BAB II

RANCANGAN BUJURSANGKAR LATIN

Dalam banyak kegiatan percobaan sering diketahui sebelumnya bahwa satuan percobaan tertentu bila diperlakukan sama sering memberikan respon yang berbeda. Misalkan pada pengamatan yang dilakukan pada hari tertentu atau pengamatan yang menggunakan alat tertentu akan lebih homogen dibandingkan dengan yang dilakukan pada hari yang berbeda atau pengamatan dengan menggunakan alat yang berbeda. Dalam keadaan yang demikian itu, dengan tingkah laku satuan individu sebagian dapat diantisipasi sehingga satuan-satuan pengamatan tersebut diklasifikasikan seperlunya. Rancangan atau lay out dapat disusun sedemikian sehingga bagian keragaman yang bersumber pada sumber yang dikenali itu dapat diukur dan dikeluarkan dari galat percobaan. Pada saat yang sama beda antar rata-rata perlakuan tidak lagi mengandung sumbangan yang berasal dari sumber yang dikenali. Model rancangan yang didalamnya ada pengelompokkan dua arah yaitu pengelompokkan menurut baris dan kolom inilah yang akan dibahas dalam penulisan berikutnya. Rancangan yang dimaksud adalah rancangan bujursangkar latin. (*Steel dan Torrie, 1991)*

Rancangan Bujursangkar Latin merupakan salah satu rancangan percobaan yang dipergunakan jika diketahui bahwa percobaan mempunyai dua sumber keragaman utama. Dan rancangan ini dipilih untuk menghilangkan dua sumber keragaman utama tersebut dengan cara melakukan pengelompokkan dalam dua arah. Bentuk rancangannya bujursangkar dan perlakuan-perlakuan yang menjadi sebab diadakannya percobaan ditandai dengan huruf latin A, B, C, D, E, F, ...

Pada Rancangan Bujursangkar Latin jumlah level (tingkat/taraf) dari kedua faktor yang ingin dihilangkan dengan melaksanakan pengelompokkan dua arah itu haruslah sama dengan jumlah faktor perlakuan yang menjadi pusat pemikiran. Kedua faktor yang ingin dihilangkan keragamannya oleh pengelompokkan disebut faktor baris dan faktor kolom. Dan setiap perlakuan hanya muncul tepat satu kali dalam tiap baris dan satu kali dalam tiap kolom. Jika terdapat t buah perlakuan maka diperlakukan t unit percobaan. Rancangan Bujursangkar Latin (Latin Square Design) disingkat RBL (LSD). Cara pencatatan secara umum adalah RBL txt yang artinya RBL dengan t buah baris dan t buah kolom. Pola tersebut telah dibakukan dan disebut dengan latin baku (standard latin).

2.1 Model Linier dan Estimasi Parameter

2.1.1 Model Linier

Model linier dari rancangan bujursangkar latin dengan t buah baris, t buah perlakuan, dan t buah kolom adalah sebagai berikut:

$$y_{i(j)k} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk} \qquad \dots \qquad (2.1)$$

i=1,2,3,...,t   j=1,2,3,...,t   k=1,2,3,...,t

(Montgomery, 1984)

dengan:

$y_{i(j)k}$ adalah nilai pengamatan pada baris ke-i, perlakuan ke-j dan

kolom ke -k

μ adalah nilai rata-rata keseluruhan

α_i adalah pengaruh dari baris ke-i

τ_j adalah pengaruh dari perlakuan ke-j

β_k adalah pengaruh dari kolom ke-k

ε_{ijk} adalah galat yang diasumsikan NID$(0, \sigma^2)$

Model linier dari rancangan bujursangkar latin seperti dalam persamaan (2.1) dapat dipandang dalam dua keadaan yaitu sebagai model tetap dan model campuran. Tapi dalam penulisan tugas akhir ini hanya dibatasi pada model tetap saja.

Dengan asumsi :

$$\sum_{i=1}^{t} \alpha_i = 0 \quad \sum_{k=1}^{t} \beta_k = 0 \quad \sum_{j=1}^{t} \tau_j = 0$$

Model persamaan (2.1) tersebut menggambarkan bahwa nilai-nilai observasi yang dihasilkan dari sebuah rataan umum μ yang mendapat pengaruh baris α_i, pengaruh perlakuan τ_j, dan pengaruh kolom β_k serta pengaruh sebuah sumber variasi yang tak terkendali ε_{ijk}

2.1.2 Estimasi Parameter model

Dari persamaan (2.1) diperoleh nilai harapan y_{ijk} adalah sebesar $E(y_{i(j)k}) = \mu + \alpha_i + \tau_j + \beta_k$. Persamaan (2.1) merupakan persamaan yang mengandung empat parameter yaitu $\mu, \alpha_i, \tau_j,$ dan β_k. Dimana dari keempat parameter tersebut dapat diestimasi atau diduga dengan menggunakan metode kuadrat terkecil (least square). Prinsip dari metode kuadrat terkecil ini adalah untuk mencari estimator-estimator bagi parameter dengan mengusahakan agar jumlah galatnya sekecil mungkin.

Estimasi parameter dari persamaan (1) adalah sebagai berikut:

$$y_{i(j)k} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk}$$

$$\varepsilon_{ijk} = y_{i(j)k} - \mu_i - \tau_j - \beta_k$$

dengan batasan bahwa:

$$\sum_{i=1}^{t} \alpha_i = 0 \qquad \sum_{k=1}^{t} \beta_k = 0 \qquad \sum_{j=1}^{t} \tau_j = 0$$

Sehingga dengan menggunakan metode kuadrat terkecil diperoleh penduga-penduga parameter sebagai berikut:

Misalkan $L = \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right)^2$

$$\frac{\partial l}{\partial \mu} = 2\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right)(-1)$$

karena $\frac{\partial L}{\partial \mu} = 0$, maka

$$-2\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right) = 0$$

$$\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right) = 0$$

$$\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - t^2\hat{\mu} - t\sum_{i=1}^{t}\alpha_i - t\sum_{j=1}^{t}\tau_j - t\sum_{k=1}^{t}\beta_k = 0$$

Karena model tetap maka $\sum_{i=1}^{t}\alpha_i = 0$, $\sum_{j=1}^{t}\tau_j = 0$ dan $\sum_{k=1}^{t}\beta_k = 0$, jadi

$$\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - t^2\hat{\mu} = 0$$

$$t^2\hat{\mu} = \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k}$$

$$\hat{\mu} = \frac{\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k}}{t^2}$$

$$\hat{\mu} = \frac{\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k}}{N}$$

$$\hat{\mu} = \frac{y\ldots}{N}$$

$$\hat{\mu} = \bar{y}\ldots$$

Syarat harga ekstrim untuk meminimumkan L adalah $\frac{\partial^2 L}{\partial \mu^2} > 0$

$$\frac{\partial^2 L}{\partial \mu^2} = 2t^2 > 0 \text{ maka } \hat{\mu} = \bar{y}\ldots$$

$$\frac{\partial L}{\partial \alpha_i} = 2\sum_{j=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right)(-1)$$

Karena $\frac{\partial L}{\partial \alpha_i} = 0$, maka

$$-2\sum_{j=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right) = 0$$

$$\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - t^2\hat{\mu} - t^2\hat{\alpha}_i - t\sum_{j=1}^{t}\tau_j - t\sum_{k=1}^{t}\beta_k = 0$$

Karena model tetap $\sum_{j=1}^{t}\tau_j = 0$ dan $\sum_{k=1}^{t}\beta_k = 0$ jadi

$$\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - t^2\hat{\mu} - t^2\hat{\alpha}_i = 0$$

$$t^2\hat{\alpha}_i = \sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - t^2\hat{\mu}$$

$$\hat{\alpha}_i = \frac{\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k}}{t^2} - \hat{\mu}$$

$$\hat{\alpha}_i = \frac{y_{i..}}{N} - \bar{y}_{...}$$

$$\hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...}$$

Syarat harga ekstrim untuk meminimumkan L adalah $\frac{\partial^2 L}{\partial \alpha_i^2} > 0$

$\frac{\partial^2 L}{\partial \alpha_i^2} = 2t > 0$ maka $\hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...}$

$$\frac{\partial L}{\partial \tau_j} = 2\sum_{i=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right)(-1)$$

Karena $\frac{\partial L}{\partial \tau_j} = 0$, maka

$$-2\sum_{i=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right) = 0$$

$$\sum_{i=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - t^2\hat{\mu} - t\sum_{i=1}^{t}\alpha_i - t^2\tau_j - t\sum_{k=1}^{t}\beta_k = 0$$

Karena model tetap $\sum_{i=1}^{t}\alpha_i = 0$ dan $\sum_{k=1}^{t}\beta_k = 0$ jadi

$$\sum_{i=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - t^2\hat{\mu} - t^2\tau_j = 0$$

$$t^2\hat{\tau}_j = \sum_{i=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - t^2\hat{\mu}$$

$$\hat{\tau}_j = \frac{\sum_{i=1}^{t}\sum_{k=1}^{t} y_{i(j)k}}{t^2} - \hat{\mu}$$

$$\hat{\tau}_j = \frac{y_{.j.}}{N} - \bar{y}_{...} = \bar{y}_{.j.} - \bar{y}_{...}$$

Syarat harga ekstrim untuk meminimumkan L adalah $\frac{\partial^2 L}{\partial \tau_j^2} > 0$

$\frac{\partial^2 L}{\partial \tau_j^2} = 2t > 0$ maka $\hat{\tau}_j = \bar{y}_{.j.} - \bar{y}_{...}$

$$\frac{\partial L}{\partial \beta_k} = 2\sum_{i=1}^{t}\sum_{j=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right)(-1)$$

Karena $\frac{\partial L}{\partial \beta_k} = 0$, maka

$$-2\sum_{i=1}^{t}\sum_{j=1}^{t}\left(y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k\right) = 0$$

$$\sum_{i=1}^{t}\sum_{j=1}^{t} y_{i(j)k} - t^2\hat{\mu} - t\sum_{i=1}^{t}\alpha_i - t\sum_{j=1}^{t}\tau_j - t^2\beta_k = 0$$

Karena model tetap $\sum_{i=1}^{t}\alpha_i = 0$ dan $\sum_{k=1}^{t}\tau_j = 0$ jadi

$$\sum_{i=1}^{t}\sum_{j=1}^{t} y_{i(j)k} - t^2\hat{\mu} - t^2\hat{\beta}_k = 0$$

$$t^2\hat{\beta}_k = \sum_{i=1}^{t}\sum_{j=1}^{t} y_{i(j)k} - t^2\hat{\mu}$$

$$\hat{\beta}_k = \frac{\sum_{i=1}^{t}\sum_{j=1}^{t} y_{i(j)k}}{t^2} - \hat{\mu}$$

$$\hat{\beta}_k = \frac{y_{..k}}{N} - \bar{y}_{...} = \bar{y}_{..k} - \bar{y}_{...}$$

Syarat harga ekstrim untuk meminimumkan L adalah $\frac{\partial^2 L}{\partial \beta_k^2} > 0$

$\frac{\partial^2 L}{\partial \beta_k^2} = 2t > 0$ maka $\hat{\beta}_k = \bar{y}_{..k} - \bar{y}_{...}$

Dari hasil penghitungan diatas diperoleh nilai estimasi parameter μ, α_i, τ_j dan β_k adalah sebagai berikut:

$$\hat{\mu} = \bar{y}_{...},\ \hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...},\ \hat{\tau}_j = \bar{y}_{.j.} - \bar{y}_{...},\ \hat{\beta}_k = \bar{y}_{..k} - \bar{y}_{...}$$

Dengan demikian,

$$\hat{y}_{i(j)k} = \bar{y}_{...} + (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{..k} - \bar{y}_{...})$$
$$\hat{y}_{i(j)k} = \bar{y}_{i..} + \bar{y}_{.j.} + \bar{y}_{..k} - 2\bar{y}_{...}$$

2.2 Analisa Statistik

Misalkan dalam suatu percobaan terdapat t buah perlakuan dan masing-masing akan diuji apakah benar perlakuan mempunyai pengaruh yang nyata terhadap variansi galat maka dengan mengggunakan kriteria-kriteria tertentu unit-unit percobaan dikelompokkan kedalam t buah baris dan t buah kolom. Lay out data untuk Rancangan Bujursangkar Latin terlihat dalam tabel A sebagai berikut :

Tabel (2.1) : Bentuk Umum Rancangan Bujursangkar Latin Standar

	Kolom						Total Baris
Baris		1	2	3	…	t	
	1	$A = y_{111}$	$B = y_{122}$	$C = y_{133}$	…	y_{1tt}	$y_{1..}$
	2	$B = y_{221}$	$C = y_{232}$	$D = y_{243}$	…	y_{2tt}	$y_{2..}$
	3	$C = y_{331}$	$D = y_{342}$	$E = y_{353}$	…	y_{3tt}	$y_{3..}$
	…	…	…	…	…	…	…
	t	y_{tt1}	y_{t12}	y_{t23}	…	$y_{t(t-1)t}$	$y_{t..}$
	Total Kolom	$y_{..1}$	$y_{..2}$	$y_{..3}$	…	$y_{..t}$	$y_{...}$

Tabel (2.2) : Total Perlakuan

Perlakuan	A	B	C	…	t	
	$y_{.1.}$	$y_{.2.}$	$y_{.3.}$	…	$y_{.t.}$	$y_{...}$

Selanjutnya didefinisikan:

$y_{i(j)k}$ menyatakan pengamatan pada baris ke-i, perlakuan ke-j, dan kolom ke-k

$y_{i..}$ menyatakan total nilai dari pengamatan pada baris ke-i

$y_{.j.}$ menyatakan total nilai dari pengamatan pada perlakuan ke-j

$y_{..k}$ menyatakan total nilai dari pengamatan pada kolom ke-k

$y_{...}$ menyatakan total nilai dari seluruh pengamatan

$\bar{y}_{i..}$ menyatakan rata-rata baris ke-i

$\bar{y}_{.j.}$ menyatakan rata-rata perlakuan ke-j

$\bar{y}_{..k}$ menyatakan rata-rata kolom ke-*k*

$\bar{y}_{...}$ menyatakan rata-rata total

Jumlah pengamatan pada Rancangan Bujursangkar Latin adalah (t x t) atau t^2 pengamatan. Meskipun pada Rancangan Bujursangkar Latin memiliki tiga subkrip yaitu *i, j, k* akan tetapi jumlah pengamatannya tetap t^2 dan bukan t^3 pengamatan *(Steel dan Torrie, 1991)*. Misalnya, perlakuan 1 muncul sekali dalam setiap baris, sekali dalam setiap kolom , tetapi semuanya hanya ada t pengamatan: jadi j=1 berpadanan dengan pasangan nilai *i, k* tertentu, yang semuanya ada t banyaknya. Begitu juga untuk nilai-nilai *j* lainnya. Jadi meskipun digunakan tiga subkrip yaitu *i, j, k* tidak berarti berlakunya klasifikasi tiga arah.

Bentuk baku dari RBL atau bujursangkar latin standard adalah bentuk yang pada baris dan kolom pertama hurufnya ditentukan menurut urutan abjad, kemudian menuliskan baris-baris berikutnya seperti baris diatasnya dengan menggeserkan satu sel ke kiri.

Contoh: Beberapa Bujursangkar Latin Standar

3 x 3

A	*B*	*C*
B	*C*	*A*
C	*A*	*B*

4 X 4

A	*B*	*C*	*D*
B	*C*	*D*	*A*
C	*D*	*A*	*B*
D	*A*	*B*	*C*

5 X 5

A	*B*	*C*	*D*	*E*
B	*C*	*D*	*E*	*A*
C	*D*	*E*	*A*	*B*
D	*E*	*A*	*B*	*C*
E	*A*	*B*	*C*	*D*

6 X 6

A	B	C	D	E	F
B	C	D	E	F	A
C	D	E	F	A	B
D	E	F	A	B	C
E	F	A	B	C	D
F	A	B	C	D	E

t x t

A	B	C	...	t
B	C	D	...	A
C	D	E	...	B
...	...	...	...	...
t	A	B	...	$(t-1)$

Dalam suatu percobaan biasanya bujursangkar latin yang digunakan dipilih secara random dari beberapa bentuk bujursangkar latin yang mempunyai ukuran yang dibutuhkan. Begitu pula cara penempatan perlakuan pada satuan-satuan percobaan juga dilakukan secara random atau acak.

2.3 Penguraian Jumlah Kuadrat

Sumber keragaman nilai-nilai observasi sebagai akibat pengaruh baris, perlakuan, kolom maupun galat dapat dilihat dari besarnya jumlah kuadrat total atau JKT. Untuk mengetahui seberapa besar jumlah kuadrat yang diakibatkan oleh baris, perlakuan, kolom serta jumlah kuadrat yang tidak terdeteksi sebagai pengaruh dari galat maka JKT diuraikan komponen-komponennya. Dimana penguraian komponen-komponen jumlah kuadrat total atau JKT adalah sebagai berikut:

$$y_{i(j)k} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk}$$

maka $\varepsilon_{ijk} = y_{i(j)k} - \mu - \alpha_i - \tau_j - \beta_k$

Sedangkan dari hasil estimasi diperoleh $\hat{\mu} = \bar{y}_{...}$, $\hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...}$, $\hat{\tau}_j = \bar{y}_{.j.} - \bar{y}_{...}$, $\hat{\beta}_k = \bar{y}_{..k} - \bar{y}_{...}$

Sehingga

$$\varepsilon_{ijk} = y_{i(j)k} - \bar{y}_{...} - (\bar{y}_{i..} - \bar{y}_{...}) - (\bar{y}_{.j.} - \bar{y}_{...}) - (\bar{y}_{..k} - \bar{y}_{...})$$

$$= y_{i(j)k} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...}$$

Jika nilai ε_{ijk} dan nilai-nilai $\hat{\mu} = \bar{y}_{...}$, $\hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...}$, $\hat{\tau}_j = \bar{y}_{.j.} - \bar{y}_{...}$, $\hat{\beta}_k = \bar{y}_{..k} - \bar{y}_{...}$ disubstitusikan kedalam persamaan (1) maka akan diperoleh:

$$y_{i(j)k} = \bar{y}_{...} + (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{..k} - \bar{y}_{...}) + (y_{i(j)k} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})$$

$$(y_{i(j)k} - \bar{y}_{...}) = (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{..k} - \bar{y}_{...}) + (y_{i(j)k} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})$$

$$\begin{aligned}(y_{i(j)k} - \bar{y}_{...})^2 = {} & (\bar{y}_{i..} - \bar{y}_{...})^2 + (\bar{y}_{.j.} - \bar{y}_{...})^2 + (\bar{y}_{..k} - \bar{y}_{...})^2 \\ & (y_{i(j)k} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})^2 + 2(\bar{y}_{i..} - \bar{y}_{...})(\bar{y}_{.j.} - \bar{y}_{...}) \\ & + 2(\bar{y}_{i..} - \bar{y}_{...})(\bar{y}_{..k} - \bar{y}_{...}) + 2(\bar{y}_{i..} - \bar{y}_{...})(y_{i(j)k} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...}) \\ & + 2(\bar{y}_{.j.} - \bar{y}_{...})(\bar{y}_{..k} - \bar{y}_{...}) + 2(\bar{y}_{.j.} - \bar{y}_{...})(y_{i(j)k} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...}) \\ & + 2(\bar{y}_{..k} - \bar{y}_{...})(y_{i(j)k} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})\end{aligned}$$

$$\begin{aligned}\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(y_{i(j)k} - \bar{y}_{...})^2 = {} & \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{i..} - \bar{y}_{...})^2 + \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{.j.} - \bar{y}_{...})^2 \\ & + \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{..k} - \bar{y}_{...})^2 + \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{i(j)k} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})^2 \\ & \underbrace{+ 2\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{i..} - \bar{y}_{...})^2}_{\textbf{A}} \underbrace{+ 2\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{i..} - \bar{y}_{...})(\bar{y}_{..k} - \bar{y}_{...})}_{\textbf{B}}\end{aligned}$$

$$+2\underbrace{\sum\sum\sum(\bar{y}_{i..}-\bar{y}_{...})(y_{i(j)k}-\bar{y}_{i..}-\bar{y}_{.j.}-\bar{y}_{..k}+2\bar{y}_{...})}_{\mathbf{C}}$$

$$+2\underbrace{\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{.j.}-\bar{y}_{...})(\bar{y}_{..k}-\bar{y}_{...})}_{\mathbf{D}}$$

$$+2\underbrace{\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{.j.}-\bar{y}_{...})(y_{i(j)k}-\bar{y}_{i..}-\bar{y}_{.j.}-\bar{y}_{..k}+2\bar{y}_{...})}_{\mathbf{E}}$$

$$+2\underbrace{\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{..k}-\bar{y}_{...})(y_{i(j)k}-\bar{y}_{i..}-\bar{y}_{.j.}-\bar{y}_{..k}+2\bar{y}_{...})}_{\mathbf{F}}$$

$$\mathbf{A}=\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{i..}-\bar{y}_{...})(\bar{y}_{.j.}-\bar{y}_{...})$$

$$=\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{i..}\bar{y}_{.j.}-\bar{y}_{i..}\bar{y}_{...}-\bar{y}_{.j.}\bar{y}_{...}+\bar{y}_{...}^{2})$$

$$=\sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{.j.}-t\bar{y}_{...}\sum_{i=1}^{t}\bar{y}_{i..}-t\bar{y}_{...}\sum_{j=1}^{t}\bar{y}_{.j.}+\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}^{2}$$

$$=\sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{.j.}-t\bar{y}_{...}\sum_{i=1}^{t}\bar{y}_{i..}-t\bar{y}_{...}\sum_{j=1}^{t}\bar{y}_{.j.}+t^{2}\bar{y}_{...}$$

$$=\frac{\sum_{i=1}^{t}y_{i..}}{t}\frac{\sum_{j=1}^{t}\sum_{k=1}^{t}y_{.j.}}{t}-t\bar{y}_{...}\frac{\sum_{i=1}^{t}y_{i..}}{t}-t\bar{y}_{...}\frac{\sum_{j=1}^{t}y_{.j.}}{t}+t^{2}\frac{y_{...}^{2}}{(t^{2})^{2}}$$

$$=\frac{y_{...}^{2}}{t^{2}}-\bar{y}_{...}\sum_{i=1}^{t}y_{i..}-\bar{y}_{...}\sum_{j=1}^{t}y_{.j.}+\frac{y_{...}^{2}}{t^{2}}$$

$$=\frac{y_{...}^{2}}{t^{2}}-\frac{y_{...}^{2}}{t^{2}}-\frac{y_{...}^{2}}{t^{2}}+\frac{y_{...}^{2}}{t^{2}}$$

$$=0$$

$$\mathbf{B} = \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(\bar{y}_{i..} - \bar{y}_{...}\right)\left(\bar{y}_{..k} - \bar{y}_{...}\right)$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(\bar{y}_{i..}\bar{y}_{..k} - \bar{y}_{i..}\bar{y}_{...} - \bar{y}_{..k}\bar{y}_{...} + \bar{y}_{...}^{2}\right)$$

$$= \sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{..k} - t\bar{y}_{...}\sum_{i=1}^{t}\bar{y}_{i..} - t\bar{y}_{...}\sum_{k=1}^{t}\bar{y}_{..k} + \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}^{2}$$

$$= \sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{..k} - t\bar{y}_{...}\sum_{i=1}^{t}\bar{y}_{i..} - t\bar{y}_{...}\sum_{k=1}^{t}\bar{y}_{..k} + t^{2}\bar{y}_{...}$$

$$= \frac{\sum_{i=1}^{t} y_{i..}}{t}\,\frac{\sum_{j=1}^{t}\sum_{k=1}^{t} y_{..k}}{t} - t\bar{y}_{...}\frac{\sum_{i=1}^{t} y_{i..}}{t} - t\bar{y}_{...}\frac{\sum_{k=1}^{t} y_{..k}}{t} + t^{2}\frac{y_{...}^{2}}{\left(t^{2}\right)^{2}}$$

$$= \frac{y_{...}^{2}}{t^{2}} - \bar{y}_{...}\sum_{i=1}^{t} y_{i..} - \bar{y}_{...}\sum_{j=1}^{t} y_{..k} + \frac{y_{...}^{2}}{t^{2}}$$

$$= \frac{y_{...}^{2}}{t^{2}} - \frac{y_{...}^{2}}{t^{2}} - \frac{y_{...}^{2}}{t^{2}} + \frac{y_{...}^{2}}{t^{2}}$$

$$= 0$$

$$\mathbf{C} = \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(\bar{y}_{i..} - \bar{y}_{...}\right)\left(y_{i(j)k} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...}\right)$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\begin{pmatrix}\bar{y}_{i..}y_{ijk} - \bar{y}_{i..}^{2} - \bar{y}_{i..}\bar{y}_{.j.} - \bar{y}_{i..}\bar{y}_{..k} + 2\bar{y}_{i..}\bar{y}_{...} - y_{i(j)k}\bar{y}_{...} + \bar{y}_{i..}\bar{y}_{...} + \bar{y}_{.j.}\bar{y}_{...} \\ + \bar{y}_{..k}\bar{y}_{...} - 2\bar{y}_{...}^{2}\end{pmatrix}$$

$$= \sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - \sum_{j=1}^{t}\sum_{k=1}^{t}\left(\sum_{i=1}^{t}\bar{y}_{i..}^{2}\right) - \sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{.j.} - \sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{..k}$$

$$+ 3\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}\sum_{i=1}^{t}\bar{y}_{i..} - \bar{y}_{...}\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k} + \sum_{i=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}\sum_{j=1}^{t}\bar{y}_{.j.}$$

$$+ \sum_{i=1}^{t}\sum_{j=1}^{t}\bar{y}_{...}\sum_{k=1}^{t}\bar{y}_{..k} - \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}^{2}$$

$$=\frac{\sum_{i=1}^{t} y_{i..}}{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{.jk}-\sum_{j=1}^{t}\sum_{k=1}^{t}\left(\frac{\sum_{i=1}^{t} y_{i..}^2}{t^2}\right)-\frac{\sum_{i=1}^{t} y_{i..}}{t}\,\frac{\sum_{j=1}^{t}\sum_{k=1}^{t} y_{.j.}}{t}-\frac{\sum_{i=1}^{t} y_{i..}}{t}\,\frac{\sum_{j=1}^{t}\sum_{k=1}^{t} y_{..k}}{t}$$

$$+3\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}\frac{\sum_{i=1}^{t}\bar{y}_{i..}}{t}-\bar{y}_{...}y_{...}+\sum_{i=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}\frac{\sum_{j=1}^{t} y_{.j.}}{t}+\sum_{i=1}^{t}\sum_{j=1}^{t}\bar{y}_{...}\frac{\sum_{k=1}^{t} y_{..k}}{t}-2t^2\frac{y_{...}^2}{\left(t^2\right)^2}$$

$$=\frac{y_{...}^2}{t}-t\frac{\sum_{i=1}^{t} y_{i..}^2}{t^2}-\frac{y_{...}^2}{t^2}-\frac{y_{...}^2}{t^2}+3t\bar{y}_{...}\frac{\sum_{i=1}^{t} y_{i..}}{t}-\frac{y_{...}^2}{t^2}+t\bar{y}_{...}\frac{\sum_{j=1}^{t} y_{.j.}}{t}+t\bar{y}_{...}\frac{\sum_{k=1}^{t} y_{..k}}{t}-2\frac{y_{...}^2}{t^2}$$

$$=\frac{y_{...}^2}{t}-\frac{y_{...}^2}{t}-\frac{y_{...}^2}{t^2}-\frac{y_{...}^2}{t^2}+3\bar{y}_{...}\sum_{i=1}^{t} y_{i..}-\frac{y_{...}^2}{t^2}+\bar{y}_{...}\sum_{j=1}^{t} y_{.j.}+\bar{y}_{...}\sum_{k=1}^{t} y_{..k}-2\frac{y_{...}^2}{t^2}$$

$$=\frac{y_{...}^2}{t}-\frac{y_{...}^2}{t}-\frac{y_{...}^2}{t^2}-\frac{y_{...}^2}{t^2}+3\frac{y_{...}^2}{t^2}-\frac{y_{...}^2}{t^2}+\frac{y_{...}^2}{t^2}+\frac{y_{...}^2}{t^2}-2\frac{y_{...}^2}{t^2}$$

$$=0$$

$$\mathbf{D}=\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(\bar{y}_{i..}-\bar{y}_{...}\right)\left(\bar{y}_{..k}-\bar{y}_{...}\right)$$

$$=\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(\bar{y}_{i..}\bar{y}_{..k}-\bar{y}_{i..}\bar{y}_{...}-\bar{y}_{..k}\bar{y}_{...}+\bar{y}_{...}^2\right)$$

$$=\sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{..k}-t\bar{y}_{...}\sum_{i=1}^{t}\bar{y}_{i..}-t\bar{y}_{...}\sum_{k=1}^{t}\bar{y}_{..k}+\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}^2$$

$$=\sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{..k}-t\bar{y}_{...}\sum_{i=1}^{t}\bar{y}_{i..}-t\bar{y}_{...}\sum_{k=1}^{t}\bar{y}_{..k}+t^2\bar{y}_{...}$$

$$=\frac{\sum_{i=1}^{t} y_{i..}}{t}\,\frac{\sum_{j=1}^{t}\sum_{k=1}^{t} y_{..k}}{t}-t\bar{y}_{...}\frac{\sum_{i=1}^{t} y_{i..}}{t}-t\bar{y}_{...}\frac{\sum_{k=1}^{t} y_{..k}}{t}+t^2\frac{y_{...}^2}{\left(t^2\right)^2}$$

$$= \frac{y_{...}^2}{t^2} - \bar{y}_{...}\sum_{i=1}^{t} y_{i..} - \bar{y}_{...}\sum_{k=1}^{t} y_{..k} + \frac{y_{...}^2}{t^2}$$

$$= \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t^2} + \frac{y_{...}^2}{t^2}$$

$$= 0$$

$$\mathbf{E} = \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(\bar{y}_{.j.} - \bar{y}_{...}\right)\left(y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...}\right)$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(\begin{array}{l}\bar{y}_{.j.}y_{ijk} - \bar{y}_{.j.}\bar{y}_{i..} - \bar{y}_{.j.}^2 - \bar{y}_{.j.}\bar{y}_{..k} + 2\bar{y}_{.j.}\bar{y}_{...} - y_{i(j)k}\bar{y}_{...} + \bar{y}_{i..}\bar{y}_{...} + \bar{y}_{.j.}\bar{y}_{...} \\ + \bar{y}_{..k}\bar{y}_{...} - 2\bar{y}_{...}^2\end{array}\right)$$

$$= \sum_{j=1}^{t}\bar{y}_{.j.}\sum_{i=1}^{t}\sum_{k=1}^{t} y_{i(j)k} - \sum_{i=1}^{t}\bar{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{.j.} - \sum_{i=1}^{t}\sum_{k=1}^{t}\left(\sum_{j=1}^{t}\bar{y}_{.j.}^2\right) - \sum_{j=1}^{t}\bar{y}_{.j.}\sum_{i=1}^{t}\sum_{k=1}^{t}\bar{y}_{..k}$$

$$+ 3\sum_{i=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}\sum_{j=1}^{t}\bar{y}_{.j.} - \bar{y}_{...}\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k} + \sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}\sum_{i=1}^{t}\bar{y}_{i..}$$

$$+ \sum_{i=1}^{t}\sum_{j=1}^{t}\bar{y}_{...}\sum_{k=1}^{t}\bar{y}_{..k} - \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}^2$$

$$= \frac{\sum_{j=1}^{1} y_{.j.}}{t}\sum_{i=1}^{t}\sum_{k=1}^{t} y_{i.k} - \frac{\sum_{i=1}^{t} y_{i..}}{t}\frac{\sum_{j=1}^{t}\sum_{k=1}^{t} y_{.j.}}{t} - t\frac{\sum_{j=1}^{t} y_{.j.}^2}{t^2} - \frac{\sum_{j=1}^{t} y_{.j.}}{t}\frac{\sum_{i=1}^{t}\sum_{k=1}^{t} y_{..k}}{t}$$

$$+ 3\sum_{i=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}\frac{\sum_{j=1}^{t} y_{.j.}}{t} - \bar{y}_{...}y_{...} + \sum_{j=1}^{t}\sum_{k=1}^{t}\bar{y}_{...}\frac{\sum_{i=1}^{t} y_{i..}}{t} + \sum_{i=1}^{t}\sum_{j=1}^{t}\bar{y}_{...}\frac{\sum_{k=1}^{t} y_{..k}}{t} - 2t^2\frac{y_{...}^2}{\left(t^2\right)^2}$$

$$= \frac{y_{...}^2}{t} - \frac{y_{...}^2}{t^2} - t\frac{\sum_{j=1}^{t} y_{.j.}^2}{t^2} - \frac{y_{...}^2}{t^2} + 3t\bar{y}_{...}\frac{\sum_{j=1}^{t} y_{.j.}}{t} - \frac{y_{...}^2}{t^2} + t\bar{y}_{...}\frac{\sum_{i=1}^{t} y_{i..}}{t} + t\bar{y}_{...}\frac{\sum_{k=1}^{t} y_{..k}}{t} - 2\frac{y_{...}^2}{t^2}$$

$$= \frac{y_{...}^2}{t} - \frac{y_{...}^2}{t} - \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t^2} + 3\bar{y}_{...}\sum_{j=1}^{t} y_{.j.} - \frac{y_{...}^2}{t^2} + \bar{y}_{...}\sum_{i=1}^{t} y_{i..} + \bar{y}_{...}\sum_{k=1}^{t} y_{..k} - 2\frac{y_{...}^2}{t^2}$$

$$= \frac{y_{...}^2}{t} - \frac{y_{...}^2}{t} - \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t^2} + 3\frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t^2} + \frac{y_{...}^2}{t^2} + \frac{y_{...}^2}{t^2} - 2\frac{y_{...}^2}{t^2}$$

$$= 0$$

$$\mathbf{F} = \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\overline{y}_{..k} - \overline{y}_{...})(y_{i(j)k} - \overline{y}_{i..} - \overline{y}_{.j.} - \overline{y}_{..k} + 2\overline{y}_{...})$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\begin{pmatrix} \overline{y}_{..k}y_{i(j)k} - \overline{y}_{..k}\overline{y}_{i..} - \overline{y}_{.j.}\overline{y}_{..k} - \overline{y}_{..k}^2 + 2\overline{y}_{..k}\overline{y}_{...} - y_{i(j)k}\overline{y}_{...} + \overline{y}_{i..}\overline{y}_{...} + \overline{y}_{.j.}\overline{y}_{...} \\ + \overline{y}_{..k}\overline{y}_{...} - 2\overline{y}_{...}^2 \end{pmatrix}$$

$$= \sum_{k=1}^{t}\overline{y}_{..k}\sum_{i=1}^{t}\sum_{j=1}^{t}y_{ij.} - \sum_{i=1}^{t}\overline{y}_{i..}\sum_{j=1}^{t}\sum_{k=1}^{t}\overline{y}_{..k} - \sum_{i=1}^{t}\sum_{j=1}^{t}\overline{y}_{.j.}\sum_{k=1}^{t}\overline{y}_{..k} - \sum_{i=1}^{t}\sum_{j=1}^{t}\left(\sum_{k=1}^{t}\overline{y}_{..k}^2\right)$$

$$+ 3\sum_{i=1}^{t}\sum_{j=1}^{t}\overline{y}_{...}\sum_{k=1}^{t}\overline{y}_{..k} - \overline{y}_{...}\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}y_{i(j)k} + \sum_{j=1}^{t}\sum_{k=1}^{t}\overline{y}_{...}\sum_{i=1}^{t}\overline{y}_{i..}$$

$$+ \sum_{i=1}^{t}\sum_{k=1}^{t}\overline{y}_{...}\sum_{j=1}^{t}\overline{y}_{.j.} - 2\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\overline{y}_{...}^2$$

$$= \frac{\sum_{k=1}^{1}y_{..k}}{t}\sum_{i=1}^{t}\sum_{j=1}^{t}y_{ij.} - \frac{\sum_{i=1}^{t}y_{i..}}{t}\frac{\sum_{j=1}^{t}\sum_{k=1}^{t}y_{..k}}{t} - \frac{\sum_{i=1}^{t}\sum_{j=1}^{t}y_{.j.}}{t}\frac{\sum_{i=1}^{t}\overline{y}_{..k}}{t} - t\frac{\sum_{k=1}^{t}y_{..k}^2}{t^2}$$

$$+ 3\sum_{i=1}^{t}\sum_{k=1}^{t}\overline{y}_{...}\frac{\sum_{j=1}^{t}y_{..k}}{t} - \overline{y}_{...}y_{...} + \sum_{j=1}^{t}\sum_{k=1}^{t}\overline{y}_{...}\frac{\sum_{i=1}^{t}y_{i..}}{t} + \sum_{i=1}^{t}\sum_{j=1}^{t}\overline{y}_{...}\frac{\sum_{j=1}^{t}y_{.j.}}{t} - 2t^2\frac{y_{...}^2}{(t^2)^2}$$

$$= \frac{y_{...}^2}{t} - \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t} + 3t\overline{y}_{...}\frac{\sum_{k=1}^{t}y_{..k}}{t} - \frac{y_{...}^2}{t^2} + t\overline{y}_{...}\frac{\sum_{i=1}^{t}y_{i..}}{t} + t\overline{y}_{...}\frac{\sum_{j=1}^{t}y_{.j.}}{t} - 2\frac{y_{...}^2}{t^2}$$

$$= \frac{y_{...}^2}{t} - \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t} + 3\overline{y}_{...}\sum_{k=1}^{t}y_{..k} - \frac{y_{...}^2}{t^2} + \overline{y}_{...}\sum_{j=1}^{t}y_{i..} + \overline{y}_{...}\sum_{k=1}^{t}y_{..k} - 2\frac{y_{...}^2}{t^2}$$

$$= \frac{y_{...}^2}{t} - \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t} + 3\frac{y_{...}^2}{t^2} - \frac{y_{...}^2}{t^2} + \frac{y_{...}^2}{t^2} + \frac{y_{...}^2}{t^2} - 2\frac{y_{...}^2}{t^2} = 0$$

Sehingga dari penjabaran rumus diatas diperoleh hasil sebagai berikut:

$$\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(y_{ijk}-\bar{y}_{...})^2=\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{i..}-\bar{y}_{...})^2+\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{.j.}-\bar{y}_{...})^2$$

$$+\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{..k}-\bar{y}_{...})^2+\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(y_{i(j)k}-\bar{y}_{i..}-\bar{y}_{.j.}-\bar{y}_{..k}+2\bar{y}_{...})^2$$

$$+0+0+0+0+0+0$$

$$\underbrace{\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(y_{i(j)k}-\bar{y}_{...})^2}_{\textbf{JKT}}=\underbrace{\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{i..}-\bar{y}_{...})^2}_{\textbf{JKBaris}}\ \underbrace{\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{.j.}-\bar{y}_{...})^2}_{\textbf{JKP}}$$

$$\underbrace{+\sum\sum\sum(\bar{y}_{..k}-\bar{y}_{...})^2}_{\textbf{JKKolom}}$$

$$\underbrace{+\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(y_{i(j)k}-\bar{y}_{i..}-\bar{y}_{.j.}-\bar{y}_{..k}+2\bar{y}_{...})^2}_{\textbf{JKG}}$$

JKT merupakan nilai besarnya variasi total yang diakibatkan oleh baris, perlakuan, kolom dan galat. JKBaris menggambarkan besarnya variasi yang disebabkan oleh baris, JKP menggambarkan besarnya variasi yang disebabkan oleh perlakuan, JKKolom menggambarkan besarnya variasi yang disebabkan oleh kolom. Sedangkan JKG menggambarkan besarnya variasi yang tidak terdeteksi. Penggunaan rumus diatas dalam praktek penghitungan akan mengalami kesulitan oleh karena itu dilakukan penyederhanaan rumus diatas yaitu sebagai berikut:

$$\textbf{JKBaris}=\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{i..}-\bar{y}_{...})^2$$

$$=t\sum_{i=1}^{t}(\bar{y}_{i...}-\bar{y}_{...})^2$$

$$= t\sum_{i=1}^{t}\left(\bar{y}_{i..}^2 - 2\bar{y}_{i..}\bar{y}_{...} + \bar{y}_{...}^2\right)$$

$$= t\sum_{i=1}^{t}\bar{y}_{i..}^2 - 2t\bar{y}_{...}\sum_{i=1}^{t}\bar{y}_{i..} + t^2\bar{y}_{...}^2$$

$$= t\frac{\sum_{i=1}^{t} y_{i..}^2}{t^2} - 2t\frac{y_{...}}{t^2}\frac{y_{...}}{t} + t^2\frac{y_{...}^2}{\left(t^2\right)^2}$$

$$= \frac{\sum_{i=1}^{t} y_{i..}^2}{t} - 2\frac{y_{...}^2}{t^2} + \frac{y_{...}^2}{t^2}$$

$$= \frac{\sum_{i=1}^{t} y_{i..}^2}{t} - \frac{y_{...}^2}{t^2}$$

JKP $$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(\bar{y}_{.j.} - \bar{y}_{...}\right)^2$$

$$= t\sum_{j=1}^{t}\left(\bar{y}_{.j.} - \bar{y}_{...}\right)^2$$

$$= t\sum_{j=1}^{t}\left(\bar{y}_{.j.}^2 - 2\bar{y}_{.j.}\bar{y}_{...} + \bar{y}_{...}^2\right)$$

$$= t\sum_{j=1}^{t}\bar{y}_{.j.}^2 - 2t\bar{y}_{...}\sum_{j=1}^{t}\bar{y}_{.j.} + t^2\bar{y}_{...}^2$$

$$= t\frac{\sum_{j=1}^{t} y_{.j.}^2}{t^2} - 2t\frac{y_{...}}{t^2}\frac{y_{...}}{t} + t^2\frac{y_{...}^2}{\left(t^2\right)^2}$$

$$= \frac{\sum_{j=1}^{t} y_{.j.}^2}{t} - 2\frac{y_{...}^2}{t^2} + \frac{y_{...}^2}{t^2}$$

$$= \frac{\sum_{j=1}^{t} y_{.j.}^2}{t} - \frac{y_{...}^2}{t^2}$$

$$\textbf{JKKolom} = \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\bar{y}_{..k} - \bar{y}_{...})^2$$

$$= t\sum_{k=1}^{t}(\bar{y}_{..k} - \bar{y}_{...})^2$$

$$= t\sum_{k=1}^{t}\left(\bar{y}_{..k}^2 - 2\bar{y}_{..k}\bar{y}_{...} + \bar{y}_{...}^2\right)$$

$$= t\sum_{k=1}^{t}\bar{y}_{..k}^2 - 2t\bar{y}_{...}\sum_{k=1}^{t}\bar{y}_{..k} + t^2\bar{y}_{...}^2$$

$$= t\frac{\sum_{k=1}^{t}y_{..k}^2}{t^2} - 2t\frac{y_{...}}{t^2}\frac{y_{...}}{t} + t^2\frac{y_{...}^2}{\left(t^2\right)^2}$$

$$= \frac{\sum_{k=1}^{t}y_{..k}^2}{t} - 2\frac{y_{...}^2}{t^2} + \frac{y_{...}^2}{t^2}$$

$$= \frac{\sum_{k=1}^{t}y_{..k}^2}{t} - \frac{y_{...}^2}{t^2}$$

$$\textbf{JKT} = \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k} - \bar{y}_{...}\right)^2$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\left(y_{i(j)k}^2 - 2y_{ijk}\bar{y}_{...} + \bar{y}_{...}^2\right)$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}y_{i(j)k}^2 - 2\bar{y}_{...}\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}y_{ijk} + t^2\bar{y}_{...}^2$$

$$= \sum\sum\sum y_{i(j)k}^2 - 2\frac{y_{...}}{t^2}y_{...} + t^2\frac{y_{...}^2}{\left(t^2\right)^2}$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}y_{i(j)k}^2 - 2\frac{y_{...}^2}{t^2} + \frac{y_{...}^2}{t^2}$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}y_{i(j)k}^2 - \frac{y_{...}^2}{t^2}$$

JKG = JKT – JKBaris – JKP – JKKolom

Derajat bebas dari **JKT** adalah (t^2-1) dan derajat bebas dari **JKBaris=JKP= JKKolom** adalah (t-1). Sedangkan derajat bebas dari **JKG** adalah (t-2)(t-1). Nilai kuadrat tengahnya adalah sebagai berikut :

$$\text{KTBaris} = \frac{JKBaris}{t-1} \text{ ; KTP} = \frac{JKP}{t-1} \text{ ; KTKolom} = \frac{JKKolom}{t-1} \text{ ;}$$

$$\text{KTG} = \frac{JKG}{(t-1)(t-2)}$$

Nilai harapan dari masing-masing kuadrat tengah adalah sebagai berikut:

$$y_{i..} = \sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k}$$

$$= \sum_{j=1}^{t}\sum_{k=1}^{t}(\mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk})$$

$$= t\mu + t\alpha_i + \sum_{j=1}^{t}\sum_{k=1}^{t}\tau_j + \sum_{j=1}^{t}\sum_{k=1}^{t}\beta_k + \sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk} \text{ karena } \sum_{j=1}^{t}\tau_j = \sum_{k=1}^{t}\beta_k = 0$$

$$= t\mu + t\alpha_i + \sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}$$

$$y_{i..}^2 = \left(t\mu + t\alpha_i + \sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}\right)^2$$

$$= t^2\mu^2 + t^2\alpha_i^2 + \sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}^2 + 2t^2\mu\alpha_i + 2t\mu\sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk} + 2t\alpha_i\sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}$$

$$E(y_{i..}^2) = t^2\mu^2 + t^2\alpha_i^2 + \sum_{j=1}^{t}\sum_{k=1}^{t}E(\varepsilon_{ijk}^2) + 2t^2\mu\alpha_i + 2t\mu\sum_{j=1}^{t}\sum_{k=1}^{t}E(\varepsilon_{ijk})$$

$$+ 2t\alpha_i\sum_{j=1}^{t}\sum_{k=1}^{t}E(\varepsilon_{ijk})$$

Karena $\varepsilon_{ijk} \sim \text{NID}\ (0, \sigma^2)$

$$E(\varepsilon_{ijk}) = 0$$

$$\mathrm{var}(\varepsilon_{ijk}) = E(\varepsilon_{ijk}^2) - E(\varepsilon_{ijk})^2$$

$$\sigma^2 = E(\varepsilon_{ijk}^2) - 0$$

$$E(\varepsilon_{ijk}^2) = \sigma^2$$

Sehingga diperoleh hasil sebagai berikut:

$$E(y_{i..}^2) = t^2\mu^2 + t^2\alpha_i^2 + t\sigma^2 2t^2\mu\alpha_i + 2t\mu.0 + 2t\alpha_i.0 + 2.0$$

$$= t^2\mu^2 + t^2\alpha_i^2 + t\sigma^2 + 2t^2\mu\alpha_i$$

$$\sum_{i=1}^{t}\frac{E(y_{i..}^2)}{t} = \sum_{i=1}^{t}\left(\frac{t^2\mu^2 + t^2\alpha_i^2 + t\sigma^2 + 2t^2\mu\alpha_i}{t}\right)$$

$$= \sum_{i=1}^{t}(t\mu^2 + t\alpha_i^2 + \sigma^2 + 2t\mu\alpha_i)$$

$$= t^2\mu^2 + t\sum_{i=1}^{t}\alpha_i^2 + t\sigma^2 + 2t\mu\sum_{i=1}^{t}\alpha_i$$

$$= t^2\mu^2 + t\sum_{i=1}^{t}\alpha_i^2 + t\sigma^2$$

$$y_{...} = \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k}$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(\mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk})$$

$$= t^2\mu + t\sum_{i=1}^{t}\alpha_i + t\sum_{j=1}^{t}\tau_j + t\sum_{k=1}^{t}\beta_k + \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}$$

Karena $\sum_{i=1}^{t}\alpha_i = \sum_{j=1}^{t}\tau_j = \sum_{k=1}^{t}\beta_k = 0$ sehingga diperoleh

$$= t^2\mu + \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}$$

$$y_{...}^2 = \left(t^2\mu + \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}\right)^2$$

$$= t^4\mu^2 + 2t^2\mu\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk} + \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}^2$$

$$E\left(y_{...}^2\right) = t^4\mu^2 + 2t^2\mu\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}E\left(\varepsilon_{ijk}\right) + \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}E\left(\varepsilon_{ijk}^2\right)$$

$$= t^4\mu^2 + t^2\sigma^2$$

$$\frac{E\left(y_{...}^2\right)}{t^2} = \frac{t^4\mu^2 + t^2\sigma^2}{t^2}$$

$$= t^2\mu^2 + \sigma^2$$

$$E(KTBaris) = E\left(\frac{JKBaris}{t-1}\right)$$

$$= \frac{1}{t-1}E(JKBaris)$$

$$= \frac{1}{t-1}E\left(\frac{\sum_{i=1}^{t} y_{i..}^2}{t} - \frac{y_{...}^2}{t^2}\right)$$

$$= \frac{1}{t-1}\left(\frac{\sum_{i=1}^{t} E\left(y_{i...}^2\right)}{t} - \frac{E\left(y_{...}^2\right)}{t^2}\right)$$

$$= \frac{1}{t-1}\left[\left(t^2\mu^2 + t\sum_{i=1}^{t}\alpha_i^2 + t\sigma^2\right) - \left(t^2\mu^2 + \sigma^2\right)\right]$$

$$= \frac{1}{t-1}\left(t\sum_{i=1}^{t}\alpha_i^2 + (t-1)\sigma^2\right)$$

$$= \sigma^2 + \frac{t\sum_{i=1}^{t}\alpha_i^2}{t-1}$$

$$y_{.j.} = \sum_{i=1}^{t}\sum_{k=1}^{t} y_{i(j)k}$$

$$= \sum_{i=1}^{t}\sum_{k=1}^{t}\left(\mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk}\right)$$

$$= t\mu + \sum_{i=1}^{t}\sum_{k=1}^{t}\alpha_i + t\tau_j + \sum_{i=1}^{t}\sum_{k=1}^{t}\beta_k + \sum_{i=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk} \text{ karena } \sum_{i=1}^{t}\alpha_i = \sum_{k=1}^{t}\beta_k = 0$$

$$= t\mu + t\tau_j + \sum_{i=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}$$

$$y_{.j.}^2 = \left(t\mu + t\tau_j + \sum_{i=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}\right)^2$$

$$= t^2\mu^2 + t^2\tau_j^2 + \sum_{i=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}^2 + 2t^2\mu\tau_j + 2t\mu\sum_{i=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk} + 2t\tau_j\sum_{i=1}^{t}\sum_{k=1}^{t}\varepsilon_{ijk}$$

$$E\left(y_{i..}^2\right) = t^2\mu^2 + t^2\tau_j^2 + \sum_{i=1}^{t}\sum_{k=1}^{t}E\left(\varepsilon_{ijk}^2\right) + 2t^2\mu\tau_j + 2t\mu\sum_{j=1}^{t}\sum_{k=1}^{t}E\left(\varepsilon_{ijk}\right)$$

$$+ 2t\tau_j\sum_{i=1}^{t}\sum_{k=1}^{t}E\left(\varepsilon_{ijk}\right)$$

Karena $\varepsilon_{ijk} \sim \text{NID}\left(0, \sigma^2\right)$

$$E\left(\varepsilon_{ijk}\right) = 0$$

$$\text{var}\left(\varepsilon_{ijk}\right) = E\left(\varepsilon_{ijk}^2\right) - E\left(\varepsilon_{ijk}\right)^2$$

$$\sigma^2 = E\left(\varepsilon_{ijk}^2\right) - 0$$

$$E\left(\varepsilon_{ijk}^2\right) = \sigma^2$$

Sehingga diperoleh hasil sebagai berikut:

$$E\left(y_{.j.}^2\right) = t^2\mu^2 + t^2\tau_j^2 + t\sigma^2 2t^2\mu\tau_j + 2t\mu.0 + 2t\alpha_i.0 + 2.0$$

$$= t^2\mu^2 + t^2\tau_j^2 + t\sigma^2 + 2t^2\mu\tau_j$$

$$\sum_{j=1}^{t}\frac{E\left(y_{.j.}^2\right)}{t} = \sum_{j=1}^{t}\left(\frac{t^2\mu^2 + t^2\tau_j^2 + t\sigma^2 + 2t^2\mu\tau_j}{t}\right)$$

$$= \sum_{j=1}^{t} \left(t\mu^2 + t\tau_j^2 + \sigma^2 + 2t\mu\tau_j \right)$$

$$= t^2\mu^2 + t\sum_{j=1}^{t} \tau_j^2 + t\sigma^2 + 2t\mu\sum_{j=1}^{t} \tau_j$$

$$= t^2\mu^2 + t\sum_{j=1}^{t} \tau_j^2 + t\sigma^2$$

$$E(KTP) = E\left(\frac{JKP}{t-1} \right)$$

$$= \frac{1}{t-1} E(JKP)$$

$$= \frac{1}{t-1} E\left(\frac{\sum_{j=1}^{t} y_{.j.}^2}{t} - \frac{y_{...}^2}{t^2} \right)$$

$$= \frac{1}{t-1} \left(\frac{\sum_{j=1}^{t} E\left(y_{.j.}^2\right)}{t} - \frac{E\left(y_{...}^2\right)}{t^2} \right)$$

$$= \frac{1}{t-1} \left[\left(t^2\mu^2 + t\sum_{j=1}^{t} \tau_j^2 + t\sigma^2 \right) - \left(t^2\mu^2 + \sigma^2 \right) \right]$$

$$= \frac{1}{t-1} \left(t\sum_{j-1}^{t} \tau_j^2 + (t-1)\sigma^2 \right)$$

$$= \sigma^2 + \frac{t\sum_{j=1}^{t} \tau_j^2}{t-1}$$

$$y_{..k} = \sum_{i=1}^{t} \sum_{j=1}^{t} y_{i(j)k}$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\left(\mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk}\right)$$

$$= t\mu + \sum_{i=1}^{t}\sum_{j=1}^{t}\alpha_i + \sum_{i=1}^{t}\sum_{j=1}^{t}\tau_j + t\beta_k + \sum_{i=1}^{t}\sum_{j=1}^{t}\varepsilon_{ijk} \text{ karena } \sum_{i=1}^{t}\alpha_i = \sum_{j=1}^{t}\tau_j = 0$$

$$= t\mu + t\beta_k + \sum_{i=1}^{t}\sum_{j=1}^{t}\varepsilon_{ijk}$$

$$y_{..k}^2 = \left(t\mu + t\beta_{kj} + \sum_{i=1}^{t}\sum_{j=1}^{t}\varepsilon_{ijk}\right)^2$$

$$= t^2\mu^2 + t^2\beta_k^2 + \sum_{i=1}^{t}\sum_{j=1}^{t}\varepsilon_{ijk}^2 + 2t^2\mu\beta_k + 2t\mu\sum_{i=1}^{t}\sum_{j=1}^{t}\varepsilon_{ijk} + 2t\beta_k\sum_{i=1}^{t}\sum_{j=1}^{t}\varepsilon_{ijk}$$

$$E\left(y_{..k}^2\right) = t^2\mu^2 + t^2\beta_k^2 + \sum_{i=1}^{t}\sum_{j=1}^{t}E\left(\varepsilon_{ijk}^2\right) + 2t^2\mu\beta_k + 2t\mu\sum_{i=1}^{t}\sum_{j=1}^{t}E\left(\varepsilon_{ijk}\right)$$

$$+ 2t\beta_k\sum_{i=1}^{t}\sum_{j=1}^{t}E\left(\varepsilon_{ijk}\right)$$

Karena $\varepsilon_{ijk} \sim \text{NID}\left(0, \sigma^2\right)$

$$E\left(\varepsilon_{ijk}\right) = 0$$

$$\text{var}\left(\varepsilon_{ijk}\right) = E\left(\varepsilon_{ijk}^2\right) - E\left(\varepsilon_{ijk}\right)^2$$

$$\sigma^2 = E\left(\varepsilon_{ijk}^2\right) - 0$$

$$E\left(\varepsilon_{ijk}^2\right) = \sigma^2$$

Sehingga diperoleh hasil sebagai berikut:

$$E\left(y_{..k}^2\right) = t^2\mu^2 + t^2\beta_k^2 + t\sigma^2 2t^2\mu\beta_k + 2t\mu.0 + 2t\beta_k 0 + 2.0$$

$$= t^2\mu^2 + t^2\beta_k^2 + t\sigma^2 + 2t^2\mu\beta_k$$

$$\sum_{k=1}^{t}\frac{E\left(y_{..k}^2\right)}{t} = \sum_{k=1}^{t}\left(\frac{t^2\mu^2 + t^2\beta_k^2 + t\sigma^2 + 2t^2\mu\beta_k}{t}\right)$$

$$= \sum_{k=1}^{t} \left(t\mu^2 + t\beta_k^2 + \sigma^2 + 2t\mu\beta_k \right)$$

$$= t^2\mu^2 + t\sum_{k=1}^{t}\beta_k^2 + t\sigma^2 + 2t\mu\sum_{k=1}^{t}\beta_k$$

$$= t^2\mu^2 + t\sum_{j=1}^{t}\beta_k^2 + t\sigma^2$$

$$E(KTKolom) = E\left(\frac{JKKolom}{t-1}\right)$$

$$= \frac{1}{t-1}E(JKKolom)$$

$$= \frac{1}{t-1}E\left(\frac{\sum_{k=1}^{t} y_{..k}^2}{t} - \frac{y_{...}^2}{t^2}\right)$$

$$= \frac{1}{t-1}\left(\frac{\sum_{k=1}^{t} E\left(y_{..k}^2\right)}{t} - \frac{E\left(y_{...}^2\right)}{t^2}\right)$$

$$= \frac{1}{t-1}\left[\left(t^2\mu^2 + t\sum_{k=1}^{t}\beta_k^2 + t\sigma^2\right) - \left(t^2\mu^2 + \sigma^2\right)\right]$$

$$= \frac{1}{t-1}\left(t\sum_{k=1}^{t}\beta_k^2 + (t-1)\sigma^2\right)$$

$$= \sigma^2 + \frac{t\sum_{k=1}^{t}\beta_k^2}{t-1}$$

$$y_{i(j)k} = \left(\mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk}\right)$$

$$y_{i(j)k}^2 = \left(\mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk}\right)^2$$

$$= \mu^2 + \alpha_i^2 + \tau_j^2 + \beta_k^2 + \varepsilon_{ijk}^2 + 2\mu\alpha_i + 2\mu\tau_j + 2\mu\beta_k + 2\alpha_i\tau_j + 2\alpha_i\beta_k$$

$$+2\alpha_i\varepsilon_{ijk}+2\tau_j\beta_k+2\tau_j\varepsilon_{ijk}+2\beta_k\varepsilon_{ijk}$$

$$E\left(y_{i(j)k}^2\right)=\mu^2+\alpha_i^2+\tau_j^2+\beta_k^2+E\left(\varepsilon_{ijk}^2\right)+2\mu\alpha_i+2\mu\tau_j+2\mu\beta_k+2\mu E\left(\varepsilon_{ijk}\right)$$

$$+2\alpha_i\tau_j+2\alpha_i\beta_k+2\alpha_i E\left(\varepsilon_{ijk}\right)+2\tau_j\beta_k+2\tau_j E\left(\varepsilon_{ijk}\right)+2\beta_k E\left(\varepsilon_{ijk}\right)$$

Karena $\varepsilon_{ijk}\sim \text{NID}\left(0,\sigma^2\right)$

$$E\left(\varepsilon_{ijk}\right)=0$$

$$\text{var}\left(\varepsilon_{ijk}\right)=E\left(\varepsilon_{ijk}^2\right)-E\left(\varepsilon_{ijk}\right)^2$$

$$\sigma^2=E\left(\varepsilon_{ijk}^2\right)-0$$

$$E\left(\varepsilon_{ijk}^2\right)=\sigma^2$$

$$E\left(y_{i(j)k}^2\right)=\mu^2+\alpha_i^2+\tau_j^2+\beta_k^2+\sigma^2+2\mu\alpha_i+2\mu\tau_j+2\mu\beta_k+2\mu.0+2\alpha_i\tau_j$$

$$+2\alpha_i\beta_k+2\alpha_i.0+2\tau_j\beta_k+2\tau_j.0+2\beta_k.0$$

$$E\left(y_{i(j)k}^2\right)=\mu^2+\alpha_i^2+\tau_j^2+\beta_k^2+\sigma^2+2\mu\alpha_i+2\mu\tau_j+2\mu\beta_k+2\alpha_i\tau_j+2\alpha_i\beta_k$$

$$+2\tau_j\beta_k$$

$$\sum_{i=1}^t\sum_{j=1}^t\sum_{k=1}^t E\left(y_{i(j)k}^2\right)=\sum_{i=1}^t\sum_{j=1}^t\sum_{k=1}^t\left(\begin{array}{l}\mu^2+\alpha_i^2+\tau_j^2+\beta_k^2+\sigma^2+2\mu\alpha_i+2\mu\tau_j+2\mu\beta_k\\+2\alpha_i\tau_j+2\alpha_i\beta_k+2\tau_j\beta_k\end{array}\right)$$

$$=t^2\mu^2+t\sum_{i=1}^t\alpha_i^2+t\sum_{j=1}^t\tau_j^2+t\sum_{k=1}^t\beta_k^2+t^2\sigma^2+2t\mu\sum_{i=1}^t\alpha_i$$

$$+2t\mu\sum_{j=1}^t\tau_j+2t\mu\sum_{k=1}^t\beta_k+2\sum_{i=1}^t\alpha_i\sum_{j=1}^t\sum_{k=1}^t\tau_j+2\sum_{i=1}^t\alpha_i\sum_{j=1}^t\sum_{k=1}^t\beta_k$$

$$+2\sum_{i=1}^t\alpha_i\sum_{j=1}^t\sum_{k=1}^t\tau_j+2\sum_{i=1}^t\alpha_i\sum_{j=1}^t\sum_{k=1}^t\beta_k+2\sum_{j=1}^t\tau_j\sum_{i=1}^t\sum_{k=1}^t\beta_k$$

$$\text{Karena } \sum_{i=1}^{t}\alpha_i = \sum_{j=1}^{t}\tau_j = \sum_{k=1}^{t}\beta_k = 0$$

$$= t^2\mu^2 + t\sum_{i=1}^{t}\alpha_i^2 + t\sum_{j=1}^{t}\tau_j + t\sum_{k=1}^{t}\beta_k + t^2\sigma^2$$

$$E(JKT) = E\left(\sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} y_{i(j)k}^2 - \frac{y_{...}^2}{t^2}\right)$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} E\left(y_{i(j)k}^2\right) - \frac{E\left(y_{...}^2\right)}{t^2}$$

$$= t^2\mu^2 + t\sum_{i=1}^{t}\alpha_i^2 + t\sum_{j=1}^{t}\tau_j^2 + t\sum_{k=1}^{t}\beta_k^2 + t^2\sigma^2 - t^2\mu^2 - \sigma^2$$

$$= t\sum_{i=1}^{t}\alpha_i^2 + t\sum_{j=1}^{t}\tau_j^2 + t\sum_{k=1}^{t}\beta_k^2 + t^2\sigma^2 - \sigma^2$$

JKG = JKT - JKBaris - JKP – JKKolom

E(JKG) = E(JKT) –E(JKBaris) – E(JKP) – E(JKKolom)

$$E(JKTG) = t\sum_{i=1}^{t}\alpha_i^2 + t\sum_{j=1}^{t}\tau_j^2 + t\sum_{k=1}^{t}\beta_k^2 + \left(t^2-1\right)\sigma^2 - t\sum_{i=1}^{t}\alpha_i^2 - (t-1)\sigma^2 - t\sum_{j=1}^{t}\tau_j^2$$

$$-(t-1)\sigma^2 - t\sum_{k=1}^{t}\beta_k^2 - (t-1)\sigma^2$$

$$= \left(t^2-1\right)\sigma^2 - (t-1)\sigma^2 - (t-1)\sigma^2 - (t-1)\sigma^2$$

$$= t^2\sigma^2 - 3t\sigma^2 + 2\sigma$$

$$= \left(t^2 - 3t + 2\right)\sigma^2$$

$$= (t-1)(t-2)\sigma^2$$

$$E(KTG) = \frac{E(JKG)}{(t-1)(t-2)} = \frac{(t-1)(t-2)\sigma^2}{(t-1)(t-2)} = \sigma^2$$

Dari hasil penghitungan diatas diperoleh nilai harapan dari kuadrat tengah adalah

$$E(KTBaris) = \sigma^2 + \frac{t\sum_{i=1}^{t}\alpha_i^2}{(t-1)}$$

$$E(KTP) = \sigma^2 + \frac{t\sum_{j=1}^{t}\tau_j^2}{(t-1)}$$

$$E(KTKolom) = \sigma^2 + \frac{t\sum_{k=1}^{t}\beta_k^2}{(t-1)}$$

$$E(KTG) = \sigma^2$$

Dari hasil pengolahan rumus diatas diperoleh tabel anova sebagai berikut:

Tabel Anova

Sumber Keragaman	Derajat Bebas	Jumlah Kuadrat	Rata-rata Kuadrat	Ekspektasi Kuadrat Tengah	F_{obs}
Baris	t-1	JKBaris	KTBaris	$\sigma^2 + \frac{t\sum_{i=1}^{t}\alpha_i^2}{(t-1)}$	
Perlakuan	t-1	JKP	KTP	$\sigma^2 + \frac{t\sum_{j=1}^{t}\tau_j^2}{(t-1)}$	KTP/KTG
Kolom	t-1	JKKolom	KTKolom	$\sigma^2 + \frac{t\sum_{k=1}^{t}\beta_k^2}{(t-1)}$	
Galat	(t-1)(t-2)	JKG	KTG	σ^2	
Total	t^2-1	JKT			

Jika benar bahwa perlakuan mempunyai pengaruh yang nyata maka hal ini akan terlihat dari besarnya jumlah kuadrat perlakuan. Sehingga untuk menguji hipotesis bahwa perlakuan mempunyai pengaruh nyata, jumlah kuadrat perlakuan merupakan komponen penting dalam uji statistik. Uji statistik yang digunakan adalah:

$$F_{obs} = \frac{KTP}{KTG} = \frac{JKP/_{t-1}}{JKG/_{(t-1)(t-2)}}$$

Dibawah asumsi H_0 benar maka F_{obs} akan berdistribusi F dengan derajat bebas (t-1) dan (t-1)(t-2). Sehingga dengan tingkat keyakinan sebesar α maka H_0 akan ditolak jika F_{obs} lebih besar dari $F_{\alpha;(t-1);((t-1)(t-2))}$.

Jika dari uji hipotesis perlakuannya ternyata H_0 ditolak maka dilakukan uji lanjut. Uji lanjut yang digunakan untuk membandingakn semua perlakuan adalah uji lanjut perbandingan ganda dengan menggunakan metode LSD (Least Significant Difference).

Langkah-langkahnya sebagai berikut:

1. Hitung galat baku dari perlakuan ke-*i* dan perlakuan ke-*j*

$$S_{\bar{y}_{.j.}-\bar{y}_{.j'.}} = \sqrt{\mathbf{KTG}\left(\frac{1}{t_{.j.}} + \frac{1}{t_{.j'.}}\right)}$$, untuk data tidak seimbang

dengan

$t_{.j.}$ = banyaknya pengulangan tiap perlakuan ke-*j*.

$t_{.j'.}$ = banyaknya pengulangan tiap perlakuan ke-j'.

$$S_{\bar{y}_{.j.}-\bar{y}_{.j'.}} = \sqrt{\frac{2(\mathbf{KTG})}{t}}$$, untuk data seimbang sebab $t_1 = t_2 = ... = t$

2. Hitung LSD yaitu

$$LSD = t_{\alpha/2;db(galat)} S_{\bar{y}_{.j.} - \bar{y}_{.j'.}}$$

dengan,

$t_{\alpha/2;db(galat)}$ = tabel t dengan α sebagai tingkat signifikan

3. Jika $\left|\bar{y}_{.j..} - \bar{y}_{.j'..}\right|$ lebih besar dari *LSD* maka pasangan mean tersebut berbeda secara signifikan dan diberi tanda *.