BAB II

ESTIMATOR TAK BIAS DAN GENERALIZED JACKKNIFE

Estimator yang mempunyai suku bias merupakan estimator yang kurang baik. Metode Generalized Jackknife yang pertama dikenalkan oleh Quenouille merupakan metode untuk mereduksi bias suatu estimator dengan penggunaan ulang sampel sehingga diperoleh estimator baru .

Pada bab ini akan diberikan beberapa pengertian tentang estimator tak bias, Metode Generalized Jackknife order satu, Metode Generalized Jackknife order dua dan cara mendapatkan estimator. -estimator dengan metode Quenouille.

2.1. Estimator tak bias

Dalam suatu penaksiran tidak mungkin selalu diperoleh estimator yang tepat, sehingga timbul beberapa kriteria tentang baiknya suatu estimator.Salah satu sifat baik estimator adalah tak bias.Pada sub bab ini akan dibahas kriteria tak bias dari estimator untuk menunjang pembahasan selanjutnya.

Definisi 2.1.

Suatu estimator titik $\hat{\theta}$ disebut estimator tak bias untuk θ jika $E\left[\hat{\theta}\right] = \theta$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

Kriteria ini menyatakan bahwa rata-rata semua harga $\hat{\theta}$ yang mungkin akan sama dengan θ . Sedangkan estimator yang tidak tak bias disebut estimator bias. Jika $\hat{\theta}$ adalah estimator bias untuk θ dan bias dari estimator ditulis $B(\hat{\theta})$,

maka :

$$E[\hat{\theta}] = \theta + B(\hat{\theta}) \tag{2.1}$$

Sehingga bias estimator :

$$B(\hat{\theta}) = E[\hat{\theta}] - \theta$$

$$= E[\hat{\theta} - \theta] \tag{2.2}$$

Dari definisi 2.1. akan diberikan contoh :

Contoh 2.1.:

Misal X berdistribusi Uniform dalam interval $(0, \theta)$ dengan fungsi densitas:

$$f(x,\theta) = \begin{cases} \dfrac{1}{\theta} & , \ 0 < x < \theta \\ 0 & , \ x \text{ lainnya} \end{cases} \tag{2.3}$$

Akan dibuktikan bahwa :

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{2.4}$$

merupakan estimator bias untuk θ .

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

Harga harapan dari X adalah :

$$E[X] = \int_0^\theta x f(x/\theta) dx \qquad (2.5)$$

$$= \int_0^\theta x \frac{1}{\theta} dx$$

$$= \frac{1}{\theta}\left[\frac{x^2}{2}\right]_0^\theta = \frac{\theta}{2} \qquad \ldots\ldots\ldots \qquad (2.6)$$

Sehingga harga harapan dari $\overline{X}$ adalah :

$$E[\overline{X}] = E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] \qquad \ldots\ldots\ldots \qquad (2.7)$$

$$= \frac{1}{n}\sum_{i=1}^{n} E[X_i]$$

$$= \frac{1}{n} n \frac{\theta}{2}$$

$$= \frac{\theta}{2}$$

$$= \theta - \frac{\theta}{2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (2.8)$$

Dari persamaan (2.8) dapat dilihat bahwa nilai yang diharapkan dari $\overline{X}$ tidak sama dengan parameter θ , sehingga $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ merupakan estimator yang bias untuk θ , dengan biasnya $-\frac{\theta}{2}$.

2.2. Generalized Jackknife order satu

Salah satu metode untuk mengurangi bias estimator adalah dengan metode Generalized Jackknife. Metode ini dapat digunakan untuk mengurangi bias estimator berorder satu. Jackknife merupakan kasus khusus dari

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

Generalized Jackknife. Dalam sub bab ini akan diberikan beberapa definisi dan theorema dari Generalized Jackknife yang merupakan bentuk umum dari Jackknife sebelum diperoleh Jackknife untuk order satu.

Definisi 2.2.1. :

Jika diberikan $\hat{\theta}_1$ dan $\hat{\theta}_2$ adalah estimator-estimator untuk θ untuk sembarang bilanggan riil $R \neq 1$, didefinisikan Generalized Jackknife :

$$G(\hat{\theta}_1, \hat{\theta}_2) = \frac{\hat{\theta}_1 - R\hat{\theta}_2}{1 - R} \qquad \text{...............} \qquad (2.9)$$

Untuk menunjukkan bahwa $G(\hat{\theta}_1, \hat{\theta}_2)$ merupakan estimator tak bias akan diberikan dengan theorema berikut:

Theorema 2.2.1.:

Jika $E[\hat{\theta}_k] = \theta + b_k(n, \theta)$ k=,1,2 (2.10)

Dengan $b_k(n, \theta)$ adalah bias dari suatu estimator untuk θ atas sampel random yang besarnya n pengamatan. Dengan $b_2(n, \theta) \neq 0$ dan :

$$R = \frac{b_1(n, \theta)}{b_2(n, \theta)} \neq 1 \qquad (2.11)$$

maka:

$$E[G(\hat{\theta}_1, \hat{\theta}_2)] = \theta \qquad (2.12)$$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

Bukti :

Dari persamaan (2.9) didefinisikan :

$$G(\hat{\theta}_1, \hat{\theta}_2) = \frac{\hat{\theta}_1 - R\hat{\theta}_2}{1-R} \qquad (2.13)$$

Kemudian harga harapan dari $G(\hat{\theta}_1, \hat{\theta}_2)$ adalah :

$$E\left[G(\hat{\theta}_1, \hat{\theta}_2)\right] = \frac{E(\hat{\theta}_1) - RE(\hat{\theta}_2)}{1-R} \qquad (2.14)$$

$$= \frac{\theta + b_1(n,\theta) - R\left[\theta + b_2(n,\theta)\right]}{1-R}$$

$$= \frac{\theta + b_1(n,\theta) - R\theta - Rb_2(n,\theta)}{1-R}$$

$$= \frac{(1-R)\theta + b_1(n,\theta) - Rb_2(n,\theta)}{1-R}$$

$$= \frac{(1-R)\theta + b_1(n,\theta) - \dfrac{b_1(n,\theta)}{b_2(n,\theta)} b_2(n,\theta)}{1-R}$$

$$= \theta \qquad \blacksquare \qquad (2.15)$$

Maksud theorema 2.2.1. adalah, jika R diketahui dan diberikan dengan persamaan (2.11) , maka $G(\hat{\theta}_1, \hat{\theta}_2)$ adalah estimator tak bias untuk θ . Dengan menganggap $G(\hat{\theta}_1, \hat{\theta}_2)$ sebagai suatu estimator untuk θ jika R diketahui, pertanyaan yang timbul adalah bagaimana sebaiknya $\hat{\theta}_1$ dan $\hat{\theta}_2$ dipilih.

Misal $\hat{\theta}_1$ adalah suatu estimator tertentu dari θ atas sampel random $X_1, X_2, X_3, \ldots, X_n$ dengan :

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$E(\hat{\theta}_1) = \theta + b_1(n,\theta) = \theta + b(\theta)f(n) \tag{2.16}$$

Selanjutnya dimisalkan $\hat{\theta}_2$ adalah estimator lain untuk θ didefinisikan atas sub sampel dari $X_1, X_2, X_3, \ldots, X_n$ yang diperoleh dengan membuang satu X_i sehingga:

$$E(\hat{\theta}_2) = \theta + b_2(n,\theta) = \theta + b(\theta)f(n-1) \tag{2.17}$$

dan

$$R = \frac{b_1(n,\theta)}{b_2(n,\theta)} = \frac{b(\theta)f(n)}{b(\theta)f(n-1)} = \frac{f(n)}{f(n-1)} \tag{2.18}$$

Sehingga jika $f(n)$ diketahui maka R diketahui. Karena pada umumnya R adalah fungsi dari n ,maka untuk selanjutnya digunakan notasi R(n) untuk R.

Untuk memilih $\hat{\theta}_1$ dan $\hat{\theta}_2$ dikerjakan dengan metode Quenouille yang akan diberikan pada sub bab berikut.

2.3. Metode Quenouille

Metode Quenouille ini digunakan untuk mencari $\hat{\theta}_2$ pada saat $\hat{\theta}_1$ telah diberikan. Untuk lebih jelasnya akan diuraikan sebagai berikut :

Misal $\hat{\theta}$ adalah suatu estimator yang didefinisikan pada sampel random $X_1, X_2, X_3, \ldots, X_n$. Kemudian sampel dipartisi ke dalam N subset dengan ukuran M, sehingga NM=n dan bentuk sampel random baru dari penghapusan sembarang subset berukuran M dari sampel yang asli.Kemudian didefinisikan

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s).also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

estimator $\hat{\theta}^{i}$ menjadi estimator. $\hat{\theta}$ yang didefinisikan pada sub sampel yang timbul pada saat subset ke-I dengan ukuran M telah dihapus.

Didefinisikan estimator : :

$$J_i(\theta) = N\hat{\theta} - (N-1)\hat{\theta}^{i} \tag{2.19}$$

yang merupakan nilai samaran Jackknife.

Dan estimator Jackknife :

$$J(\hat{\theta}) = \frac{1}{N}\sum_{i=1}^{N} J_i(\hat{\theta}) \tag{2.20}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left[N\hat{\theta} - (N-1)\hat{\theta}^{i}\right]$$

$$= N\hat{\theta} - (N-1)\bar{\hat{\theta}}^{i} \tag{2.21}$$

Jackknife merupakan kejadian khusus dari Generalized Jackknife.Akan ditunjukkan bahwa Jackknife adalah kejadian khusus dari $G(\hat{\theta}_1, \hat{\theta}_2)$ yaitu sebagai berikut:

$$J(\hat{\theta}) = N\hat{\theta} - (N-1)\bar{\hat{\theta}}^{i}$$

$$= N\left[\hat{\theta} - \frac{(N-1)}{N}\bar{\hat{\theta}}^{i}\right]$$

$$= \frac{\hat{\theta} - \frac{N-1}{N}\bar{\hat{\theta}}^{i}}{\frac{1}{N}}$$

$$= \frac{\hat{\theta} - \frac{N-1}{N}\bar{\hat{\theta}}^{i}}{1 - \frac{N-1}{NN}} \tag{2.22}$$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

Dari persamaan (2.22) dapat dilihat bahwa $J(\hat{\theta})$ dapat disajikan dalam bentuk $G(\hat{\theta}_1, \hat{\theta}_2)$ dimana :

$$\hat{\theta}_1 = \hat{\theta} \tag{2.23}$$

$$\hat{\theta}_2 = \bar{\hat{\theta}}^i = \frac{1}{N}\sum_{i=1}^{N}\hat{\theta}^i \tag{2.24}$$

$$R = \frac{N-1}{N} \tag{2.25}$$

2.4. Generalized Jackknife order dua

Generalized Jackknife dapat dikerjakan untuk estimator dengan order bias yang lebih dari satu. Pada sub bab ini akan dibahas Generalized Jackknife dengan order bias yang lebih dari satu.

Mengingat persamaan Generalized Jackknife order satu :

$$G(\hat{\theta}_1, \hat{\theta}_2) = \frac{\hat{\theta}_1 - R\hat{\theta}_2}{1-R}$$

dan dengan mengambil:

$$R(n) = \frac{a_{11}(n)}{a_{12}(n)} \tag{2.26}$$

dimana:

$$B_k(n, \theta) = a_{1k}(n)b_1(\theta) \qquad k = 1, 2. \tag{2.27}$$

merupakan bias estimator ke-k yang tergantung dari n sampel dengan parameter θ yang dapat ditunjukkan dalam bentuk perkalian dari $a_{1k}(n)$ yang merupakan fungsi yang tergantung dari n dari order pertama pada estimator

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

ke-k dengan $b_1(\theta)$ yang merupakan fungsi yang tergantung dari parameter θ dengan order satu.

Kemudian penekanan pada n dapat dituliskan :

$$G(\hat{\theta}_1, \hat{\theta}_2) = \frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 \\ a_{11} & a_{12} \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ a_{11} & a_{12} \end{vmatrix}} \tag{2.28}$$

dengan $$\begin{vmatrix} 1 & 1 \\ a_{11} & a_{12} \end{vmatrix} \neq 0 \tag{2.29}$$

Kemudian dapat diperluas dengan memperhatikan definisi berikut.

Definisi 2.4.1. :

Misalkan $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1}$ ada k+1 estimator untuk θ yang didefinisikan pada sampel random $X_1, X_2, X_3, \ldots, X_n$. Selanjutnya, misalkan a_{ij} , i= 1,2,3,...,k. j= 1,2,..., k+1 bilangan riil sedemikian hingga

$$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ a_{11} & a_{12} & \cdots & a_{1,k+1} \\ \vdots & \vdots & & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{k,k+1} \end{vmatrix} \neq 0 \tag{2.30}$$

sehingga $G(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1})$ didefinisikan sebagai :

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$G\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{k+1}\right) = \frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \cdots & \hat{\theta}_{k+1} \\ a_{11} & a_{12} & \cdots & a_{1,k+1} \\ \vdots & \vdots & & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{k,k+1} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ a_{11} & a_{12} & \cdots & a_{1,k+1} \\ \vdots & \vdots & & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{k,k+1} \end{vmatrix}} \tag{2.31}$$

Bias dari estimator $G\left(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1}\right)$ dapat ditulis sebagai hasilkali dari fungsi n dan fungsi θ, sehingga dapat diberikan dalam theorema berikut:

Theorema 2.4.1. :

Jika : $E\left[\hat{\theta}_j\right] = \theta + \sum_{i=1}^{\infty} f_{ij}(n) b_i(\theta)$, j=1,2,...,k+1 (2.32)

dan $a_{ij} = f_{ij}(n)$ (2.33)

dengan n merupakan bilangan bulat positif.

Maka

$$E\left[G\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{k+1}\right)\right] = \theta + B_G(n, \theta) \tag{2.34}$$

dengan

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$B_G(\underline{n},\theta)=\frac{\begin{vmatrix} B_1 & B_2 & \cdots & B_{k+1} \\ f_{11}(n) & f_{12}(n) & \cdots & f_{1,k+1}(n) \\ \vdots & \vdots & & \vdots \\ f_{k1}(n) & f_{k2}(n) & \cdots & f_{k,k+1}(n) \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ f_{11}(n) & f_{12}(n) & \cdots & f_{1,k+1}(n) \\ \vdots & \vdots & & \vdots \\ f_{k1}(n) & f_{k2}(n) & \cdots & f_{k,k+1}(n) \end{vmatrix}} \tag{2.35}$$

dan:

$$B_j=\sum_{i=1}^{\infty} f_{ij}(n) b_i(\theta) \qquad j=1,2,\ldots,k+1 \tag{2.36}$$

Bukti:

Dari persamaan (2.31) diketahui :

$$G(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{k+1})=\frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \cdots & \hat{\theta}_{k+1} \\ a_{11} & a_{12} & \cdots & a_{1,k+1} \\ \vdots & \vdots & & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{k,k+1} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ a_{11} & a_{12} & \cdots & a_{1,k+1} \\ \vdots & \vdots & & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{k,k+1} \end{vmatrix}}$$

Karena $a_{ij}=f_{ij}(n)$ sehingga nilai harapan dari

$G(\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3,\ldots,\hat{\theta}_{k+1})$ dapat ditulis sebagai berikut:

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$E\left[G\left(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{k+1}\right)\right]=\frac{\begin{vmatrix} E\left[\hat{\theta}_1\right] & E\left[\hat{\theta}_2\right] & \cdots & E\left[\hat{\theta}_{k+1}\right] \\ f_{11}(n) & f_{12}(n) & \cdots & f_{1,k+1} \\ \vdots & \vdots & & \vdots \\ f_{k1}(n) & f_{k2}(n) & \cdots & f_{k,k+1}(n) \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ f_{11}(n) & f_{12}(n) & \cdots & f_{1,k+1}(n) \\ \vdots & \vdots & & \vdots \\ f_{k1}(n) & f_{k2}(n) & \cdots & f_{k,k+1}(n) \end{vmatrix}} \qquad (2.38)$$

Karena : $E\left[\hat{\theta}_j\right]=\theta+\sum_{i=1}^{\infty} f_{ij}(n) b_i(\theta)$ $\qquad$ $j=1,2,\ldots,k+1$

sehingga $E\left[G\left(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{k+1}\right)\right]$ dapat ditulis dalam bentuk :

$$E\left[G\left(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{k+1}\right)\right]=\frac{\begin{vmatrix} \theta+\sum_{i=1}^{\infty} f_{i1}(n) b_i(\theta) & \cdots & \theta+\sum_{i=1}^{\infty} f_{i,k+1}(n) b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,k+1}(n) \\ \vdots & & \vdots \\ f_{k1}(n) & \cdots & f_{k,k+1}(n) \end{vmatrix}}{\begin{vmatrix} 1 & \cdots & 1 \\ f_{11}(n) & \cdots & f_{1,k+1}(n) \\ \vdots & & \vdots \\ f_{k1}(n) & \cdots & f_{k,k+1}(n) \end{vmatrix}}$$

$$E\left[G\left(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{k+1}\right)\right]=\theta+\frac{\begin{vmatrix} \sum_{i=1}^{\infty} f_{i1}(n) b_i(\theta) & \cdots & \sum_{i=1}^{\infty} f_{i,k+1}(n) b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,k+1}(n) \\ \vdots & & \vdots \\ f_{k1}(n) & \cdots & f_{k,k+1}(n) \end{vmatrix}}{\begin{vmatrix} 1 & \cdots & 1 \\ f_{11}(n) & \cdots & f_{1,k+1}(n) \\ \vdots & & \vdots \\ f_{k1}(n) & \cdots & f_{k,k+1}(n) \end{vmatrix}}$$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

Karena :

$$B_G(n,\theta)=\frac{\begin{vmatrix}\sum_{i=1}^{\infty}f_{i1}(n)b_i(\theta) & \cdots & \sum_{i=1}^{\infty}f_{i,k+1}(n)b_i(\theta)\\ f_{11}(n) & \cdots & f_{1,k+1}(n)\\ \vdots & & \vdots\\ f_{k1}(n) & \cdots & f_{k,k+1}(n)\end{vmatrix}}{\begin{vmatrix}1 & \cdots & 1\\ f_{11}(n) & \cdots & f_{1,k+1}(n)\\ \vdots & & \vdots\\ f_{k1}(n) & \cdots & f_{k,k+1}(n)\end{vmatrix}}$$

maka $$E\left[G\left(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{k+1}\right)\right]=\theta+B_G(n,\theta) \qquad \blacksquare \qquad (2.39)$$

Akibat theorema 2.4.1. :

Jika : $E\left[\hat{\theta}_j\right]=\theta+\sum_{i=1}^{k}f_{ij}(n)b_i(\theta)$, j=1,2,... ,k+1

maka : $E\left[G\left(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{k+1}\right)\right]=\theta$

Bukti:

Dengan menggunakan induksi matematik.

■ Untuk k=1

$$G\left(\hat{\theta}_1,\hat{\theta}_2\right)=\frac{\begin{vmatrix}\hat{\theta}_1 & \hat{\theta}_2\\ f_{11}(n) & f_{12}(n)\end{vmatrix}}{\begin{vmatrix}1 & 1\\ f_{11}(n) & f_{12}(n)\end{vmatrix}} \qquad (2.40)$$

$$E\left[G\left(\hat{\theta}_1,\hat{\theta}_2\right)\right]=\theta+\frac{\begin{vmatrix}f_{11}(n)b_1(\theta) & f_{12}(n)b_2(\theta)\\ f_{11}(n) & f_{12}(n)\end{vmatrix}}{\begin{vmatrix}1 & 1\\ f_{11}(n) & f_{12}(n)\end{vmatrix}} \qquad (2.41)$$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$= \theta + \frac{f_{11}(n)b_1(\theta)f_{12}(n) - f_{12}(n)b_1(\theta)f_{11}(n)}{f_{12}(n) - f_{11}(n)}$$

$$= \theta \qquad (2.42)$$

Andaikan benar untuk k=m , akan dibuktikan benar untuk k=m+1

- Untuk k=m

$$G\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{m+1}\right) = \frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \cdots & \hat{\theta}_{m+1} \\ f_{11}(n) & f_{12}(n) & \cdots & f_{1,m+1}(n) \\ \vdots & \vdots & & \vdots \\ f_{m1}(n) & f_{m2}(n) & \cdots & f_{m,m+1}(n) \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ f_{11}(n) & f_{12}(n) & \cdots & f_{1,m+1}(n) \\ \vdots & \vdots & & \vdots \\ f_{m1}(n) & f_{m2}(n) & \cdots & f_{m,m+1}(n) \end{vmatrix}} \qquad (2.43)$$

$$E\left[G\left(\hat{\theta}_1, \ldots, \hat{\theta}_{m+1}\right)\right] = \theta + \frac{\begin{vmatrix} \sum_{i=1}^{m} f_{i1}(n)b_i(\theta) & \cdots & \sum_{i=1}^{m} f_{i,m+1}(n)b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,m+1}(n) \\ \vdots & & \vdots \\ f_{m1}(n) & \cdots & f_{m,m+1}(n) \end{vmatrix}}{\begin{vmatrix} 1 & \cdots & 1 \\ f_{11}(n) & \cdots & f_{1,m+1}(n) \\ \vdots & & \vdots \\ f_{m1}(n) & \cdots & f_{m,m+1}(n) \end{vmatrix}} \qquad (2.44)$$

$$= \theta \qquad (2.45)$$

karena tidak mempunyai baris yang merupakan kelipatan dari baris lainnya ,sehingga :

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$\begin{vmatrix} 1 & \cdots & 1 \\ f_{11}(n) & \cdots & f_{1,m+1}(n) \\ \vdots & & \vdots \\ f_{m,1}(n) & \cdots & f_{m,m+1}(n) \end{vmatrix} \neq 0 \tag{2.46}$$

sehingga:

$$\begin{vmatrix} \sum_{i=1}^{m} f_{i1}(n) b_i(\theta) & \cdots & \sum_{i=1}^{m} f_{i,m+1}(n) b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,m+1}(n) \\ \vdots & & \vdots \\ f_{m,1}(n) & \cdots & f_{m,m+1}(n) \end{vmatrix} = 0 \tag{2.47}$$

■ Untuk k=m+1

$$G\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{m+2}\right) = \frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \cdots & \hat{\theta}_{m+2} \\ f_{11}(n) & f_{12}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & \vdots & & \vdots \\ f_{m+1,1}(n) & f_{m+1,2}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ f_{11}(n) & f_{12}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & \vdots & & \vdots \\ f_{m+1,1}(n) & f_{m+1,2}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix}} \tag{2.48}$$

$$E\left[G\left(\hat{\theta}_1, \ldots, \hat{\theta}_{m+2}\right)\right] = \theta + \frac{\begin{vmatrix} \sum_{i=1}^{m+1} f_{i1}(n) b_i(\theta) & \cdots & \sum f_{im+2}(n) b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ & & \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix}}{\begin{vmatrix} 1 & \cdots & 1 \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & & \vdots \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix}} \tag{2.49}$$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

karena:

$$\begin{vmatrix} 1 & \cdots & 1 \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & & \vdots \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix} \neq 0 \qquad (2.50)$$

Berarti harus dibuktikan bahwa:

$$\begin{vmatrix} \sum_{i=1}^{m+1} f_{i1}(n)b_i(\theta) & \cdots & \sum_{i=1}^{m+1} f_{im+2}(n)b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & & \vdots \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix} = 0 \qquad (2.51)$$

Karena:

$$\begin{vmatrix} \sum_{i=1}^{m+1} f_{i1}(n)b_i(\theta) & \cdots & \sum_{i=1}^{m+1} f_{im+2}(n)b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & & \vdots \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix} = \begin{vmatrix} \sum_{i=1}^{m} f_{i1}(n)b_i(\theta) & \cdots & \sum_{i=1}^{m} f_{i,m+2}(n)b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & & \vdots \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix}$$

$$+ \begin{vmatrix} f_{m+1,1}(n)b_{m+1}(\theta) & \cdots & f_{m+1,m+2}(n)b_{m+1}(\theta) \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & & \vdots \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix}$$

.... (2.52)

dengan:

$$\begin{vmatrix} f_{m+1,1}(n)b_{m+1}(\theta) & \cdots & f_{m+1,m+2}(n)b_{m+1}(\theta) \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & & \vdots \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix} = 0 \qquad (2.53)$$

karena baris m+2 merupakan kelipatan dari baris pertama, maka:

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$\begin{vmatrix} \sum_{i=1}^{m+1} f_{i1}(n) b_i(\theta) & \cdots & \sum_{i=1}^{m+1} f_{i,m+2}(n) b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & & \vdots \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix} = 0 \tag{2.54}$$

Sehingga

$$E\left[G\left(\hat{\theta}_1, \ldots, \hat{\theta}_{m+2}\right)\right] = \theta + \frac{\begin{vmatrix} \sum_{i=1}^{m+1} f_{i1}(n) b_i(\theta) & \cdots & \sum f_{im+2}(n) b_i(\theta) \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix}}{\begin{vmatrix} 1 & \cdots & 1 \\ f_{11}(n) & \cdots & f_{1,m+2}(n) \\ \vdots & & \vdots \\ f_{m+1,1}(n) & \cdots & f_{m+1,m+2}(n) \end{vmatrix}}$$

$$E\left[G\left(\hat{\theta}_1, \ldots, \hat{\theta}_{m+2}\right)\right] = \theta + 0 = \theta \tag{2.55}$$

Bila berlaku untuk k=m maka berlaku untuk k=m+1 ■

Yang dimaksud dari theorema 2.4.1 adalah, jika $f_{ij}(n)$ diketahui dan a_{ij} didefinisikan dengan $a_{ij} = f_{ij}(n)$, maka $G(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1})$ yang sesuai dengan persamaan (2.31) merupakan estimator untuk θ.

Theorema 2.4.2. :

Jika bias dalam $\hat{\theta}_j$, j=1,2,...,k+1 seperti dalam theorema 2.4.1 dan untuk semua i dan j, $f_{ij} = \dfrac{1}{(n-j+1)^i}$, maka:

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$E\left[G\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{k+1}\right)\right] = \theta + 0\left(n^{-k-1}\right) \tag{2.56}$$

yaitu bias dari $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1}$ mempunyai order n^{-1} sedangkan bias dari $G\left(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1}\right)$ mempunyai order n^{-k-1}.

Bukti :

Dari theorema 2.4.1. dan dengan mengambil $f_{ij} = \dfrac{1}{(n-j+1)^i}$,

kemudiaan dapat ditulis :

$$E\left[\hat{\theta}_j\right] = \theta + \sum_{i=1}^{\infty} \frac{b_i(\theta)}{(n-j+1)^i} \tag{2.57}$$

Misal diambil untuk j=3, sehingga :

$$E\left[\hat{\theta}_3\right] = \theta + \frac{b_1(\theta)}{(n-2)} + \frac{b_2(\theta)}{(n-2)^2} + \frac{b_3(\theta)}{(n-2)^3} + \ldots \tag{2.58}$$

dari persamaan diatas dapat dilihat bahwa bias dari $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1}$ mempunyai order n^{-1}. Kemudian akan ditunjukkan bahwa $E\left[G\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{k+1}\right) - \theta\right]$ mempunyai order n^{-k-1}.

Dari persamaan (2.31):

$$G\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{k+1}\right) = \frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \cdots & \hat{\theta}_{k+1} \\ a_{11} & a_{12} & \cdots & a_{1,k+1} \\ \vdots & \vdots & & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{k,k+1} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ a_{11} & a_{12} & \cdots & a_{1,k+1} \\ \vdots & \vdots & & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{k,k+1} \end{vmatrix}}$$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

dimana $a_{ij} = f_{ij}(n)$.

Dengan mengambil $f_{ij} = \dfrac{1}{(n-j+1)^i}$ sehingga diperoleh harga harapannya adalah :

$$E\left[G(\hat{\theta}_1,\ldots,\hat{\theta}_{k+1})\right] = \frac{\begin{vmatrix} E[\hat{\theta}_1] & E[\hat{\theta}_2] & \cdots & E[\hat{\theta}_{k+1}] \\ \frac{1}{n} & \frac{1}{n-1} & \cdots & \frac{1}{n-k} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^k} & \frac{1}{(n-1)^k} & \cdots & \frac{1}{(n-k)^k} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{n-1} & \cdots & \frac{1}{n-k} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^k} & \frac{1}{(n-1)^k} & \cdots & \frac{1}{(n-k)^k} \end{vmatrix}} \tag{2.59}$$

Misal diambil untuk k=2, sehingga dapat ditulis:

$$E\left[G(\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3)\right] = \theta + \frac{\begin{vmatrix} \sum_{i=1}^{\infty}\frac{b_i(\theta)}{n^i} & \sum_{i=1}^{\infty}\frac{b_i(\theta)}{(n-1)^i} & \sum_{i=1}^{\infty}\frac{b_i(\theta)}{(n-2)^i} \\ \frac{1}{n} & \frac{1}{n-1} & \frac{1}{n-2} \\ \frac{1}{n^2} & \frac{1}{(n-1)^2} & \frac{1}{(n-2)^2} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{n} & \frac{1}{n-1} & \frac{1}{n-2} \\ \frac{1}{n^2} & \frac{1}{(n-1)^2} & \frac{1}{(n-2)^2} \end{vmatrix}} \tag{2.60}$$

dengan memisahkan biasnya dapat ditulis:

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$
E\left[G\left(\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3\right)\right]
$$

$$
=\theta+\frac{\begin{vmatrix}\sum_{i=1}^{3}\frac{b_i(\theta)}{n^i} & \sum_{i=1}^{3}\frac{b_i(\theta)}{(n-1)^i} & \sum_{i=1}^{3}\frac{b_i(\theta)}{(n-2)^i}\\ \frac{1}{n} & \frac{1}{n-1} & \frac{1}{n-2}\\ \frac{1}{n^2} & \frac{1}{(n-1)^2} & \frac{1}{(n-2)^2}\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\ \frac{1}{n} & \frac{1}{n-1} & \frac{1}{n-2}\\ \frac{1}{n^2} & \frac{1}{(n-1)^2} & \frac{1}{(n-2)^2}\end{vmatrix}}+\frac{\begin{vmatrix}\sum_{i=4}^{\infty}\frac{b_i(\theta)}{n^i} & \sum_{i=4}^{\infty}\frac{b_i(\theta)}{(n-1)^i} & \sum_{i=4}^{\infty}\frac{b_i(\theta)}{(n-2)^i}\\ \frac{1}{n} & \frac{1}{n-1} & \frac{1}{n-2}\\ \frac{1}{n^2} & \frac{1}{(n-1)^k} & \frac{1}{(n-2)^k}\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\ \frac{1}{n} & \frac{1}{n-1} & \frac{1}{n-2}\\ \frac{1}{n^k} & \frac{1}{(n-1)^k} & \frac{1}{(n-2)^k}\end{vmatrix}}
$$

$$
=\theta+\frac{\left\{\begin{array}{l}\left[\sum_{i=1}^{3}\frac{b_i(\theta)}{n^i}\frac{1}{(n-1)(n-2)^2}+\sum_{i=1}^{3}\frac{b_i(\theta)}{(n-1)^i}\frac{1}{n^2(n-2)}+\sum_{i=1}^{3}\frac{b_i(\theta)}{(n-2)^i}\frac{1}{n(n-1)^2}\right]\\ -\left[\sum_{i=1}^{3}\frac{b_i(\theta)}{n^i}\frac{1}{(n-1)^2(n-2)}+\sum_{i=1}^{3}\frac{b_i(\theta)}{(n-1)^i}\frac{1}{n(n-2)^2}+\sum_{i=1}^{3}\frac{b_i(\theta)}{(n-2)^i}\frac{1}{n^2(n-1)}\right]\end{array}\right\}}{\left\{\begin{array}{l}\left[\frac{1}{(n-1)^2(n-2)}+\frac{1}{n(n-2)^2}+\frac{1}{n^2(n-1)}\right]\\ -\left[\frac{1}{(n-1)(n-2)^2}+\frac{1}{n^2(n-2)}+\frac{1}{n(n-1)^2}\right]\end{array}\right\}}
$$

$$
+\frac{\left\{\begin{array}{l}\left[\sum_{i=4}^{\infty}\frac{b_i(\theta)}{n^i}\frac{1}{(n-1)(n-2)^2}+\sum_{i=4}^{\infty}\frac{b_i(\theta)}{(n-1)^i}\frac{1}{n^2(n-2)}+\sum_{i=4}^{\infty}\frac{b_i(\theta)}{(n-2)^i}\frac{1}{n(n-1)^2}\right]\\ -\left[\sum_{i=4}^{\infty}\frac{b_i(\theta)}{n^i}\frac{1}{(n-1)^2(n-2)}+\sum_{i=4}^{\infty}\frac{b_i(\theta)}{(n-1)^i}\frac{1}{n(n-2)^2}+\sum_{i=4}^{\infty}\frac{b_i(\theta)}{(n-2)^i}\frac{1}{n^2(n-1)}\right]\end{array}\right\}}{\left\{\begin{array}{l}\left[\frac{1}{(n-1)^2(n-2)}+\frac{1}{n(n-2)^2}+\frac{1}{n^2(n-1)}\right]\\ -\left[\frac{1}{(n-1)(n-2)^2}+\frac{1}{n^2(n-2)}+\frac{1}{n(n-1)^2}\right]\end{array}\right\}}
$$

.... 2.61)

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

dari perhitungan tersebut , kemudian diperoleh bentuk:

$$E\left[G\left(\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3\right)\right]=\theta+\frac{b_3(\theta)}{n(n-1)(n-2)}+\dots \tag{2.62}$$

$$=\theta+O\left(n^{-3}\right) \tag{2.63}$$

Dari penyelesaian tersebut dapat disimpulkan bahwa bias dari

$G\left(\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3\right)$ mempunyai order n $^{-3}$

Sehingga: $E\left[G\left(\hat{\theta}_1,\hat{\theta}_2,\dots,\hat{\theta}_{k+1}\right)\right]=\theta+0\left(n^{-k-1}\right)$.. ∎

Akibat dari theorema 2.4.2.

Jika $$E\left[\hat{\theta}_j\right]=\theta+\sum_{i=1}^{k}\frac{b_i(\theta)}{(n-j+1)^i} \qquad j=1,2,\dots,k+1 \tag{2.64}$$

maka $G\left(\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3,\dots,\hat{\theta}_{k+1}\right)$ adalah tak bias .

Bukti:

Dengan menggunakan induksi matematik.

- dibuktikan benar untuk k=1

$$G\left(\hat{\theta}_1,\hat{\theta}_2\right)=\frac{\begin{vmatrix}\hat{\theta}_1 & \hat{\theta}_2\\ \frac{1}{n} & \frac{1}{(n-1)}\end{vmatrix}}{\begin{vmatrix}1 & 1\\ \frac{1}{n} & \frac{1}{(n-1)}\end{vmatrix}} \tag{2.65}$$

Akan dibuktikan bahwa Generalized Jackknife order dua $G\left(\hat{\theta}_1,\hat{\theta}_2\right)$ adalah estimator tak bias

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$E\left[G\left(\hat{\theta}_1,\hat{\theta}_2\right)\right]=\frac{\begin{vmatrix} E\left[\hat{\theta}_1\right] & E\left[\hat{\theta}_2\right] \\ \frac{1}{n} & \frac{1}{(n-1)} \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} \end{vmatrix}} \tag{2.66}$$

$$E\left[G\left(\hat{\theta}_1,\hat{\theta}_2\right)\right]=\theta+\frac{\begin{vmatrix} \frac{b_i(\theta)}{n^i} & \frac{b_i(\theta)}{(n-1)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} \end{vmatrix}}$$

$$=\theta+0=\theta \tag{2.67}$$

Andaikan benar untuk k=m akan dibuktikan benar untuk k=m+1

- Untuk k=m.

$$G\left(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{m+1}\right)=\frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \cdots & \hat{\theta}_{m+1} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix}} \tag{2.68}$$

Kemudian harga harapan dari $G\left(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{m+1}\right)$ adalah:

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$E\left[G\left(\hat{\theta}_1,\ldots,\hat{\theta}_{m+1}\right)\right]=\frac{\begin{vmatrix} E\left[\hat{\theta}_1\right] & E\left[\hat{\theta}_2\right] & \cdots & E\left[\hat{\theta}_{m+1}\right] \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix}} \tag{2.69}$$

$$E\left[G\left(\hat{\theta}_1,\ldots,\hat{\theta}_{m+1}\right)\right]=\frac{\begin{vmatrix} \sum_{i=1}^{m}\frac{b_i(\theta)}{n^i} & \sum_{i=1}^{m}\frac{b_i(\theta)}{(n-1)^i} & \cdots & \sum_{i=1}^{m}\frac{b_i(\theta)}{(n-m)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix}}$$

karena tidak mempunyai baris yang merupakan kelipatan dari baris lainnya,

sehingga: $$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix} \neq 0 \tag{2.70}$$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

Kemudian dibuktikan :

$$\begin{vmatrix} \sum_{i=1}^{m} \frac{b_i(\theta)}{n^i} & \sum_{i=1}^{m} \frac{b_i(\theta)}{(n-1)^i} & \cdots & \sum_{i=1}^{m} \frac{b_i(\theta)}{(n-m)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix} = 0 \qquad (2.71)$$

Dengan memisahkan baris pertama persamaan(2.72), kemudian diperoleh:

$$\begin{vmatrix} \sum_{i=1}^{m} \frac{b_i(\theta)}{n^i} & \sum_{i=1}^{m} \frac{b_i(\theta)}{(n-1)^i} & \cdots & \sum_{i=1}^{m} \frac{b_i(\theta)}{(n-m)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix} = \begin{vmatrix} \frac{b_1(\theta)}{n} & \frac{b_1(\theta)}{(n-1)} & \cdots & \frac{b_1(\theta)}{(n-m)} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix} +$$

$$\begin{vmatrix} \frac{b_2(\theta)}{n^2} & \frac{b_2(\theta)}{(n-1)^2} & \cdots & \frac{b_2(\theta)}{(n-m)^2} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix} + \dots + \begin{vmatrix} \frac{b_m(\theta)}{n^m} & \frac{b_m(\theta)}{(n-1)^m} & \cdots & \frac{b_m(\theta)}{(n-m)^m} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix}$$

....(2.72)

Dari pemisahan penjumlahan baris pertama dapat dilihat bahwa sekarang tiap baris pertama mempunyai kelipatan dari baris-baris dibawahnya sehingga dapat disimpulkan bahwa :

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$\begin{vmatrix} \sum_{i=1}^{m}\frac{b_i(\theta)}{n^i} & \sum_{i=1}^{m}\frac{b_i(\theta)}{(n-1)^i} & \cdots & \sum_{i=1}^{m}\frac{b_i(\theta)}{(n-m)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m)^m} \end{vmatrix} = 0$$

Sehingga : $E\left[G(\hat{\theta}_1,\ldots,\hat{\theta}_{m+1})\right] = \theta + 0$ (2.73)

- untuk k=m+1

$$G(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{m+2}) = \frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \cdots & \hat{\theta}_{m+2} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix}} \qquad (2.74)$$

Kemudian harga harapannya dapat ditulis:

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$E\left[G\left(\hat{\theta}_1,\ldots,\hat{\theta}_{m+2}\right)\right]=\frac{\begin{vmatrix} E\left[\hat{\theta}_1\right] & E\left[\hat{\theta}_2\right] & \cdots & E\left[\hat{\theta}_{m+2}\right] \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix}} \tag{2.75}$$

$$E\left[G\left(\hat{\theta}_1,\ldots,\hat{\theta}_{m+2}\right)\right]=\frac{\begin{vmatrix} \sum_{i=1}^{m+1}\frac{b_i(\theta)}{n^i} & \sum_{i=1}^{m+1}\frac{b_i(\theta)}{(n-1)^i} & \cdots & \sum_{i=1}^{m+1}\frac{b_i(\theta)}{(n-m-1)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix}}$$

karena:

$$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix} \neq 0 \tag{2.76}$$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

kemudian harus dibuktikan bahwa:

$$\begin{vmatrix} \sum_{i=1}^{m+1} \frac{b_i(\theta)}{n^i} & \sum_{i=1}^{m+1} \frac{b_i(\theta)}{(n-1)^i} & \cdots & \sum_{i=1}^{m+1} \frac{b_i(\theta)}{(n-m-1)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix} = 0 \qquad (2.77)$$

Dari pemisahan :

$$\begin{vmatrix} \sum_{i=1}^{m+1} \frac{b_i(\theta)}{n^i} & \sum_{i=1}^{m+1} \frac{b_i(\theta)}{(n-1)^i} & \cdots & \sum_{i=1}^{m+1} \frac{b_i(\theta)}{(n-m-1)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix}$$

$$= \begin{vmatrix} \sum_{i=1}^{m} \frac{b_i(\theta)}{n^i} & \sum_{i=1}^{m} \frac{b_i(\theta)}{(n-1)^i} & \cdots & \sum_{i=1}^{m} \frac{b_i(\theta)}{(n-m-1)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix}$$

$$+ \begin{vmatrix} \frac{b_i(\theta)}{n} & \frac{b_i(\theta)}{(n-1)} & \cdots & \frac{b_i(\theta)}{(n-m-1)} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix} \qquad (2.78)$$

Analog dari persamaan (2.72) sehingga diperoleh :

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$\begin{vmatrix} \sum_{i=1}^{m} \frac{b_i(\theta)}{n^i} & \sum_{i=1}^{m} \frac{b_i(\theta)}{(n-1)^i} & \cdots & \sum_{i=1}^{m} \frac{b_i(\theta)}{(n-m-1)^i} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^m} & \frac{1}{(n-1)^m} & \cdots & \frac{1}{(n-m-1)^m} \end{vmatrix} = 0 \qquad (2.79)$$

dan

$$\begin{vmatrix} \frac{b_i(\theta)}{n} & \frac{b_i(\theta)}{(n-1)} & \cdots & \frac{b_i(\theta)}{(n-m-1)} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-m-1)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^{m+1}} & \frac{1}{(n-1)^{m+1}} & \cdots & \frac{1}{(n-m-1)^{m+1}} \end{vmatrix} = 0 \qquad (2.80)$$

karena baris ke 2 merupakan kelipatan baris pertama.

Sehingga: $$E\left[G\left(\hat{\theta}_1, \ldots, \hat{\theta}_{m+2}\right)\right] = \theta + 0 \qquad (2.81)$$

Bila berlaku untuk k=m maka berlaku juga untuk k=m+1 ■

Maka dapat disimpulkan bahwa Generalized Jackknife order k $G(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1})$ adalah estimator tak bias.

Dalam theorema diatas tidak disebutkan bagaimana k estimator $\hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1}$ akan dipilih untuk $\hat{\theta}_1$ yang telah diberikan.Untuk mendapatkan estimator $\hat{\theta}_2, \hat{\theta}_3, \ldots, \hat{\theta}_{k+1}$ digunakan metode Quenouille pada order yang lebih dari satu,yang akan diberikan pada sub bab berikut.

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

2.5. Metode Quenouille pada order lebih dari dua

Metode Quenouille pada order lebih dari satu digunakan untuk mendapatkan k estimator $\hat{\theta}_2,\hat{\theta}_3,\ldots,\hat{\theta}_{k+1}$ yang akan dipilih untuk $\hat{\theta}_1$ yang telah diberikan.Metode ini merupakan perluasan dari metode Quenouille yang lebih sederhana. Perluasan ini diberikan pada definisi berikut ini :

Definisi 2.5.1. :

Misal k<n dan misalkan $\hat{\theta}_1^{\ i_2},\hat{\theta}_1^{\ i_2 i_3},\ldots,\hat{\theta}_1^{\ i_2\ldots i_{k+1}}$, estimator yang dihasilkan dari pembatasan $\hat{\theta}_1$ yang telah diberikan untuk sampel yang didapatkan dengan penghapusan pada pengamatan pengamatan random 1,2,...,k .

Maka. didefinisikan :

$$\hat{\theta}_j = \overline{\hat{\theta}_1^{\ i_2,\ldots,i_j}} \qquad j=2,3,\ldots,k+1 \tag{2.82}$$

Berdasarkan theorema 2.5.1. dengan mengambil

$$E\left[\hat{\theta}_j\right] = \theta + \sum_{i=1}^{k} \frac{b_i(\theta)}{(n-j+1)^i}$$

$$= \theta + B(n-j+1)$$

Sehingga dapat ditulis:

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$E[\hat{\theta}_1] = \theta + \sum_{i=1}^{k} \frac{b_i}{(n-1+1)^i}$$

$$= \theta + B(n,\theta)$$

$$= \theta + \frac{b_1}{n} + \frac{b_2}{n^2} + \ldots \qquad (2.83)$$

$$E[\hat{\theta}_2] = \theta + B(n-1,\theta)$$

$$= \theta + \frac{b_1}{(n-j+1)} + \frac{b_2}{(n-j+i)^2} + \ldots \qquad (2.84)$$

Dari persamaan diatas menghasilkan

$$G(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{k+1}) = \frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \cdots & \hat{\theta}_{k+1} \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-k)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^k} & \frac{1}{(n-1)^k} & \cdots & \frac{1}{(n-k)^k} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-k)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n^k} & \frac{1}{(n-1)^k} & \cdots & \frac{1}{(n-k)^k} \end{vmatrix}} \qquad (2.85)$$

Untuk selanjutnya akan digunakan notasi $G^k(\hat{\theta})$ untuk menuliskan Generalized Jackknife order k $G(\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3,\ldots,\hat{\theta}_{k+1})$.

Pada saat bentuk dasar dari Generalized Jackknife order k $G(\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3,\ldots,\hat{\theta}_{k+1})$ dapat disajikan dalam bentuk

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$G\left(\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{k+1}\right)=\frac{\begin{vmatrix}\hat{\theta}_1 & \hat{\theta}_2 & \cdots & \hat{\theta}_{k+1}\\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-k)}\\ \vdots & \vdots & & \vdots\\ \frac{1}{n^k} & \frac{1}{(n-1)^k} & \cdots & \frac{1}{(n-k)^k}\end{vmatrix}}{\begin{vmatrix}1 & 1 & \cdots & 1\\ \frac{1}{n} & \frac{1}{(n-1)} & \cdots & \frac{1}{(n-k)}\\ \vdots & \vdots & & \vdots\\ \frac{1}{n^k} & \frac{1}{(n-1)^k} & \cdots & \frac{1}{(n-k)^k}\end{vmatrix}} \tag{2.86}$$

dimana $\hat{\theta}_1=\hat{\theta}$, akan digunakan notasi $J^k\left(\hat{\theta}\right)$ yaitu Jackknife order k. Sehingga hubungannya dapat dituliskan sebagai berikut :

Generalized Jackknife order k sama dengan Jackknife order k pada

saat $$f_{ij}=\frac{1}{\left(n-j+1\right)^i}. \tag{2.87}$$

Dari uraian diatas dengan mengambil k=2 sehingga akan diperoleh bentuk Jackknife order dua sebagai berikut:

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)

$$G^2(\hat{\theta}) = G(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3) = \theta + \frac{\begin{vmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \hat{\theta}_3 \\ \frac{1}{n} & \frac{1}{n-1} & \frac{1}{n-2} \\ \frac{1}{n^2} & \frac{1}{(n-1)^2} & \frac{1}{(n-2)^2} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{n} & \frac{1}{n-1} & \frac{1}{n-2} \\ \frac{1}{n^2} & \frac{1}{(n-1)^2} & \frac{1}{(n-2)^2} \end{vmatrix}}$$

$$= \frac{\hat{\theta}_1}{(n-1)(n-2)^2} - \frac{\hat{\theta}_1}{(n-1)^2(n-2)} + \frac{\hat{\theta}_2}{n^2(n-2)} - \frac{\hat{\theta}_2}{n(n-2)^2} + \frac{\hat{\theta}_3}{n(n-1)^2} - \frac{\hat{\theta}_3}{n^2(n-1)}$$

$$/ \quad \frac{1}{(n-1)(n-2)^2} - \frac{1}{(n-1)^2(n-2)} + \frac{1}{n^2(n-2)} - \frac{1}{n(n-2^2)} + \frac{1}{n(n-1)^2} - \frac{1}{n^2(n-1)}$$

$$= \frac{n^2\hat{\theta}_1 - 2(n-1)^2\hat{\theta}_2 + (n-2)^2\hat{\theta}_3}{n^2 - 2(n-1)^2 + (n-2)^2}$$

$$= \frac{1}{2}\left[n^2\hat{\theta}_1 - 2(n-1)^2\hat{\theta}_2 + (n-2)^2\hat{\theta}_3\right] \tag{2.88}$$

Sehingga dapat ditulis :

$$G^2(\hat{\theta}) = J^{(2)}(\hat{\theta}) = \frac{1}{2}\left[n^2\hat{\theta}_1 - 2(n-1)^2\hat{\theta}_2 + (n-2)^2\hat{\theta}_3\right] \tag{2.89}$$

Dari persamaan (2.89), Jackknife order dua didefinisikan sebagai:

$$J^{(2)}(\hat{\theta}) = \frac{1}{2}\left[n^2\hat{\theta}_1 - 2(n-1)^2\hat{\theta}_2 + (n-2)^2\hat{\theta}_3\right] \tag{2.90}$$

This document is Undip Institutional Repository Collection. The author(s) or copyright owner(s) agree that UNDIP-IR may, without changing the content, translate the submission to any medium or format for the purpose of preservation. The author(s) or copyright owner(s) also agree that UNDIP-IR may keep more than one copy of this submission for purpose of security, back-up and preservation:
(http://eprints.undip.ac.id)