

Table A.1 General properties of the Laplace-Stieltjes transforms.

$F(t)$	$F^*(s) = \int_0^{\infty} e^{-st} dF(t)$
$F_1(t) + F_2(t)$	$F_1^*(s) + F_2^*(s)$
$aF(t)$	$aF^*(s)$
$F(t-a)$ ( $a > 0$ )	$e^{-sa} F^*(s)$
$F(at)$ ( $a > 0$ )	$F^*(s/a)$
$e^{-at} F(t)$ ( $a > 0$ )	$\frac{s}{s+a} F^*(s+a)$
$F'(t) = \frac{dF(t)}{dt}$	$s[F^*(s) - F(0)]$
$tF'(t)$	$-s \frac{dF^*(s)}{ds}$
$\int_0^t F(x) dx$	$\frac{1}{s} F^*(s)$
$\int_0^t \dots \int_0^t F(t) (dt)^n$	$\frac{1}{s^n} F^*(s)$
$\lim_{t \rightarrow +0} F(t)$	$\lim_{s \rightarrow \infty} F^*(s)$
$\lim_{t \rightarrow \infty} F(t)$	$\lim_{s \rightarrow +0} F^*(s)$

Table A.2 Formulas of the Laplace-Stieltjes Transforms

$F(t)$	$F^*(s) = \int_0^{\infty} e^{-st} dF(t)$
$\delta(t-a)^\dagger$ ( $a > 0$ )	$se^{-sa}$
$1(t-a)^\ddagger$ ( $a > 0$ )	$e^{-sa}$
$1(t)$	$\frac{1}{s}$
$t$	$\frac{1}{s^2}$
$t^n$ ( $n$ : a positive integer)	$\frac{1}{s^{n+1}}$
$t^\alpha$ ( $\alpha > -1$ )	$\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}$
$e^{-\alpha t}$ ( $\alpha > 0$ )	$\frac{s}{s+\alpha}$
$te^{-\alpha t}$ ( $\alpha > 0$ )	$\frac{s}{(s+\alpha)^2}$
$t^n e^{-\alpha t}$ ( $\alpha > 0$ )	$\frac{n!s}{(s+\alpha)^{n+1}}$
$t^\beta e^{-\alpha t}$ ( $\alpha > 0, \beta > -1$ )	$\frac{s\Gamma(\beta+1)}{(s+\alpha)^{\beta+1}}$
$\cos \alpha t$	$\frac{s}{s^2 + \alpha^2}$ ( $\Re(s) >  \alpha $ )
$\sin \alpha t$	$\frac{\alpha}{s^2 + \alpha^2}$ ( $\Re(s) >  \alpha $ )
$\cosh \alpha t$	$\frac{s}{s^2 - \alpha^2}$ ( $\Re(s) >  \alpha $ )
$\sinh \alpha t$	$\frac{\alpha}{s^2 - \alpha^2}$ ( $\Re(s) >  \alpha $ )
$\log t$	$-\gamma - \log s$ §

- † Dirac's delta function.
- ‡ Heaviside's unit function.
- §  $\gamma = 0.57721 \dots$ , Euler's constant.