APPENDIX

Apabila fungsi ϕ didefinisikan dalam suatu domain D yang dibatasi kontur C, maka $\frac{\partial \phi}{\partial n}$ adalah derivatif normal dari ϕ, dengan :

$$\frac{\partial \phi}{\partial n} = \nabla \phi \cdot \bar{N}$$

$\frac{\partial \phi}{\partial n}$ = derivatif normal dari ϕ

$\nabla \phi$ = normal dari ϕ

$\bar{N}$ = normal satuan dari kontur C.

Berdasarkan pendefinisian diatas buktikanlah derivatif normal dari fungsi U(x,y) atau U(r,θ) (dalam koordinat polar) akan sama dengan :

1. $U_r(r,\theta)$

   Untuk suatu fungsi U(x,y) atau U(r,θ) yang didefinisikan didalam daerah yang dibatasi lingkaran dengan pusat pada titik asal.

2. $U_y(x,y)$

   Untuk suatu fungsi U(x,y) yang didefinisikan dalam daerah setengah bidang atas yang dibatasi oleh sumbu x.

Bukti :

1. Untuk suatu fungsi U(x,y) yang didefinisikan didalam daerah yang dibatasi lingkaran dengan pusat pada titik asal, maka:

$$\nabla U(x,y) = \frac{\partial U}{\partial x}\mathbf{i} + \frac{\partial U}{\partial y}\mathbf{j}$$

atau apabila diubah kebentuk polar dengan substitusi

$$x = r\cos\theta \text{ dan } y = r\sin\theta$$

dideferensialkan secara parsial terhadap x

$$1 = \cos\theta \frac{\partial r}{\partial x} - r\sin\theta \frac{\partial \theta}{\partial x}$$

$$0 = \sin\theta \frac{\partial r}{\partial x} + r\cos\theta \frac{\partial \theta}{\partial x}$$

$$\frac{\partial r}{\partial x} = \frac{\begin{vmatrix} 1 & -r\sin\theta \\ 0 & r\cos\theta \end{vmatrix}}{\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}} = \frac{r\cos\theta}{r} = \cos\theta$$

$$\frac{\partial \theta}{\partial x} = \frac{\begin{vmatrix} \cos\theta & 1 \\ \sin\theta & 0 \end{vmatrix}}{\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}} = \frac{-\sin\theta}{r}$$

dideferensialkan secara parsial terhadap y

$$0 = \cos\theta \frac{\partial r}{\partial y} - r\sin\theta \frac{\partial \theta}{\partial y}$$

$$1 = \sin\theta \frac{\partial r}{\partial y} + r\cos\theta \frac{\partial \theta}{\partial y}$$

$$\frac{\partial r}{\partial y} = \frac{\begin{vmatrix} 0 & -r\sin\theta \\ 1 & r\cos\theta \end{vmatrix}}{\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}} = \frac{r\sin\theta}{r} = \sin\theta$$

$$\frac{\partial\theta}{\partial y} = \frac{\begin{vmatrix}\cos\theta & 0\\ \sin\theta & 1\end{vmatrix}}{\begin{vmatrix}\cos\theta & -r\sin\theta\\ \sin\theta & r\cos\theta\end{vmatrix}} = \frac{\cos\theta}{r}$$

$$\frac{\partial U}{\partial x} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial U}{\partial\theta}\frac{\partial\theta}{\partial x} = \cos\theta\,\frac{\partial U}{\partial r} - \frac{\sin\theta}{r}\frac{\partial U}{\partial\theta}$$

$$\frac{\partial U}{\partial y} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial U}{\partial\theta}\frac{\partial\theta}{\partial y} = \sin\theta\,\frac{\partial U}{\partial r} + \frac{\cos\theta}{r}\frac{\partial U}{\partial\theta}$$

$$\nabla U(r,\theta) = \left(\cos\theta\,\frac{\partial U}{\partial r} - \frac{\sin\theta}{r}\frac{\partial U}{\partial\theta}\right)i + \left(\sin\theta\,\frac{\partial U}{\partial r} + \frac{\cos\theta}{r}\frac{\partial U}{\partial\theta}\right)j$$

$$\bar{N} = \cos\theta i + \sin\theta j$$

$$\frac{\partial U}{\partial n} = \left(\cos\theta\,\frac{\partial U}{\partial r} - \frac{\sin\theta}{r}\frac{\partial U}{\partial\theta}\right)\cos\theta + \left(\sin\theta\,\frac{\partial U}{\partial r} + \frac{\cos\theta}{r}\frac{\partial U}{\partial\theta}\right)\sin\theta$$

$$= \cos^2\theta\,\frac{\partial U}{\partial r} - \frac{\sin\theta\cos\theta}{r}\frac{\partial U}{\partial\theta} + \sin^2\theta\,\frac{\partial U}{\partial r} + \frac{\sin\theta\cos\theta}{r}\frac{\partial U}{\partial\theta}$$

$$= \left(\cos^2\theta + \sin^2\theta\right)\frac{\partial U}{\partial r}$$

$$= \frac{\partial U}{\partial r}$$

$$= U_r(r,\theta)$$

2. Untuk suatu fungsi U(x,y) yang didefinisikan dalam daerah setengah bidang atas yang dibatasi sumbu x.

$$\nabla U(x,y) = \frac{\partial U}{\partial x}i + \frac{\partial U}{\partial y}j$$

$$\bar{N} = 0i + 1j = j$$

$$\frac{\partial U}{\partial n} = \nabla U(x,y).\bar{N}$$

$$= \left(\frac{\partial U}{\partial x}i + \frac{\partial U}{\partial y}j\right).j = \frac{\partial U}{\partial y} = U_y(x,y)$$

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