1} (-1)^3\ 1^4$$

$$+ \binom{4}{2} (-1)^2\ 2^4 + \binom{4}{3} (-1)^1\ 3^4$$

$$+ \binom{4}{4} (-1)^0\ 4^4$$

$$= 0 - 4 + 96 - 324 + 256 = 24$$

$$A = \frac{\theta^4\ \lambda e^{-\theta}}{4!} + \frac{14\ \theta^4\ (\lambda e^{-\theta})^2}{2!\ \ 4!} + \frac{36\ \theta^4\ (\lambda e^{-\theta})^3}{3!\ \ 4!}$$

$$+ \frac{24\ \theta^4\ (\lambda e^{-\theta})^4}{4!\ \ 4!}$$

$$= \frac{\theta^4\ \lambda e^{-\theta}}{4!} [1 + 7\lambda e^{-\theta} + 6(\lambda e^{-\theta})^2 + (\lambda e^{-\theta})^3$$

$$P\ (4) = P\ (0)\ \frac{\theta^4\ \lambda e^{-\theta}}{4!} [1 + 7\lambda e^{-\theta} + 6(\lambda e^{-\theta})^2 + (\lambda e^{-\theta})^3$$

dan seterusnya.

Mean dan variansi dapat diperoleh dari fungsi pembangkit G(s) persamaan (3.4.2), diperoleh

$$\text{Mean }(k) = \mu_1'(k) = \left.\frac{dG(s)}{ds}\right|_{s=1} = \lambda\theta \qquad (3.4.5)$$

$$E\,(X\,(X-1)\,) = \mu_2'(k) = \left.\frac{d^2G(s)}{ds}\right|_{s=1} = \lambda\theta^2 + \lambda^2\theta^2$$

$$\text{Variansi }(k) = \mu_2(k) = \mu_2'(k) + \mu_1'(k) - (\mu_1'(k))^2$$

$$= \lambda\theta(\,1 + \theta\,) \qquad (3.4.6)$$

Parameter λ dan θ diperoleh dari mean dan variansi pengamatan, dan dari sini distribusi peluang dapat dihitung.

2. Distribusi Pendek

Dalam distibusi Panjang kita berasumsi bahwa kecelakaan hanya terjadi dalam Spells, jadi kecelakaan yang tercela saja yang diperhitungkan. Jika asumsi ini diperlemah, sambil berpegangan pada asumsi tambahan yakni kecelakaan dapat terjadi di luar Spells untuk semua pengemudi, meski jarang terjadi.

Kecelakaan di luar Spells dan kecelakaan dalam Spells saling bebas, juga kecelakaan di luar Spells dan Spells saling bebas. Kita dapat menarik kesimpulan bahwa ada distribusi peluang yang lain dinamakan distribusi Pendek . Misalkan $\emptyset$ rata-rata (mean) banyaknya kecelakaan di luar Spells, maka fungsi pembangkit menjadi :

$$G(s) = \sum_{k=o}^{\infty} \frac{\lambda^k\ e^{-\lambda}}{k\ !} \left[\ e^{k\theta(s-1)}\ e^{\emptyset(s-1)}\ \right]$$

$$= e^{-\lambda} e^{\emptyset(s-1)} \sum_{k=0}^{\infty} \frac{(\lambda e^{\theta(s-1)})^k}{k!}$$

$$= e^{-\lambda} e^{\lambda e^{\theta(s-1)}} e^{\emptyset(s-1)} \qquad (3.4.7)$$

$$= e^{\lambda(e^{-\theta}-1)-\emptyset} e^{\lambda e^{-\theta}(e^{\theta s}-1)+\emptyset s}$$

$$= P(0)\, e^{\lambda e^{-\theta}(e^{\theta s}-1)} e^{\emptyset s}$$

dengan

$$p(0) = e^{\lambda(e^{-\theta}-1)-\emptyset}$$

Telah terbukti bahwa :

$$e^{\lambda e^{-\theta}(e^{\theta s}-1)} = \sum_{j=0}^{\infty} \frac{s^j \theta^j}{j!} \sum_{k=0}^{j} \frac{\Delta^k(0^j)}{k!} (\lambda e^{-\theta})^k$$

$$G(s) = P(0) \sum_{j=0}^{\infty} \frac{s^j \theta^j}{j!} \sum_{k=0}^{j} \frac{\Delta^k(0^j)}{k!} (\lambda e^{-\theta})^k \sum_{r=0}^{\infty} \frac{(\emptyset s)^r}{r!}$$

$$= P(0) \sum_{j=0}^{\infty} \frac{s^j \theta^j}{j!} \sum_{r=0}^{\infty} \frac{\emptyset^r s^r}{r!} \sum_{k=0}^{j} \frac{\Delta^k(0^j)}{k!} (\lambda e^{-\theta})^k$$

$$= P(0) \sum_{r=0}^{\infty} \sum_{j=0}^{r} \frac{s^j \theta^j}{j!} \frac{\emptyset^{r-j} s^{r-j}}{(r-j)!} \sum_{k=0}^{j} \frac{\Delta^k(0^j)}{k!} (\lambda e^{-\theta})^k$$

$$= P(0) \sum_{r=0}^{\infty} \sum_{j=0}^{r} \frac{\theta^j \emptyset^{r-j}}{j!\,(r-j)!} s^r \sum_{k=0}^{j} \frac{\Delta^k(0^j)}{k!} (\lambda e^{-\theta})^k \qquad (3.4.8)$$

Distribusi peluang P(r) dapat diperoleh dari fungsi pembangkit (3.4.8),

$$P(r) = P(0) \sum_{j=0}^{r} \left[\frac{\theta^j \emptyset^{r-j}}{j!\,(r-j)!} \sum_{k=0}^{j} \frac{\Delta^k(0^j)}{k!} (\lambda e^{-\theta})^k \right] \qquad (3.4.9)$$

Distribusi ini dinamakan distribusi Pendek (Short distribution).

$$P(0) = P(0) \underbrace{\sum_{j=0}^{0} \left[\frac{\theta^j \phi^{0-j}}{j!(0-j)!} \sum_{k=0}^{j} \frac{\Delta^k (0^j)}{k!} (\lambda e^{-\theta})^k \right]}_{A}$$

$$= P(0) = e^{\lambda (e^{-\theta} - 1) - \phi}$$

$$P(1) = P(0) \underbrace{\sum_{j=0}^{1} \left[\frac{\theta^j \phi^{1-j}}{j!(1-j)!} \sum_{k=0}^{j} \frac{\Delta^k (0^j)}{k!} (\lambda e^{-\theta})^k \right]}_{A}$$

$$A = \frac{\theta^0 \phi^1}{0!\ 1!} \sum_{k=0}^{0} \frac{\Delta^k (0^0)}{k!} (\lambda e^{-\theta})^k + \frac{\theta^1 \phi^0}{1!\ 0!}$$

$$\times \sum_{k=0}^{1} \frac{\Delta^k (0^1)}{k!} (\lambda e^{-\theta})^k$$

$$\sum_{k=0}^{0} \frac{\Delta^k (0^0)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^0)}{0!} (\lambda e^{-\theta})^0$$

$$= 1$$

$$\sum_{k=0}^{1} \frac{\Delta^k (0^1)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^1)}{0!} (\lambda e^{-\theta})^0 + \frac{\Delta^1 (0^1)}{1!} (\lambda e^{-\theta})^1$$

$$= 0 + \lambda e^{-\theta} = \lambda e^{-\theta}$$

$$A = \phi + \theta \lambda e^{-\theta}$$

$$P(1) = P(0) (\phi + \theta \lambda e^{-\theta})$$

$$P(2) = P(0) \underbrace{\sum_{j=0}^{2} \left[\frac{\theta^j \phi^{2-j}}{j!(2-j)!} \sum_{k=0}^{j} \frac{\Delta^k (0^j)}{k!} (\lambda e^{-\theta})^k \right]}_{A}$$

$$A = \frac{\theta^0 \phi^2}{0!\ 2!} \sum_{k=0}^{0} \frac{\Delta^k (0^0)}{k!} (\lambda e^{-\theta})^k + \frac{\theta^1 \phi^1}{1!\ 1!}$$

$$\times \sum_{k=0}^{1} \frac{\Delta^k (0^1)}{k!} (\lambda e^{-\theta})^k + \frac{\theta^2 \phi^0}{2!\ 0!} \sum_{k=0}^{2} \frac{\Delta^k (0^2)}{k!} (\lambda e^{-\theta})^k$$

$$\sum_{k=0}^{0} \frac{\Delta^k (0^0)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^0)}{0!} (\lambda e^{-\theta})^0$$

$$= 1$$

$$\sum_{k=0}^{1} \frac{\Delta^k (0^1)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^1)}{0!} (\lambda e^{-\theta})^0 + \frac{\Delta^1 (0^1)}{1!} (\lambda e^{-\theta})^1$$

$$= 0 + \lambda e^{-\theta} = \lambda e^{-\theta}$$

$$\sum_{k=0}^{2} \frac{\Delta^k (0^2)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^2)}{0!} (\lambda e^{-\theta})^0 + \frac{\Delta^1 (0^2)}{1!} (\lambda e^{-\theta})^1 + \frac{\Delta^2 (0^2)}{2!} (\lambda e^{-\theta})^2$$

$$= 0 + \lambda e^{-\theta} + (\lambda e^{-\theta})^2$$

$$A = \frac{\phi}{2!} + \theta\phi\lambda e^{-\theta} + \frac{\theta}{2!} [\lambda e^{-\theta} + (\lambda e^{-\theta})^2]$$

$$P(2) = P(0) \left\{ \frac{\phi}{2!} + \theta\phi\lambda e^{-\theta} + \frac{\theta}{2!} [\lambda e^{-\theta} + (\lambda e^{-\theta})^2] \right\}$$

$$P(3) = P(0) \underbrace{\sum_{j=0}^{3} \left[\frac{\theta^j \phi^{3-j}}{j!(3-j)!} \sum_{k=0}^{j} \frac{\Delta^k (0^j)}{k!} (\lambda e^{-\theta})^k \right]}_{A}$$

$$A = \frac{\theta^0 \phi^3}{0!\ 3!} \sum_{k=0}^{0} \frac{\Delta^k (0^0)}{k!} (\lambda e^{-\theta})^k + \frac{\theta^1 \phi^2}{1!\ 2!} \times \sum_{k=0}^{1} \frac{\Delta^k (0^1)}{k!} (\lambda e^{-\theta})^k + \frac{\theta^2 \phi^1}{2!\ 1!} \sum_{k=0}^{2} \frac{\Delta^k (0^2)}{k!} (\lambda e^{-\theta})^k + \frac{\theta^3 \phi^0}{3!\ 0!} \sum_{k=0}^{3} \frac{\Delta^k (0^3)}{k!} (\lambda e^{-\theta})^k$$

$$\sum_{k=0}^{0} \frac{\Delta^k (0^0)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^0)}{0!} (\lambda e^{-\theta})^0$$

$$= 1$$

$$\sum_{k=0}^{1} \frac{\Delta^k (0^1)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^1)}{0!} (\lambda e^{-\theta})^0 + \frac{\Delta^1 (0^1)}{1!} (\lambda e^{-\theta})^1$$

$$= 0 + \lambda e^{-\theta} = \lambda e^{-\theta}$$

$$\sum_{k=0}^{2} \frac{\Delta^k (0^2)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^2)}{0!} (\lambda e^{-\theta})^0 + \frac{\Delta^1 (0^2)}{1!} (\lambda e^{-\theta})^1 + \frac{\Delta^2 (0^2)}{2!} (\lambda e^{-\theta})^2$$

$$= 0 + \lambda e^{-\theta} + (\lambda e^{-\theta})^2$$

$$\sum_{k=0}^{3} \frac{\Delta^k (0^3)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^3)}{0!} (\lambda e^{-\theta})^0 + \frac{\Delta^1 (0^3)}{1!} (\lambda e^{-\theta})^1 + \frac{\Delta^2 (0^3)}{2!} (\lambda e^{-\theta})^2 + \frac{\Delta^3 (0^3)}{3!} (\lambda e^{-\theta})^3$$

$$= 0 + \lambda e^{-\theta} + 3(\lambda e^{-\theta})^2 + (\lambda e^{-\theta})^3$$

$$A = \frac{\phi^3}{3!} + \frac{\theta\phi^2}{2!} (\lambda e^{-\theta}) + \frac{\theta^2\phi}{2!} [\, \lambda e^{-\theta} + (\lambda e^{-\theta})^2 \,] + \frac{\theta^3}{3!} [\, \lambda e^{-\theta} + 3(\lambda e^{-\theta})^2 + (\lambda e^{-\theta})^3 \,]$$

$$P(3) = P(0) \left\{ \frac{\phi^3}{3!} + \frac{\theta\phi^2}{2!} (\lambda e^{-\theta}) + \frac{\theta^2\phi}{2!} [\lambda e^{-\theta} + (\lambda e^{-\theta})^2] + \frac{\theta^3}{3!} [\, \lambda e^{-\theta} + 3(\lambda e^{-\theta})^2 + (\lambda e^{-\theta})^3 \,] \right\}$$

$$P(4) = P(0) \underbrace{\sum_{j=0}^{4} \left[\frac{\theta^j \phi^{4-j}}{j!(4-j)!} \sum_{k=0}^{j} \frac{\Delta^k (0^j)}{k!} (\lambda e^{-\theta})^k \right]}_{A}$$

$$A = \frac{\theta^0 \ \emptyset^4}{0! \ 4!} \sum_{k=0}^{0} \frac{\Delta^k \ (0^0)}{k!} (\lambda e^{-\theta})^k + \frac{\theta^1 \ \emptyset^3}{1! \ 3!}$$

$$\times \sum_{k=0}^{1} \frac{\Delta^k \ (0^1)}{k!} (\lambda e^{-\theta})^k + \frac{\theta^2 \ \emptyset^2}{2! \ 2!} \sum_{k=0}^{2} \frac{\Delta^k \ (0^2)}{k!} (\lambda e^{-\theta})^k$$

$$+ \frac{\theta^3 \ \emptyset^1}{3! \ 1!} \sum_{k=0}^{3} \frac{\Delta^k \ (0^3)}{k!} (\lambda e^{-\theta})^k + \frac{\theta^4 \ \emptyset^0}{4! \ 1!}$$

$$\times \sum_{k=0}^{4} \frac{\Delta^k \ (0^4)}{k!} (\lambda e^{-\theta})^k$$

$$\sum_{k=0}^{0} \frac{\Delta^k \ (0^0)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 \ (0^0)}{0!} (\lambda e^{-\theta})^0$$

$$= 1$$

$$\sum_{k=0}^{1} \frac{\Delta^k \ (0^1)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 \ (0^1)}{0!} (\lambda e^{-\theta})^0$$

$$+ \frac{\Delta^1 \ (0^1)}{1!} (\lambda e^{-\theta})^1$$

$$= 0 + \lambda e^{-\theta} = \lambda e^{-\theta}$$

$$\sum_{k=0}^{2} \frac{\Delta^k \ (0^2)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 \ (0^2)}{0!} (\lambda e^{-\theta})^0$$

$$+ \frac{\Delta^1 \ (0^2)}{1!} (\lambda e^{-\theta})^1$$

$$+ \frac{\Delta^2 \ (0^2)}{2!} (\lambda e^{-\theta})^2$$

$$= 0 + \lambda e^{-\theta} + (\lambda e^{-\theta})^2$$

$$\sum_{k=0}^{3} \frac{\Delta^k \ (0^3)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 \ (0^3)}{0!} (\lambda e^{-\theta})^0$$

$$+ \frac{\Delta^1 \ (0^3)}{1!} (\lambda e^{-\theta})^1$$

$$+ \frac{\Delta^2 \ (0^3)}{2!} (\lambda e^{-\theta})^2$$

$$+ \frac{\Delta^3 \ (0^3)}{3!} (\lambda e^{-\theta})^3$$

$$= 0 + \lambda e^{-\theta} + 3(\lambda e^{-\theta})^2 + (\lambda e^{-\theta})^3$$

$$\sum_{k=0}^{4} \frac{\Delta^k (0^4)}{k!} (\lambda e^{-\theta})^k = \frac{\Delta^0 (0^4)}{0!} (\lambda e^{-\theta})^0 + \frac{\Delta^1 (0^4)}{1!} (\lambda e^{-\theta})^1 + \frac{\Delta^2 (0^4)}{2!} (\lambda e^{-\theta})^2 + \frac{\Delta^3 (0^4)}{3!} (\lambda e^{-\theta})^3 + \frac{\Delta^4 (0^4)}{4!} (\lambda e^{-\theta})^4$$

$$= 0 + \lambda e^{-\theta} + 7(\lambda e^{-\theta})^2 + 6(\lambda e^{-\theta})^3 + (\lambda e^{-\theta})^4$$

$$A = \frac{\emptyset^4}{4!} + \frac{\theta\,\emptyset^3}{3!} \lambda e^{-\theta} + \frac{\theta^2\emptyset^2}{2!2!} [\, \lambda e^{-\theta} + (\lambda e^{-\theta})^2 \,] + \frac{\theta^3\emptyset}{3!} [\, \lambda e^{-\theta} + 3(\lambda e^{-\theta})^2 + (\lambda e^{-\theta})^3] + \frac{\theta^4}{4!} [\lambda e^{-\theta} + 7(\lambda e^{-\theta})^2 + 6(\lambda e^{-\theta})^3 + (\lambda e^{-\theta})^4 \,]$$

$$P\,(4) = P\,(0) \; \frac{\emptyset^4}{4!} + \frac{\theta\,\emptyset^3}{3!} \lambda e^{-\theta} + \frac{\theta^2\emptyset^2}{2!2!} [\, \lambda e^{-\theta} + (\lambda e^{-\theta})^2] + \frac{\theta^3\emptyset}{3!} [\, \lambda e^{-\theta} + 3(\lambda e^{-\theta})^2 + (\lambda e^{-\theta})^3] + \frac{\theta^4}{4!} [\, \lambda e^{-\theta} + 7(\lambda e^{-\theta})^2 + 6(\lambda e^{-\theta})^3 + (\lambda e^{-\theta})^4 \,]$$

dan seterusnya.

Mean dan variansi diperoleh dengan mendeferensiasi fungsi pembangkit (3.4.7) dengan mengambil s = 1

$$\text{Mean } (k) = \mu_1{}'(k) = E(x) = \left.\frac{d\,G(s)}{ds}\right|_{s=1}$$

$$= \lambda\theta + \emptyset \qquad (3.4.10)$$

$$E(\,X\,(X - 1)) = \mu_2'\,(k) = \left.\frac{d\,G(s)}{ds^2}\right|_{s=1}$$

$$= \lambda^2\theta^2 + \lambda\theta^2 + 2\lambda\theta\emptyset + \emptyset^2$$

$$\text{Variansi (k)} = \mu'_2(k) + \mu'_1(k) - \mu'_1(k))^2$$

$$= \lambda\theta(1 + \theta) + \emptyset \qquad (3.4.11)$$

$$E(X(X - 1)(X - 2)) = \mu'_3(k) = \left.\frac{d^3G(s)}{ds^3}\right|_{s=1}$$

$$= \theta^3 + 3\lambda\theta\emptyset^2 + 3\lambda\theta^2\emptyset + 3\lambda^2\theta^2\emptyset + 3\lambda^2\theta^3 + \lambda^3\theta^3 + \lambda\theta^3$$

$$\text{Momen sentral ketiga (k)} = E(X - \mu'_1)^3$$

$$= \lambda\theta\ (1+3\theta+\theta^2) + \emptyset \qquad (3.4.12)$$

Dari mean variansi momen sentral ketiga kita memperoleh parameter λ, θ dan $\emptyset$. Dari sini distribusi peluang dapat dihitung.