LAMPIRAN

Lampiran 1 : Matriks informasi $\mathfrak{I}$ untuk model probit

$$\mathfrak{I}_{00} = E(U_0U_0)$$

$$=E\left\{\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1-n_1\pi_1}{\pi_1(1-\pi_1)}\right)+\cdots+\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N-n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right]\cdot\right.$$

$$\left.\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1-n_1\pi_1}{\pi_1(1-\pi_1)}\right)+\cdots+\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N-n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right]\right\}$$

$$=E\left\{\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\left(\frac{y_1-n_1\pi_1}{\pi_1(1-\pi_1)}\right)^2+\cdots+\right.$$

$$\left.\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\left(\frac{y_N-n_N\pi_N}{\pi_N(1-\pi_N)}\right)^2\right\}$$

$$=E\left[\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\left(\frac{y_1-n_1\pi_1}{\pi_1(1-\pi_1)}\right)^2\right]+\cdots+$$

$$E\left[\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\left(\frac{y_N-n_N\pi_N}{\pi_N(1-\pi_N)}\right)^2\right]$$

$$=\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\frac{n_1\pi_1(1-\pi_1)}{(\pi_1(1-\pi_1))^2}+\cdots+\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\frac{n_N\pi_N(1-\pi_N)}{(\pi_N(1-\pi_N))^2}$$

$$=\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\frac{n_1}{\pi_1(1-\pi_1)}+\cdots+\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\frac{n_N}{\pi_N(1-\pi_N)}$$

$$\mathfrak{I}_{00}=\sum_{i=1}^{N}\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{ij}\right)\right]^2\frac{n_i}{\pi_i(1-\pi_i)}$$

Dalam hal ini, $E\left[\left(\frac{y_i - n_i\pi_i}{\pi_i(1-\pi_i)}\right)\cdot\left(\frac{y_l - n_l\pi_l}{\pi_l(1-\pi_l)}\right)\right] = 0$, $\forall\ i \neq l,\ i = 1, \ldots, N$ dan $l = 1,\ldots,N$

Bukti

$$E\left[\left(\frac{y_i - n_i\pi_i}{\pi_i(1-\pi_i)}\right)\cdot\left(\frac{y_l - n_l\pi_l}{\pi_l(1-\pi_l)}\right)\right] = E\left[\frac{1}{\pi_i\pi_l(1-\pi_i)(1-\pi_l)}(y_i y_l - y_i n_l \pi_l - y_l n_i \pi_i + n_i\pi_i n_l\pi_l)\right] = \frac{1}{\pi_i\pi_l(1-\pi_i)(1-\pi_l)} E(y_i y_l) - E(y_i n_l \pi_l) - E(y_l n_i \pi_i) + E(n_i \pi_i n_l \pi_l)$$

$$= \frac{1}{\pi_i\pi_l(1-\pi_i)(1-\pi_l)} E(y_i)E(y_l) - n_l\pi_l E(y_i) - n_i\pi_i E(y_l) + n_i n_l \pi_i \pi_l$$

$$= \frac{1}{\pi_i\pi_l(1-\pi_i)(1-\pi_l)} (n_i\pi_i n_l\pi_l - n_i\pi_i n_l\pi_l - n_i\pi_i n_l\pi_l + n_i\pi_i n_l\pi_l) = 0$$

$$\Im_{01} = \Im_{10} = E(U_0 U_1)$$

$$= E\left\{\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right) + \cdots + \phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right]\cdot\right.$$

$$\left.\left[x_{11}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right) + \cdots + x_{N1}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right]\right\}$$

$$= E\left\{x_{11}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right)^2 + \cdots +\right.$$

$$\left. x_{N1}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)^2\right\}$$

$$= x_{11}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2 \frac{n_1\pi_1(1-\pi_1)}{\left(\pi_1(1-\pi_1)\right)^2} + \cdots +$$

$$x_{N1}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2 \frac{n_N\pi_N(1-\pi_N)}{\left(\pi_N(1-\pi_N)\right)^2}$$

$$\Im_{01} = \sum_{i=1}^{N} x_{i1}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{ij}\right)\right]^2 \frac{n_i}{\pi_i(1-\pi_i)}$$

$$\vdots$$

$$\Im_{0p} = \Im_{p0} = E\left(U_0 U_p\right)$$

$$= E\left\{\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right) + \cdots + \phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right].\right.$$

$$\left.\left[x_{1p}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right) + \cdots + x_{Np}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right]\right\}$$

$$= E\left\{x_{1p}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right)^2 + \cdots +\right.$$

$$\left.x_{Np}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)^2\right\}$$

$$= x_{1p}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2 \frac{n_1\pi_1(1-\pi_1)}{\left(\pi_1(1-\pi_1)\right)^2} + \cdots +$$

$$x_{Np}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2 \frac{n_N\pi_N(1-\pi_N)}{\left(\pi_N(1-\pi_N)\right)^2}$$

$$\Im_{0p} = \sum_{i=1}^{N} x_{ip}\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{ij}\right)\right]^2 \frac{n_i}{\pi_i(1-\pi_i)}$$

$$\Im_{11} = E\left(U_1 U_1\right)$$

$$= E\left\{\left[x_{11}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right) + \cdots + x_{N1}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right].\right.$$

$$\left.\left[x_{11}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right) + \cdots + x_{N1}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right]\right\}$$

$$= E\left\{x_{11}^2\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right)^2 + \cdots + \right.$$

$$\left. x_{N1}^2\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)^2\right\}$$

$$= x_{11}^2\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2 \frac{n_1\pi_1(1-\pi_1)}{\left(\pi_1(1-\pi_1)\right)^2} + \cdots +$$

$$x_{N1}^2\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2 \frac{n_N\pi_N(1-\pi_N)}{\left(\pi_N(1-\pi_N)\right)^2}$$

$$\Im_{11} = \sum_{i=1}^{N} x_{i1}^2\left[\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{ij}\right)\right]^2 \frac{n_i}{\pi_i(1-\pi_i)}$$

$$\Im_{12} = \Im_{21} = E\left(U_1 U_2\right)$$

$$= E\left\{\left[x_{11}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1 - n_1\pi_1}{\pi_1(1-\pi_1)}\right) + \cdots + x_{N1}.\phi\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N - n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right].\right.$$

$$\left[x_{12}.\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1-n_1\pi_1}{\pi_1(1-\pi_1)}\right)+\cdots+x_{N2}.\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N-n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right]\Bigg\}$$

$$=E\left\{x_{11}.x_{12}.\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\left(\frac{y_1-n_1\pi_1}{\pi_1(1-\pi_1)}\right)^2+\cdots+\right.$$

$$\left.x_{N1}.x_{N2}.\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\left(\frac{y_N-n_N\pi_N}{\pi_N(1-\pi_N)}\right)^2\right\}$$

$$=x_{11}.x_{12}.\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\frac{n_1\pi_1(1-\pi_1)}{(\pi_1(1-\pi_1))^2}+\cdots+$$

$$x_{N1}.x_{N2}.\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\frac{n_N\pi_N(1-\pi_N)}{(\pi_N(1-\pi_N))^2}$$

$$\Im_{12}=\sum_{i=1}^{N}x_{i1}.x_{i2}.\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{ij}\right)\right]^2\frac{n_i}{\pi_i(1-\pi_i)}$$

$\vdots$

$$\Im_{pp}=E\left(U_pU_p\right)$$

$$=E\left\{\left[x_{1p}.\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1-n_1\pi_1}{\pi_1(1-\pi_1)}\right)+\cdots+x_{Np}.\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N-n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right].\right.$$

$$\left.\left[x_{1p}.\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\left(\frac{y_1-n_1\pi_1}{\pi_1(1-\pi_1)}\right)+\cdots+x_{Np}.\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\left(\frac{y_N-n_N\pi_N}{\pi_N(1-\pi_N)}\right)\right]\right\}$$

$$=E\left\{x_{1p}^2\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\left(\frac{y_1-n_1\pi_1}{\pi_1(1-\pi_1)}\right)^2+\cdots+\right.$$

$$x_{Np}^{2}\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\left(\frac{y_N-n_N\pi_N}{\pi_N(1-\pi_N)}\right)^2\Bigg\}$$

$$=x_{1p}^{2}\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{1j}\right)\right]^2\frac{n_1\pi_1(1-\pi_1)}{(\pi_1(1-\pi_1))^2}+\cdots+$$

$$x_{Np}^{2}\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{Nj}\right)\right]^2\frac{n_N\pi_N(1-\pi_N)}{(\pi_N(1-\pi_N))^2}$$

$$\Im_{pp}=\sum_{i=1}^{N}x_{ip}^{2}\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{ij}\right)\right]^2\frac{n_i}{\pi_i(1-\pi_i)}$$

Secara umum, elemen matriks informasi untuk model probit dapat dituliskan :

$$\Im_{sk}=E(U_sU_k)=E\left[\frac{\partial l}{\partial\beta_s}\frac{\partial l}{\partial\beta_k}\right]=\sum_{i=1}^{N}x_{is}.x_{ik}\left[\phi\left(\beta_0+\sum_{j=1}^{p}\beta_j x_{ij}\right)\right]^2\frac{n_i}{\pi_i(1-\pi_i)}$$

dengan $s = 0, 1, \ldots, p$ dan $k = 0, 1, \ldots, p$.

Lampiran 2 : Matriks informasi $\Im$ untuk model logistik

$$\Im_{00} = E(U_0U_0)$$

$$= E[(y_1 - n_1\pi_1) + \cdots + (y_N - n_N\pi_N) \cdot (y_1 - n_1\pi_1) + \cdots + (y_N - n_N\pi_N)]$$

$$= E[(y_1 - n_1\pi_1)^2 + \cdots + (y_N - n_N\pi_N)^2]$$

$$= E((y_1 - n_1\pi_1)^2) + \cdots + E((y_N - n_N\pi_N)^2)$$

$$\Im_{00} = n_1\pi_1(1-\pi_1) + \cdots + n_N\pi_N(1-\pi_N) = \sum_{i=1}^{N} n_i\pi_i(1-\pi_i)$$

Dalam hal ini, $E[(y_i - n_i\pi_i)\cdot(y_l - n_l\pi_l)] = 0$, $\forall\ i \neq l$, $i = 1, \ldots, N$ dan $l = 1, \ldots, N$

$$\Im_{01} = \Im_{10} = E(U_0U_1)$$

$$= E[(y_1 - n_1\pi_1) + \cdots + (y_N - n_N\pi_N) \cdot x_{11}.(y_1 - n_1\pi_1) + \cdots + x_{N1}.(y_N - n_N\pi_N)]$$

$$= E[x_{11}.(y_1 - n_1\pi_1)^2 + \cdots + x_{N1}.(y_N - n_N\pi_N)^2]$$

$$= x_{11}.E((y_1 - n_1\pi_1)^2) + \cdots + x_{N1}.E((y_N - n_N\pi_N)^2)$$

$$\Im_{01} = x_{11}n_1\pi_1(1-\pi_1) + \cdots + x_{N1}n_N\pi_N(1-\pi_N) = \sum_{i=1}^{N} x_{i1}n_i\pi_i(1-\pi_i)$$

$$\vdots$$

$$\Im_{0p} = \Im_{p0} = E(U_0U_p)$$

$$= E[(y_1 - n_1\pi_1) + \cdots + (y_N - n_N\pi_N) \cdot x_{1p}.(y_1 - n_1\pi_1) + \cdots + x_{Np}.(y_N - n_N\pi_N)]$$

$$= E[x_{1p}.(y_1 - n_1\pi_1)^2 + \cdots + x_{Np}.(y_N - n_N\pi_N)^2]$$

$$= x_{1p}.E((y_1 - n_1\pi_1)^2) + \cdots + x_{Np}.E((y_N - n_N\pi_N)^2)$$

$$\Im_{0p} = x_{1p}n_1\pi_1(1-\pi_1) + \cdots + x_{Np}n_N\pi_N(1-\pi_N) = \sum_{i=1}^{N} x_{ip}n_i\pi_i(1-\pi_i)$$

$$\Im_{11} = E(U_1U_1)$$

$$= E[x_{11}(y_1 - n_1\pi_1) + \cdots + x_{N1}(y_N - n_N\pi_N) \cdot x_{11}(y_1 - n_1\pi_1) + \cdots + x_{N1}(y_N - n_N\pi_N)]$$

$$= E\left[x_{11}^2.(y_1 - n_1\pi_1)^2 + \cdots + x_{N1}^2.(y_N - n_N\pi_N)^2\right]$$

$$= x_{11}^2.E\left((y_1 - n_1\pi_1)^2\right) + \cdots + x_{N1}^2.E\left((y_N - n_N\pi_N)^2\right)$$

$$\Im_{11} = x_{11}^2 n_1\pi_1(1-\pi_1) + \cdots + x_{N1}^2 n_N\pi_N(1-\pi_N) = \sum_{i=1}^{N} x_{i1}^2 n_i\pi_i(1-\pi_i)$$

$$\Im_{12} = \Im_{21} = E(U_1U_2)$$

$$= E[x_{11}(y_1 - n_1\pi_1) + \cdots + x_{N1}(y_N - n_N\pi_N) \cdot x_{12}(y_1 - n_1\pi_1) + \cdots + x_{N2}(y_N - n_N\pi_N)]$$

$$= E\left[x_{11}.x_{12}.(y_1 - n_1\pi_1)^2 + \cdots + x_{N1}.x_{N2}.(y_N - n_N\pi_N)^2\right]$$

$$= x_{11}.x_{12}.E\left((y_1 - n_1\pi_1)^2\right) + \cdots + x_{N1}.x_{N2}.E\left((y_N - n_N\pi_N)^2\right)$$

$$\Im_{12} = x_{11}..x_{12}.n_1\pi_1(1-\pi_1) + \cdots + x_{N1}.x_{N2}.n_N\pi_N(1-\pi_N) = \sum_{i=1}^{N} x_{i1}.x_{i2}.n_i\pi_i(1-\pi_i)$$

$$\vdots$$

$$\Im_{pp} = E(U_pU_p)$$

$$= E[x_{1p}(y_1 - n_1\pi_1) + \cdots + x_{Np}(y_N - n_N\pi_N) \cdot x_{1p}(y_1 - n_1\pi_1) + \cdots + x_{Np}(y_N - n_N\pi_N)]$$

$$= E\left[x_{1p}^2.(y_1 - n_1\pi_1)^2 + \cdots + x_{Np}^2.(y_N - n_N\pi_N)^2\right]$$

$$= x_{1p}^2.E\left((y_1 - n_1\pi_1)^2\right) + \cdots + x_{Np}^2.E\left((y_N - n_N\pi_N)^2\right)$$

$$\Im_{0p} = x_{1p}^2.n_1\pi_1(1-\pi_1) + \cdots + x_{Np}^2.n_N\pi_N(1-\pi_N) = \sum_{i=1}^{N} x_{ip}^2.n_i\pi_i(1-\pi_i)$$

Secara umum, elemen matriks informasi untuk model probit dapat dituliskan :

$$\Im_{sk} = E\left(U_s U_k\right) = E\left[\frac{\partial l}{\partial \beta_s}\frac{\partial l}{\partial \beta_k}\right] = \sum_{i=1}^{N} x_{is}.x_{ik}.n_i \pi_i \left(1 - \pi_i\right)$$

dengan $s = 0, 1, \ldots, p$ dan $k = 0, 1, \ldots, p$.

Lampiran 3 : Matriks informasi $\Im$ untuk model nilai ekstrim

$$\Im_{00} = E(U_0U_0)$$

$$= E\left\{\left[\log(1-\pi_1)\left(n_1 - \frac{y_1}{\pi_1}\right) + \cdots + \log(1-\pi_N)\left(n_N - \frac{y_N}{\pi_N}\right)\right]\cdot\right.$$

$$\left.\left[\log(1-\pi_1)\left(n_1 - \frac{y_1}{\pi_1}\right) + \cdots + \log(1-\pi_N)\left(n_N - \frac{y_N}{\pi_N}\right)\right]\right\}$$

$$= E\left\{\left[\log(1-\pi_1)\left(\frac{n_1\pi_1 - y_1}{\pi_1}\right) + \cdots + \log(1-\pi_N)\left(\frac{n_N\pi_N - y_N}{\pi_N}\right)\right]\cdot\right.$$

$$\left.\left[\log(1-\pi_1)\left(\frac{n_1\pi_1 - y_1}{\pi_1}\right) + \cdots + \log(1-\pi_N)\left(\frac{n_N\pi_N - y_N}{\pi_N}\right)\right]\right\}$$

$$= E\left[(\log(1-\pi_1))^2\left(\frac{n_1\pi_1 - y_1}{\pi_1}\right)^2 + \cdots + (\log(1-\pi_N))^2\left(\frac{n_N\pi_N - y_N}{\pi_N}\right)^2\right]$$

$$= E\left[(\log(1-\pi_1))^2\left(\frac{n_1\pi_1 - y_1}{\pi_1}\right)^2\right] + \cdots + E\left[(\log(1-\pi_N))^2\left(\frac{n_N\pi_N - y_N}{\pi_N}\right)^2\right]$$

$$= (\log(1-\pi_1))^2\,\frac{n_1\pi_1(1-\pi_1)}{\pi_1^{\,2}} + \cdots + (\log(1-\pi_N))^2\,\frac{n_N\pi_N(1-\pi_N)}{\pi_N^{\,2}}$$

Dalam hal ini, $E\left[\left(\frac{n_i\pi_i - y_i}{\pi_i}\right)\cdot\left(\frac{n_l\pi_l - y_l}{\pi_l}\right)\right] = 0,\ \forall\ i \neq l,\ i = 1, \ldots, N$ dan $l = 1, \ldots, N$

$$\Im_{00} = \sum_{i=1}^{N}(\log(1-\pi_i))^2\,\frac{n_i(1-\pi_i)}{\pi_1}$$

$$\Im_{01} = \Im_{10} = E(U_0U_1)$$

$$= E\left\{\left[\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\cdot\right.$$

$$\left.\left[x_{11}\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+x_{N1}\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\right\}$$

$$= E\left[x_{11}(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2+\cdots+x_{N1}(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$= E\left[x_{11}(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2\right]+\cdots+E\left[x_{N1}(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$= x_{11}(\log(1-\pi_1))^2\,\frac{n_1\pi_1(1-\pi_1)}{\pi_1^{\,2}}+\cdots+x_{N1}(\log(1-\pi_N))^2\,\frac{n_N\pi_N(1-\pi_N)}{\pi_N^{\,2}}$$

$$\Im_{01} = \sum_{i=1}^{N} x_{i1}(\log(1-\pi_i))^2\,\frac{n_i(1-\pi_i)}{\pi_1}$$

$$\vdots$$

$$\Im_{0p} = \Im_{p0} = E\left(U_0U_p\right)$$

$$= E\left\{\left[\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\cdot\right.$$

$$\left.\left[x_{1p}\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+x_{Np}\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\right\}$$

$$= E\left[x_{1p}(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2+\cdots+x_{Np}(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$= E\left[x_{1p}(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2\right]+\cdots+E\left[x_{Np}(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$= x_{1p}(\log(1-\pi_1))^2 \frac{n_1\pi_1(1-\pi_1)}{\pi_1^{\,2}} + \cdots + x_{Np}(\log(1-\pi_N))^2 \frac{n_N\pi_N(1-\pi_N)}{\pi_N^{\,2}}$$

$$\Im_{0p} = \sum_{i=1}^{N} x_{ip}(\log(1-\pi_i))^2 \frac{n_i(1-\pi_i)}{\pi_1}$$

$$\Im_{11} = E(U_1U_1)$$

$$=E\left\{\left[x_{11}\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+x_{N1}\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\cdot\right.$$

$$\left.\left[x_{11}\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+x_{N1}\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\right\}$$

$$=E\left[x_{11}^{\,2}.(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2+\cdots+x_{N1}^{\,2}.(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$=E\left[x_{11}^{\,2}(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2\right]+\cdots+E\left[x_{N1}^{\,2}(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$= x_{11}^{\,2}.(\log(1-\pi_1))^2 \frac{n_1\pi_1(1-\pi_1)}{\pi_1^{\,2}} + \cdots + x_{N1}^{\,2}.(\log(1-\pi_N))^2 \frac{n_N\pi_N(1-\pi_N)}{\pi_N^{\,2}}$$

$$\Im_{11} = \sum_{i=1}^{N} x_{i1}^{\,2}.(\log(1-\pi_i))^2 \frac{n_i(1-\pi_i)}{\pi_1}$$

$$\Im_{12} = \Im_{21} = E(U_1U_2)$$

$$=E\left\{\left[x_{11}\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+x_{N1}\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\cdot\right.$$

$$\left.\left[x_{12}\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+x_{N2}\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\right\}$$

$$=E\left[x_{11}.x_{12}(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2+\cdots+x_{N1}.x_{N2}(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$=E\left[x_{11}.x_{12}(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2\right]+\cdots+E\left[x_{N1}.x_{N2}(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$=x_{11}.\,x_{12}(\log(1-\pi_1))^2\,\frac{n_1\pi_1(1-\pi_1)}{\pi_1^{\,2}}+\cdots+x_{N1}.\,x_{N2}(\log(1-\pi_N))^2\,\frac{n_N\pi_N(1-\pi_N)}{\pi_N^{\,2}}$$

$$\Im_{12}=\sum_{i=1}^{N}x_{i1}.\,x_{i2}(\log(1-\pi_i))^2\,\frac{n_i(1-\pi_i)}{\pi_1}$$

$$\vdots$$

$$\Im_{pp}=E\left(U_pU_p\right)$$

$$=E\left\{\left[x_{1p}\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+x_{Np}\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\cdot\right.$$

$$\left.\left[x_{1p}\log(1-\pi_1)\left(n_1-\frac{y_1}{\pi_1}\right)+\cdots+x_{Np}\log(1-\pi_N)\left(n_N-\frac{y_N}{\pi_N}\right)\right]\right\}$$

$$=E\left[x_{1p}^{\,2}(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2+\cdots+x_{Np}^{\,2}(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$=E\left[x_{1p}^{\,2}(\log(1-\pi_1))^2\left(\frac{n_1\pi_1-y_1}{\pi_1}\right)^2\right]+\cdots+E\left[x_{Np}^{\,2}(\log(1-\pi_N))^2\left(\frac{n_N\pi_N-y_N}{\pi_N}\right)^2\right]$$

$$=x_{1p}^{\,2}(\log(1-\pi_1))^2\,\frac{n_1\pi_1(1-\pi_1)}{\pi_1^{\,2}}+\cdots+x_{Np}^{\,2}(\log(1-\pi_N))^2\,\frac{n_N\pi_N(1-\pi_N)}{\pi_N^{\,2}}$$

$$\Im_{pp}=\sum_{i=1}^{N}x_{ip}^{\,2}(\log(1-\pi_i))^2\,\frac{n_i(1-\pi_i)}{\pi_1}$$

Secara umum, elemen matriks informasi untuk model nilai ekstrim dapat dituliskan :

$$\Im_{sk} = E(U_s U_k) = E\left[\frac{\partial l}{\partial \beta_s}\frac{\partial l}{\partial \beta_k}\right] = \sum_{i=1}^{N} x_{is}.x_{ik}\left[\log(1-\pi_i)\right]^2 \frac{n_i(1-\pi_i)}{\pi_i}$$

dengan $s = 0, 1, \ldots, p$ dan $k = 0, 1, \ldots, p$.

Lampiran 4 : Pengolahan data pada contoh penerapan untuk model probit

```
> # Input Data
> y <- c(0,0,0,0,0,0,0,2,0,0,1,4,0,1,1,3,0,0,0,0)
> x1<- c(7,7,7,7,7,14,14,14,14,14,27,27,27,27,27,51,51,51,51,51)
> x2 <- c(1.0,1.7,2.2,2.8,4.0,1.0,1.7,2.2,2.8,4.0,1.0,1.7,2.2,2.8,4.0,1.0,1.7,2.2,2.8,4.0)
> n <- c(10,17,7,12,9,31,43,33,31,19,56,44,21,22,16,13,1,1,0,1)
> # Mencari taksiran parameter untuk model reduksi dari model probit
> redmodprobit <- glm (y/n~x1, family = binomial (link = probit), weights = n,
  na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
GLM   linear loop 1: deviance = 61.8305
GLM   linear loop 2: deviance = 21.7025
GLM   linear loop 3: deviance = 14.2238
GLM   linear loop 4: deviance = 13.4411
GLM   linear loop 5: deviance = 13.4246
GLM   linear loop 6: deviance = 13.4246
> summary (redmodprobit)
Call: glm (formula = y/n ~ x1, family = binomial(link = probit), weights = n,
na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
Deviance Residuals:
      Min          1Q       Median          3Q        Max
-1.317159  -0.7777843  -0.4429983  -0.08387085  1.821454
Coefficients:
                  Value   Std. Error    t value
(Intercept) -2.80032215   0.32824128  -8.531292
         x1  0.03907466   0.01129991   3.457963
(Dispersion Parameter for Binomial family taken to be 1 )
Deviance: 13.42458 on 17 degrees of freedom
1 observations deleted due to missing values
Number of Fisher Scoring Iterations: 6
```

```
> # Mencari taksiran parameter untuk model lengkap dari model probit
>  fullmodprobit <- glm (y/n~x1+x2, family = binomial (link = probit), weights = n,
   na.action = na.exclude, epsilon = 0.0001, maxit = 20,trace = T)
GLM   linear loop 1: deviance = 61.8279
GLM   linear loop 2: deviance = 21.6895
GLM   linear loop 3: deviance = 14.186
GLM   linear loop 4: deviance = 13.3855
GLM   linear loop 5: deviance = 13.3669
GLM   linear loop 6: deviance = 13.3669
> summary (fullmodprobit)
Call: glm(formula = y/n ~ x1 + x2, family = binomial(link = probit), weights = n,
na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
Deviance Residuals:
      Min          1Q         Median        3Q          Max
 -1.332306   -0.7799029   -0.4272628   -0.1165677   1.823711
Coefficients:
                  Value        Std. Error    t value
(Intercept)   -2.89327035    0.50026718   -5.7834503
         x1    0.03995286    0.01184215    3.3737848
         x2    0.03621661    0.14672721    0.2468296
(Dispersion Parameter for Binomial family taken to be 1 )
Deviance: 13.36691 on 16 degrees of freedom
1 observations deleted due to missing values
Number of Fisher Scoring Iterations: 6
> #  Mencari selisih nilai deviansi antara deviansi model reduksi dengan model
lengkap
>  deltaD <- redmodprobit $ deviance – fullmodprobit $ deviance
>  deltaD
[1] 0.05767007
```

Lampiran 5 : Pengolahan data pada contoh penerapan untuk model logistik

```
> # Input Data
> y <- c(0,0,0,0,0,0,0,2,0,0,1,4,0,1,1,3,0,0,0,0)
> x1 <- c(7,7,7,7,7,14,14,14,14,14,27,27,27,27,27,51,51,51,51,51)
> x2 <- c(1.0,1.7,2.2,2.8,4.0,1.0,1.7,2.2,2.8,4.0,1.0,1.7,2.2,2.8,4.0,1.0,1.7,2.2,2.8,4.0)
> n <- c(10,17,7,12,9,31,43,33,31,19,56,44,21,22,16,13,1,1,0,1)
> # Mencari taksiran parameter untuk model reduksi dari model logit
> redmodlogit <- glm (y/n~x1, family = binomial (link = logit), weights = n,
  na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
GLM   linear loop 1: deviance = 70.9985
GLM   linear loop 2: deviance = 25.175
GLM   linear loop 3: deviance = 15.1187
GLM   linear loop 4: deviance = 13.8189
GLM   linear loop 5: deviance = 13.7816
GLM   linear loop 6: deviance = 13.7816
> summary (redmodlogit)
Call: glm (formula = y/n ~ x1, family = binomial(link = logit), weights = n, na.action
= na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
Deviance Residuals:
      Min         1Q      Median         3Q       Max
-1.272488 -0.7957226 -0.514829 -0.0931117 1.718862
Coefficients:
                  Value   Std. Error   t value
(Intercept) -5.41517208  0.72683241 -7.450372
         x1  0.08069587  0.02234502  3.611358
(Dispersion Parameter for Binomial family taken to be 1 )
Deviance: 13.78157 on 17 degrees of freedom
1 observations deleted due to missing values
Number of Fisher Scoring Iterations: 6
```

```
> # Mencari taksiran parameter untuk model lengkap dari model logit
> fullmodlogit <- glm (y/n~x1+x2, family = binomial (link = logit), weights = n,
  na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
GLM   linear loop 1: deviance = 70.9966
GLM   linear loop 2: deviance = 25.1677
GLM   linear loop 3: deviance = 15.0992
GLM   linear loop 4: deviance = 13.7908
GLM   linear loop 5: deviance = 13.7527
GLM   linear loop 6: deviance = 13.7526
> summary (fullmodlogit)
Call: glm(formula = y/n ~ x1 + x2, family = binomial(link = logit), weights = n,
na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
Deviance Residuals:
      Min         1Q       Median          3Q         Max
-1.283108  -0.7818358  -0.5051426  -0.09701591  1.719227
Coefficients:
                   Value   Std. Error    t value
(Intercept)  -5.55915959   1.11865123  -4.9695199
         x1   0.08203067   0.02372086   3.4581654
         x2   0.05677077   0.33098209   0.1715222
(Dispersion Parameter for Binomial family taken to be 1 )
Deviance: 13.75263 on 16 degrees of freedom
1 observations deleted due to missing values
Number of Fisher Scoring Iterations: 6
> # Mencari selisih nilai deviansi antara deviansi model reduksi dengan model
lengkap
> deltaD <- redmodlogit $ deviance – fullmodlogit $ deviance
> deltaD
[1] 0.02894484
```

Lampiran 6 : Pengolahan data pada contoh penerapan untuk model nilai ekstrim

```
> # Input Data
> y <- c(0,0,0,0,0,0,0,2,0,0,1,4,0,1,1,3,0,0,0,0)
> x1 <-c(7,7,7,7,7,14,14,14,14,14,27,27,27,27,27,51,51,51,51,51)
> x2 <-c(1.0,1.7,2.2,2.8,4.0,1.0,1.7,2.2,2.8,4.0,1.0,1.7,2.2,2.8,4.0,1.0,1.7,2.2,2.8,4.0)
> n <-c(10,17,7,12,9,31,43,33,31,19,56,44,21,22,16,13,1,1,0,1)
> # Mencari taksiran parameter untuk model reduksi dari model nilai ekstrim
> redmodcloglog <- glm (y/n~x1, family = binomial (link = cloglog), weights = n,
  na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
GLM    linear loop 1: deviance = 93.9407
GLM    linear loop 2: deviance = 31.2294
GLM    linear loop 3: deviance = 16.3597
GLM    linear loop 4: deviance = 13.97
GLM    linear loop 5: deviance = 13.8578
GLM    linear loop 6: deviance = 13.8575
> summary (redmodcloglog)
Call: glm (formula = y/n ~ x1, family = binomial(link = cloglog), weights = n,
na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
Deviance Residuals:
      Min         1Q        Median         3Q        Max
 -1.262664  -0.7894766  -0.5265397  -0.0943614  1.699398
Coefficients:
                  Value      Std. Error    t value
(Intercept)  -5.34751574   0.69555761   -7.688099
      x1      0.07689975   0.02063424    3.726804
(Dispersion Parameter for Binomial family taken to be 1 )
Deviance: 13.85746 on 17 degrees of freedom
1 observations deleted due to missing values
Number of Fisher Scoring Iterations: 6
```

```
> # Mencari taksiran parameter untuk model lengkap dari model nilai ekstrim
> fullmodcloglog <- glm (y/n~x1, family = binomial (link = cloglog), weights = n,
  na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
GLM   linear loop 1: deviance = 93.9394
GLM   linear loop 2: deviance = 31.2249
GLM   linear loop 3: deviance = 16.3476
GLM   linear loop 4: deviance = 13.9522
GLM   linear loop 5: deviance = 13.8394
GLM   linear loop 6: deviance = 13.8391
> summary (fullmodcloglog)
Call: glm (formula = y/n ~ x1 + x2, family = binomial(link = cloglog), weights = n,
na.action = na.exclude, epsilon = 0.0001, maxit = 20, trace = T)
Deviance Residuals:
     Min        1Q       Median       3Q          Max
 -1.270846  -0.7788217  -0.519223  -0.08836523  1.69904
Coefficients:
                 Value  Std. Error    t value
(Intercept)  -5.45579783  1.07227284  -5.0880687
        x1    0.07786334  0.02198591   3.5415109
        x2    0.04326497  0.31968414   0.1353366
(Dispersion Parameter for Binomial family taken to be 1 )
Deviance: 13.83906 on 16 degrees of freedom
1 observations deleted due to missing values
Number of Fisher Scoring Iterations: 6
> #  Mencari selisih nilai deviansi antara deviansi model reduksi dengan model
lengkap
>  deltaD <- redmodcloglog $ deviance – fullmodcloglog $ deviance
>  deltaD
[1] 0.0184052
```

Lampiran 7 : Tabel Distribusi Chi-Kuadrat

TABLE II

*The Chi-Square Distribution**

$$Pr(X \le x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2}\, dw$$

r	Pr (X ≤ x) 0.01	0.025	0.050	0.95	0.975	0.99
1	0.000	0.001	0.004	3.84	5.02	6.63
2	0.020	0.051	0.103	5.99	7.38	9.21
3	0.115	0.216	0.352	7.81	9.35	11.3
4	0.297	0.484	0.711	9.49	11.1	13.3
5	0.554	0.831	1.15	11.1	12.8	15.1
6	0.872	1.24	1.64	12.6	14.4	16.8
7	1.24	1.69	2.17	14.1	16.0	18.5
8	1.65	2.18	2.73	15.5	17.5	20.1
9	2.09	2.70	3.33	16.9	19.0	21.7
10	2.56	3.25	3.94	18.3	20.5	23.2
11	3.05	3.82	4.57	19.7	21.9	24.7
12	3.57	4.40	5.23	21.0	23.3	26.2
13	4.11	5.01	5.89	22.4	24.7	27.7
14	4.66	5.63	6.57	23.7	26.1	29.1
15	5.23	6.26	7.26	25.0	27.5	30.6
16	5.81	6.91	7.96	26.3	28.8	32.0
17	6.41	7.56	8.67	27.6	30.2	33.4
18	7.01	8.23	9.39	28.9	31.5	34.8
19	7.63	8.91	10.1	30.1	32.9	36.2
20	8.26	9.59	10.9	31.4	34.2	37.6
21	8.90	10.3	11.6	32.7	35.5	38.9
22	9.54	11.0	12.3	33.9	36.8	40.3
23	10.2	11.7	13.1	35.2	38.1	41.6
24	10.9	12.4	13.8	36.4	39.4	43.0
25	11.5	13.1	14.6	37.7	40.6	44.3
26	12.2	13.8	15.4	38.9	41.9	45.6
27	12.9	14.6	16.2	40.1	43.2	47.0
28	13.6	15.3	16.9	41.3	44.5	48.3
29	14.3	16.0	17.7	42.6	45.7	49.6
30	15.0	16.8	18.5	43.8	47.0	50.9

*This table is abridged and adapted from "Tables of Percentage Points of the Incomplete Beta Function and of the Chi-Square Distribution," *Biometrika*, 32 (1941). It is published here with the kind permission of Professor E. S. Pearson on behalf of the author, Catherine M. Thompson, and of the Biometrika Trustees.

Lampiran 8 : Tabel Distribusi Normal Standar

TABLE III

The Normal Distribution

$$\Pr(X \le x) = \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[\Phi(-x) = 1 - \Phi(x)]$$

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.500	1.10	0.864	2.05	0.980
0.05	0.520	1.15	0.875	2.10	0.982
0.10	0.540	1.20	0.885	2.15	0.984
0.15	0.560	1.25	0.894	2.20	0.986
0.20	0.579	1.282	0.900	2.25	0.980
0.25	0.599	1.30	0.903	2.30	0.989
0.30	0.618	1.35	0.911	2.326	0.990
0.35	0.637	1.40	0.919	2.35	0.991
0.40	0.655	1.45	0.926	2.40	0.992
0.45	0.674	1.50	0.933	2.45	0.993
0.50	0.691	1.55	0.939	2.50	0.994
0.55	0.709	1.60	0.945	2.55	0.995
0.60	0.726	1.645	0.950	2.576	0.995
0.65	0.742	1.65	0.951	2.60	0.995
0.70	0.758	1.70	0.955	2.65	0.996
0.75	0.773	1.75	0.960	2.70	0.997
0.80	0.788	1.80	0.964	2.75	0.997
0.85	0.802	1.85	0.968	2.80	0.997
0.90	0.816	1.90	0.971	2.85	0.998
0.95	0.829	1.95	0.974	2.90	0.998
1.00	0.841	1.960	0.975	2.95	0.998
1.05	0.853	2.00	0.977	3.00	0.999

Lampiran 9 : Tabel Distribusi t

TABLE IV

*The t-Distribution**

$$\Pr(T \le t) = \int_{-\infty}^{t} \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\,\Gamma(r/2)(1 + w^2/r)^{(r+1)/2}}\,dw$$

$$[\Pr(T \le -t) = 1 - \Pr(T \le t)]$$

	$\Pr(T \le t)$				
r	0.90	0.95	0.975	0.99	0.995
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

*This table is abridged from Table III of Fisher and Yates, *Statistical Tables for Biological, Agricultural, and Medical Research*, published by Oliver and Boyd, Ltd., Edinburgh, by permission of the authors and publishers.