LAMPIRAN-A

PERSAMAAN TAMPANG LINTANG DIFERENSIAL DALAM SUKU MULTIPOL ELEKTROMAGNETIK

Perumusan tampang lintang hamburan elektron-inti dengan menggunakan formula Rosenbluth adalah sebagai berikut :

$$d^2\sigma = \frac{4\alpha^2 m^2}{q_\mu^4 \varepsilon' |\vec{k}|} \left|\overline{u}_{\lambda'}(k')\gamma_\mu u_\lambda(\vec{k})\right|^2 (|k'|\varepsilon' d\varepsilon' d\Omega)$$

$$\otimes \frac{Vd^3 p}{(2\pi)^3}\delta(E'+\varepsilon'-E-\varepsilon)\, J_\mu(q)\, J_\nu^*(q). \qquad \text{(A.1)}$$

Adapun bentuk persamaan tampang lintang persatuan ruang fase yang tersedia untuk inti recoil, dapat diperoleh dengan mengintegrasikan $d^2\sigma$ meliputi $d\varepsilon'$ dengan faktor delta Dirac masih tetap ada. Hasilnya adalah :

$$(d\sigma)_{tp} = d\Omega \frac{\alpha^2}{q_\mu^4}\frac{\varepsilon'}{\varepsilon}\tfrac{1}{2}Tr\{\gamma_\mu(m-i\gamma.k)\gamma_\nu(m-i\gamma.k')\}J_\mu(q)\, J_\nu^*(q).$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{q_\mu^4}.\frac{\varepsilon'}{\varepsilon}2\{k_\mu k_{\nu'} + k_{\mu'}k_\nu + \tfrac{1}{2}q_\mu^2\delta_{\mu\nu}\}J_\mu(q)\, J_\nu^*(q). \qquad \text{(A.2)}$$

Dengan mendefinisikan :

$$Q_\mu \equiv \tfrac{1}{2}(k'+k)_\mu$$
$$q_\mu \equiv (k'-k)_\mu \qquad \text{(A.3)}$$

diperoleh :

$$4Q_\mu Q_\nu - q_\mu q_\nu = 2\{k_\mu k_{\nu'} + k_{\mu'} k_\nu\} \qquad \text{(A.4)}$$

substitusi persamaan (4) ke dalam persamaan (2) menghasilkan :

$$\left(\frac{d\sigma}{d\Omega}\right)_{tp} = \frac{2\alpha^2}{q_\mu^4}\cdot\frac{\varepsilon'}{\varepsilon}\left\{2Q_\mu Q_\nu + \tfrac{1}{2}q_\mu q_\nu + \tfrac{1}{2}q_\mu^2 \delta_{\mu\nu}\right\} J_\mu(q)\, J_\nu^*(q)$$
$$= \frac{2\alpha^2}{q_\mu^4}\cdot\frac{\varepsilon'}{\varepsilon}\{\, 2\,(J_\mu(q)\, Q_\mu)\,(J_\nu^*(q)\, Q_\nu)$$
$$- \tfrac{1}{2} q_\mu J_\mu(q)\, q_\nu J_\nu^*(q) + \tfrac{1}{2} q_\mu^2 J_\mu(q)\, J_\nu^*(q) \,\}$$

(A.5)

Dari persamaan kontinyuitas arus inti yang berbentuk :

$$\frac{\partial}{\partial x_\mu} J_\mu(x) = 0$$

akan diperoleh :

$$
\begin{aligned}
q_\mu J_\mu(\vec{q}) &= q_\mu \int \exp(-i\vec{q}.\vec{x})\, J_\mu(\vec{x})\, d^3x \\
&= \frac{i}{|\vec{q}|} q_\mu \frac{\partial}{\partial x_\mu} \int \exp(-i\vec{q}.\vec{x})\, J_\mu(\vec{x})\, d^3x \\
&= \frac{i}{|\vec{q}|} q_\mu \int \exp(-i\vec{q}.\vec{x}) \left\{ \frac{\partial}{\partial x_\mu} J_\mu(\vec{x}) \right\} d^3x \\
&= 0,
\end{aligned}
$$

sehingga persamaan (A.5) menjadi :

$$
\left(\frac{d\sigma}{d\Omega}\right)_{lp} = \frac{2\alpha^2}{q_\mu^4} \cdot \frac{\varepsilon'}{\varepsilon} \left\{ 2\,(J_\mu(q)\, Q_\mu)\,(J_\nu^*(q)\, Q_\nu) + \tfrac{1}{2} q_\mu^2 J_\nu(q)\, J_\nu^*(q) \right\} \quad \text{(A.6)}
$$

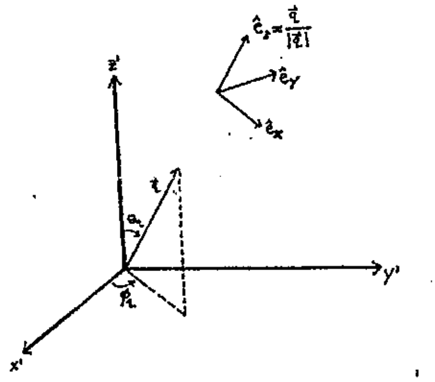
rerata yang melingkupi state awal dan jumlahan yang melingkupi state akhir inti dengaan bilangan kuantum (JM) :

$$
\sum_i \sum_f \leftrightarrow \frac{1}{2J_i+1} \sum_{M_i} \sum_{M_f}
$$
, memberikan :

$$
\begin{aligned}
m &= \frac{1}{2J_i+1} \sum_{M_i} \sum_{M_f} \left\{ 2\,(J_\mu(q)\, Q_\mu)\,(J_\nu^*(q)\, Q_\nu) + \tfrac{1}{2} q_\mu^2 J_\nu(q)\, J_\nu^*(q) \right\} \\
&= \frac{1}{2J_i+1} \sum_{M_i} \sum_{M_f} \left\{ 2\,(\vec{J}.\vec{Q} - Q_0)\,(\vec{J}^*.\vec{Q} - \rho^*\, Q_0) + \tfrac{1}{2} q_\mu^2 (\vec{J}.\vec{J}^*\, \rho\rho^*) \right\}
\end{aligned}
$$

(A.7)

dalam basis sferik, didefinisikan :



Gambar A.1 Basis.basis vektor sferik

$$\hat{e}_\lambda \begin{cases} \hat{e}_{\pm} = \mp \frac{1}{\sqrt{2}}(\hat{e}_x \pm i\hat{e}_y) \\ \hat{e}_0 = \hat{e}_z = \hat{e}_0^* \end{cases} \tag{A.8}$$

dengan sifat.sifat : $\hat{e}_\lambda^* . \hat{e}_{\lambda'} = \delta_{\lambda\lambda'}$ (A.9)

Sehingga komponen J dapat dituliskan dalam basis tersebut sebagai :

$$\begin{aligned} J_\mu(\vec{q}) &= \sum_\lambda J_\lambda(\vec{q})\, \hat{e}_\lambda^* \\ J_\lambda(\vec{q}) &= \hat{e}_\lambda . J_\mu(\vec{x}) \int \exp(-i\vec{q}.\vec{x})\, \hat{J}\ (\vec{x})\, d^3x \end{aligned} \tag{A.10}$$

Sehingga :

$$\vec{J}.\vec{Q} = \sum_{\lambda=0,\pm1} J_\lambda Q_\lambda^* = \sum_{\lambda=\pm1} J_\lambda Q_\lambda^* + J_0 Q_0$$
$$= \sum_{\lambda=\pm1} J_\lambda Q_\lambda^* + \frac{\omega}{|\vec{q}|}\rho Q_0 \qquad \text{(A.11)}$$

dimana : $J_0 = \frac{\omega}{|\vec{q}|}\rho$, berasal dari kontinyuitas arus inti.

Dengan substitusi persamaan (A.11) ke dalam persamaan (A.7) didapat :

$$m = \frac{1}{2J_i + 1}\sum_{M_i}\sum_{M_f}\Bigg[2\{ \sum_{\lambda=\pm1} J_\lambda Q_\lambda^* + (\frac{\omega}{|\vec{q}|} - Q_0)\ \}\{ \sum_{\lambda=\pm1} J_{\lambda'}^* Q_{\lambda'}^* + (\frac{\omega}{|\vec{q}|} - Q_0)\rho^*\ \} + \tfrac{1}{2}q_\mu^2 \{ \sum_{\lambda=\pm1} J_\lambda J_\lambda^* - (1 - \frac{\omega^2}{|\vec{q}|^2})\rho\rho^*\ \} \Bigg]$$

(A.12)

dimana bentuk $\sum_{M_i}\sum_{M_f}\sum_{\lambda=\pm1} J_\lambda \rho^* = \sum_{M_i}\sum_{M_f}\sum_{\lambda=\pm1} J_{\lambda'}^* \rho = 0$,

karena arus tranversal membawa momentum sudut ± 1 sepanjang $\vec{q}$ sementara arus longitudinal membawa momentum 0 sepanjang $\vec{q}$ sehingga keduanya tidak dapat berinterferensi dalam $\sum_{M_i}\sum_{M_f}$ karena keduanya memiliki state akhir yang berbeda.

Oleh karenanya, persamaan (A.12) menjadi :

$$\vec{J}.\vec{Q} = \sum_{\lambda=0,\pm1} J_\lambda Q_\lambda^* = \sum_{\lambda=\pm1} J_\lambda Q_\lambda^* + J_0 Q_0$$
$$= \sum_{\lambda=\pm1} J_\lambda Q_\lambda^* + \frac{\omega}{|\vec{q}|}\rho Q_0 \qquad \text{(A.11)}$$

dimana : $J_0 = \frac{\omega}{|\vec{q}|}\rho$, berasal dari kontinyuitas arus inti.

Dengan substitusi persamaan (A.11) ke dalam persamaan (A.7) didapat :

$$m = \frac{1}{2J_i+1}\sum_{M_i}\sum_{M_f}\Big[2\{\sum_{\lambda=\pm1} J_\lambda Q_\lambda^* + (\frac{\omega}{|\vec{q}|} - Q_0)\}\{\sum_{\lambda=\pm1} J_{\lambda'}^* Q_{\lambda'}^* + (\frac{\omega}{|\vec{q}|} - Q_0)\rho^*\} + \tfrac{1}{2}q_\mu^2\{\sum_{\lambda=\pm1} J_\lambda J_\lambda^* - (1 - \frac{\omega^2}{|\vec{q}|^2})\rho\rho^*\}\Big] \qquad \text{(A.12)}$$

dimana bentuk $\sum_{M_i}\sum_{M_f}\sum_{\lambda=\pm1} J_\lambda \rho^* = \sum_{M_i}\sum_{M_f}\sum_{\lambda=\pm1} J_{\lambda'}^* \rho = 0$,

karena arus tranversal membawa momentum sudut ± 1 sepanjang $\vec{q}$ sementara arus longitudinal membawa momentum 0 sepanjang $\vec{q}$ sehingga keduanya tidak dapat berinterferensi dalam $\sum_{M_i}\sum_{M_f}$ karena keduanya memiliki state akhir yang berbeda.

Oleh karenanya, persamaan (A.12) menjadi :

$$m = \frac{1}{2J_i + 1}\sum_{M_i}\sum_{M_f}\left[2\{\sum_{\lambda}\sum_{\lambda'} J_{\lambda}J^*_{\lambda}Q^*_{\lambda'}Q^*_{\lambda'} + (\frac{\omega}{|\vec{q}|} - Q_0)\rho\rho^* \right.$$

$$\left. + \tfrac{1}{2}q^2_{\mu}\{\sum_{\lambda} J_{\lambda'}J^*_{\lambda} - (1 - \frac{\omega^2}{|\vec{q}|^2})\rho\rho^* \}\right] \qquad \text{(A.13)}$$

sumbangan komponen waktu rapat arus

Dari tranformasi Fourier :

$$\begin{aligned}\rho(\vec{q}) &= \langle J_f M_f|\rho(\vec{q})|J_i M_i\rangle \\ &= \int \exp(-i\vec{q}.\vec{x})\,\langle J_f M_f|\hat{\rho}(\vec{x})|J_i M_i\rangle d^3x,\end{aligned} \qquad \text{(A.14)}$$

$\exp(-i\vec{q}.\vec{x})$ jika diekspansikan ke dalam gelombang sferik, akan memiliki bentuk :

$$\exp(-i\vec{q}.\vec{x}) = 4\pi\sum_{J}\sum_{M}(-i)^J\int j_J(qx)Y_{JM}(\Omega_x)Y^*_{JM}(\Omega_q), \qquad \text{(A.15)}$$

sehingga persamaan (A.14) menjadi :

$$\rho(\vec{q}) = 4\pi \sum_{JM} (-i)^J \int j_J(qx) Y_{JM}(\Omega_x) Y^*_{JM}(\Omega_q)$$

$$\otimes \left\langle J_f M_f \left| \hat{\rho}(\vec{x}) \right| J_i M_i \right\rangle d^3x \qquad \text{(A.16)}$$

$$= 4\pi \sum_{JM} (-i)^J Y^*_{JM}(\Omega_q) \left\langle J_f M_f \left| \hat{M}^{coul}_{JM}(q) \right| J_i M_i \right\rangle,$$

dimana didefinisikan operator multipol Coulomb :

$$\hat{M}^{coul}_{JM} \equiv \int j_J(qx) Y_{JM}(\Omega_x)\, \hat{\rho}(\vec{x})\, d^3x. \qquad \text{(A.17)}$$

Dari teorema Wigner.Eckart :

$$\left\langle J_f M_f \left| \hat{M}^{coul}_{JM}(q) \right| J_i M_i \right\rangle = (-1)^{J_f - M_f} \begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix} \qquad \text{(A.18)}$$

$$\otimes \left\langle J_f \left\| \hat{M}^{coul}_{J}(q) \right\| J_i \right\rangle,$$

persamaan (A.16) menjadi :

$$\rho(\vec{q}) = 4\pi \sum_{JM} (-i)^J Y^*_{JM}(\Omega_q) (-1)^{J_f - M_f} \begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix} \qquad \text{(A.19)}$$

$$\otimes \left\langle J_f \left\| \hat{M}^{coul}_{J}(q) \right\| J_i \right\rangle,$$

sehingga rerata yang melingkupi *state* awal dan jumlahan yang melingkupi state akhir memberikan :

$$\frac{1}{2J_i+1}\sum_{M_i}\sum_{M_f}\rho(\vec{q})\rho^*(\vec{q})=\frac{(4\pi)^2}{2J_i+1}\sum_{JM}\sum_{J'M'}(-1)^{J-J'}Y^*_{JM}(\Omega_q)$$
$$\otimes Y_{J'M'}(\Omega_q)(-1)^{J_f-M_f}\begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix}\begin{pmatrix} J_f & J' & J_i \\ -M_f & M' & M_i \end{pmatrix} \quad \text{(A.20)}$$
$$\otimes\left\langle J_f\left\|\hat{M}^{coul}_J(q)\right\|J_i\right\rangle\left\langle J_f\left\|\hat{M}^{coul}_{J'}(q)\right\|J_i\right\rangle$$

Dari hubungan ortogonalitas :

$$\sum_{JM}\sum_{J'M'}\begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix}\begin{pmatrix} J_f & J' & J_i \\ -M_f & M' & M_i \end{pmatrix}=\frac{1}{2J_i+1}\delta_{JJ'}\delta_{MM'} \quad \text{(A.21)}$$

dari teorama Legendre :

$$\sum_J\sum_M Y^*_{JM}(\Omega_q)Y_{JM}(\Omega_q)=\frac{2J+1}{4\pi}, \quad \text{(A.22)}$$

maka persamaan (A.20) menjadi :

$$\frac{1}{2J_i+1}\sum_{M_i}\sum_{M_f}\rho(\vec{q})\rho^*(\vec{q})=\frac{(4\pi)^2}{2J_i+1}\sum_{J\geq 0}\left|\left\langle J_f\left\|\hat{M}^{coul}_J(q)\right\|J_i\right\rangle\right|^2. \quad \text{(A.23)}$$

sumbangan komponen ruang rapat arus

Dari definisi basis vektor sferik (persamaan A.8 dan A.9) dan persamaan (A.10), elemen matrik operator rapat arus dapat dituliskan sebagai :

$$J_\lambda(\vec{q}) = \left\langle J_f M_f \left| \hat{J}_\lambda(\vec{q}) \right| J_i M_i \right\rangle$$
$$= \int \hat{e}_\lambda . \exp(-i\vec{q}.\vec{x}) \left\langle J_f M_f \left| \hat{J}_\lambda(\vec{x}) \right| J_i M_i \right\rangle d^3x, \qquad \text{(A.24)}$$

Faktor $\hat{e}_\lambda . \exp(-i\vec{q}.\vec{x})$ dapat diekspansikan dalam gelombang sferik dengan langkah.langkah sebagai berikut :

Dari definisi vektor harmonik sferik :

$$\vec{Y}^M_{Jl1} = \sum_m \sum_\lambda \langle lm1\lambda | l1JM \rangle \vec{Y}_{lm}(\Omega) \hat{e}_\lambda , \qquad \text{(A.25)}$$

maka:

$$\sum_J \sum_M \vec{Y}^M_{Jl1} \langle lm'1\lambda' | l1JM \rangle$$
$$= \sum_{JM} \sum_{m\lambda} \langle lm'1\lambda' | l1JM \rangle \langle lm1\lambda | l1JM \rangle Y_{lm}(\Omega) \hat{e}_\lambda$$
$$= \sum_m \sum_\lambda \delta_{mm'} \delta_{\lambda\lambda'} Y_{lm}(\Omega) \hat{e}_\lambda \qquad \text{(A.26)}$$
$$= Y_{lm'}(\Omega) \hat{e}_{\lambda'} ,$$

sehingga :

$$\vec{Y}_{\ell m}(\Omega) \hat{e}_\lambda = \sum_J \sum_M \vec{Y}^M_{Jl1} \langle l\, m 1\lambda | l1JM \rangle . \qquad \text{(A.27)}$$

Dengan memilih sumbu.z sejajar dengan $\vec{q}$, didapat :

$$\begin{aligned}\hat{e}_\lambda.\exp(-i\vec{q}.\vec{x}) &= \sum_l (-1)^l \{4\pi(2l+1)\}^{\frac{1}{2}} j_l(qx) Y_{l0}(\Omega_x)\,\hat{e}_\lambda \\ &= \sum_l (-i)^l \{4\pi(2l+1)\}^{\frac{1}{2}} j_l(qx) \langle l 0 1 \lambda | l 1 J\lambda\rangle \vec{Y}^{\lambda}_{Jl1}\end{aligned} \qquad \text{(A.28)}$$

l kopling dengan $1 \Rightarrow J$, maka : $l = J-1; J; J+1$, sehingga :

(a). sumbangan $l = J-1$ pada penjumlahan ke l :

$$\begin{aligned}\hat{e}_\lambda.\exp(-i\vec{q}.\vec{x}) &= \sum_J (-i)^{J-1} \{4\pi(2J-1)\}^{\frac{1}{2}} j_{J-1}(qx) \\ &\quad \otimes \langle J-1\ 0\ 1\lambda | J-1\ 1\ J\lambda\rangle \vec{Y}^{\lambda}_{JJ-11} \\ &= i\sum_J (-i)^J \{2\pi(2J+1)\}^{\frac{1}{2}} j_{J-1}(qx) \vec{Y}^{\lambda}_{JJ-11},\end{aligned} \qquad \text{(A.29)}$$

dimana :

$$\langle J-1\ 0\ 1\lambda | J-1\ 1\ J\lambda\rangle = \left\{\frac{J+1}{2(2J-1)}\right\}^{\frac{1}{2}}. \qquad \text{(A.30)}$$

(b). sumbangan l=J pada penjumlahan ke l :

$$\hat{e}_{\lambda}.\exp(-i\vec{q}.\vec{x}) = \sum_{J} (-i)^{J+1}\{4\pi(2J+1)\}^{\frac{1}{2}} j_J(qx)\langle J\ 0\ 1\lambda | J\ 1\ J\lambda\rangle \vec{Y}^{\lambda}_{JJ1}$$
$$= i\sum_{J} (-i)^{J}\{2\pi(2J+1)\}^{\frac{1}{2}} j_J(qx)\vec{Y}^{\lambda}_{JJ1}, \qquad \text{(A.31)}$$

dimana :

$$\langle J\ 0\ 1\lambda | J\ 1\ J\lambda\rangle = \lambda\left\{\frac{1}{2}\right\}^{\frac{1}{2}}. \qquad \text{(A.32)}$$

(c). sumbangan l=J+1 pada penjumlahan ke l :

$$\hat{e}_{\lambda}.\exp(-i\vec{q}.\vec{x}) = \sum_{J} (-i)^{J+1}\{4\pi(2J+1)\}^{\frac{1}{2}} j_{J+1}(qx)\langle J+1\ 0\ 1\lambda | J+1\ 1\ J\lambda\rangle \vec{Y}^{\lambda}_{JJ+11}$$
$$= i\sum_{J} (-i)^{J}\{2\pi J\}^{\frac{1}{2}} j_{J+1}(qx)\vec{Y}^{\lambda}_{JJ+11},$$

(A.33)

dimana :

$$\langle J+1\ 0\ 1\lambda | J+1\ 1\ J\lambda\rangle = \lambda\left\{\frac{J}{2(2J+3)}\right\}^{\frac{1}{2}}. \qquad \text{(A.34)}$$

Dengan mengingat hubungan rekurensi yang memberikan :

$$\frac{1}{q}\nabla x(j_J(qx)\vec{Y}^{\lambda}_{JJ1}) = -i\left(\frac{J}{2J+1}\right)^{\frac{1}{2}} j_{J+1}(qx)\vec{Y}^{\lambda}_{JJ+11} + i\left(\frac{J+1}{2J+1}\right)^{\frac{1}{2}} j_{J+1}(qx)\vec{Y}^{\lambda}_{JJ-11} \qquad \text{(A.35)}$$

maka sumbangan total l pada penjumlahan meliputi l adalah jumlahan persamaan (A.29), (A.31) dan (A.33) dan substutusi persamaan (A.35) pada penjumlahan tersebut, sehingga di dapat hasil :

$$\hat{e}_{\lambda}.\exp(-i\vec{q}.\vec{x}) = -(2\pi)^{\frac{1}{2}}\sum_{J}(-i)^{J+1}(2J+1)^{\frac{1}{2}}\left\{j_J(qx)\vec{Y}^{\lambda}_{JJ1} + \frac{1}{q}\nabla x(j_J(qx)\vec{Y}^{\lambda}_{JJ1})\right\}$$

(A.36)

sehingga persamaan dapat dituliskan sebagai :

$$J_{\lambda}(\vec{q}) = -(2\pi)^{\frac{1}{2}}\sum_{J\geq 1}(-i)^{J}(2J+1)^{\frac{1}{2}} \otimes \left\langle J_f M_f \middle| \lambda\hat{T}^{mag}_{JM}(q) + \hat{T}^{el}_{JM}(q) \middle| J_i M_i \right\rangle, \tag{A.37}$$

di mana didefinisikan operator.operator multipol tranversal listrik dan magnetik, sebagai :

$$\hat{T}^{el}_{J\lambda}(q) \equiv \frac{1}{q}\int \nabla x(j_J(qx)\vec{Y}^{\lambda}_{JJ1}).\hat{\vec{J}}(\vec{x})\,d^3x, \tag{A.38}$$

$$\hat{T}^{mag}_{J\lambda}(q) \equiv \int j_J(qx)\vec{Y}^{\lambda}_{JJ1}.\hat{\vec{J}}(\vec{x})\,d^3x. \tag{A.39}$$

dalam koordinat sembarang, persamaan (A.37) harus diputar melalui sudut-sudut EULER dengan rotator $D^J_{M\lambda}(-\phi_q\ \theta_q\ \phi_q)$:

$$\hat{T}^{el}_{J\lambda}(q) \equiv \sum_M D^J_{M\lambda}(-\phi_q\ \theta_q\ \phi_q)\hat{T}^{mag}_{JM}(q), \qquad \text{(A.40)}$$

$$\hat{T}^{mag}_{J\lambda}(q) \equiv \sum_M D^J_{M\lambda}(-\phi_q\ \theta_q\ \phi_q)\hat{T}^{el}_{JM}(q), \qquad \text{(A.41)}$$

sehingga persamaan (A.37), dengan menerapkan teorema Wigner-Eckart akan menjadi :

$$J_\lambda(\vec{q}) = -(2\pi)^{\frac{1}{2}} \sum_{J\geq 1} (-i)^J (2J+1)^{\frac{1}{2}} (-1)^{J_f - M_f} \sum_{M=-J}^{J} \begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_f \end{pmatrix}$$
$$\otimes \left\langle J_f M_f \middle| \lambda \hat{T}^{mag}_{JM}(q) + \hat{T}^{el}_{JM}(q) \middle| J_i M_i \right\rangle D^J_{M\lambda}(-\phi_q\ \theta_q\ \phi_q), \qquad \text{(A.42)}$$

Rerata yang melingkupi *state* awal dan jumlahan yang meliputi *state* akhir, memberikan bentuk :

$$\frac{1}{2J_i+1}\sum_{M_i}\sum_{M_f} J_\lambda(\vec{q})J_{\lambda'}^*(\vec{q}) = S_{\lambda\lambda'}$$

$$= \frac{2\pi}{2J_i+1}\sum_{JJ'}\sum_{MM'}(-1)^{J-J'}(2J+1)^{\frac{1}{2}}(2J'+1)^{\frac{1}{2}}\sum_{M_i}\sum_{M_f}\begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix}$$

$$\otimes\begin{pmatrix} J_f & J' & J_i \\ -M_f & M' & M_i \end{pmatrix}\langle J_f \| \lambda\hat{T}_J^{mag} + \hat{T}_J^{el} \| J_i\rangle\langle J_f \| \lambda T_{J'}^{mag}(q) + T_{J'}^{el}(q) \| J_i\rangle^* \quad \text{(A.43)}$$

$$\otimes D_{M\lambda}^{J}(-\phi_q\,\theta_q\,\phi_q)\,D_{M'\lambda'}^{J'}(-\phi_q\,\theta_q\,\phi_q)^*$$

$$= \delta_{\lambda\lambda'}\frac{2\pi}{2J_i+1}\sum_{J\geq 1}\left\{\left|\langle J_f \| \hat{T}_J^{mag}(q) \| J_i\rangle\right|^2 + \left|\langle J_f \| T_J^{el}(q) \| J_i\rangle\right|^2\right\}.$$

Untuk memperoleh persamaan (A.43) dipakai hubungan :

$$\sum_{M_i}\sum_{M_f}\begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix}\begin{pmatrix} J_f & J' & J_i \\ -M_f & M' & M_i \end{pmatrix} = \frac{1}{2J_i+1}\delta_{JJ'}\delta_{MM'}, \quad \text{(A.44)}$$

$$\sum_{M}\sum_{M'} D_{M\lambda}^{J}(-\phi_q\theta_q\phi_q)D_{M'\lambda'}^{J'}(-\phi_q\theta_q\phi_q)^*\delta_{MM'} = \delta_{MM'}\delta_{\lambda\lambda'}. \quad \text{(A.45)}$$

substitusi persamaan (A.43) dan (A.23) ke dalam persamaan (A.13) serta hasil penjabaran pada lampiran-B, diperoleh :

$$m=\frac{1}{2J_i+1}\sum_{M_i}\sum_{M_f}\left[2\left\{\sum_{\lambda}\sum_{\lambda'} J_\lambda J^*_\lambda Q^*_{\lambda'} Q^*_{\lambda'}+\left(\frac{\omega}{|\vec{q}|}-Q_0\right)\rho\rho^*\right.\right.$$

$$\left.+\tfrac{1}{2}q^2_\mu\left\{\sum_{\lambda} J_\lambda J^*_\lambda-\left(1-\frac{\omega^2}{|\vec{q}|^2}\right)\rho\rho^*\right\}\right]$$

$$=\frac{4\pi}{2J_i+1}\left[\sum_{J\geq 0} V_L(\theta)\left|\left\langle J_f\left\|\hat{M}^{cuol}_J(q)\right\|J_i\right\rangle\right|^2\right.$$

$$\left.+\sum_{J\geq 1} V_T(\theta)\left\{\left|\left\langle J_f\left\|\hat{T}^{mag}_J(q)\right\|J_i\right\rangle\right|^2+\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2\right\}\right],$$

(A.46)

dimana didefinisikan faktor bentuk kinematika longitudinal dan tranversal :

$$V_L(\theta)\equiv\frac{q^4_\mu}{|\vec{q}|^4}\left(2Q^2_0-\tfrac{1}{2}|\vec{q}|^2\right), \qquad \text{(A.47)}$$

$$V_T(\theta)\equiv\left(\vec{Q}^2-\frac{(q.Q)^2}{|\vec{q}|^4}+\tfrac{1}{2}q^2_\mu\right). \qquad \text{(A.48)}$$

Dari hasil persamaan (A.46), maka tampang lintang hamburan dalam suku multipol dapat disajikan sebagai :

$$\left(\frac{d\sigma}{d\Omega}\right)_{tp}=\frac{2\alpha^2}{q^4_\mu}\frac{\varepsilon'}{\varepsilon}\frac{4\pi}{2J_i+1}\left[\sum_{J\geq 0} V_L(\theta)\left|\left\langle J_f\left\|\hat{M}^{cuol}_J(q)\right\|J_i\right\rangle\right|^2\right.$$

$$\left.+\sum_{J\geq 1} V_T(\theta)\left\{\left|\left\langle J_f\left\|\hat{T}^{mag}_J(q)\right\|J_i\right\rangle\right|^2+\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2\right\}\right]. \qquad \text{(A.49)}$$

Dari pendekatan relativistik , didapat :

$$V_L(\theta) \equiv \frac{q_\mu^4}{|\vec{q}|^4}(2Q_0^2 - \tfrac{1}{2}|\vec{q}|^2) = \frac{q_\mu^4}{|\vec{q}|^4} \cdot \tfrac{1}{2}\left\{(\varepsilon' + \varepsilon)^2 - (\vec{k}' - \vec{k})^2\right\} \tag{A.50}$$
$$= 2\varepsilon\varepsilon' \cos^2 \tfrac{1}{2}\theta,$$

$$V_T(\theta) \equiv (\vec{Q}^2 - \frac{(q.Q)^2}{|\vec{q}|^4} + \tfrac{1}{2}q_\mu^2) = \tfrac{1}{4}(\vec{k}' + \vec{k})^2 - \frac{(\vec{k}' - \vec{k})^2.(\vec{k}' + \vec{k})^2}{|\vec{k}' - \vec{k}|^2} + (k' - k)^2 \tag{A.51}$$
$$\approx 4\varepsilon\varepsilon' \sin^2 \tfrac{1}{2}\theta + 2\varepsilon\varepsilon' \cos^2 \tfrac{1}{2}\theta,$$

sehingga diperoleh hasil akhir tampang lintang hamburan dalam suku multipol :

$$\left(\frac{d\sigma}{d\Omega}\right)_{tp} = \frac{4\pi\sigma_M}{1 + (2\varepsilon \sin^2 \frac{1}{2}\theta)/M} \frac{1}{2J_i + 1}\left[\sum_{J \geq 0} \left|\langle J_f \| \hat{M}_J^{cuol}(q) \| J_i \rangle\right|^2 \right.$$
$$\left. + (\tfrac{1}{2} + \tan^2 \tfrac{1}{2}\theta) \sum_{J \geq 1} \left\{\left|\langle J_f \| \hat{T}_J^{mag}(q) \| J_i \rangle\right|^2 + \left|\langle J_f \| \hat{T}_J^{el}(q) \| J_i \rangle\right|^2\right\}\right]$$

(A.52)

LAMPIRAN-B

FAKTOR STRUKTUR TRANSVERSAL DAN LONGITUDINAL

$$J_\mu(\vec{q})Q_\mu = \sum_{\lambda=\pm1}\left\{J_\lambda(\vec{q})Q_\lambda^* + J_3(\vec{q})Q_3 + J_4(\vec{q})Q_4\right\}$$

$$\xrightarrow[x_3 /\!/ \vec{q}]{} \sum_{\lambda=\pm1}\left\{J_\lambda(\vec{q})Q_\lambda^* + \left(\frac{\omega}{|\vec{q}|^2}\vec{q}.\vec{Q} - Q_0\right)\rho(\vec{q})\right\} \qquad \text{(B.1)}$$

$$J_\mu^*(\vec{q})Q_\mu^* = \sum_{\lambda'=\pm1}\left\{J_{\lambda'}(\vec{q})Q_{\lambda'}^* + J_3^*(\vec{q})Q_3^* + J_4^*(\vec{q})Q_4^*\right\}$$

$$\xrightarrow[x_3 /\!/ \vec{q}]{} \sum_{\lambda'=\pm1}\left\{J_{\lambda'}(\vec{q})Q_{\lambda'} + \left(\frac{\omega}{|\vec{q}|^2}\vec{q}.\vec{Q} - Q_0\right)\rho^*(\vec{q})\right\} \qquad \text{(B.2)}$$

$$J_\nu\ (\vec{q})J_\nu^*(\vec{q}) = \sum_{\lambda'=\pm1}\left\{J_\lambda(\vec{q})J_\lambda^*(\vec{q}) + \left(\frac{\omega}{|\vec{q}|^2}\vec{q}.\vec{Q} - 1\right)\rho(\vec{q})\rho(\vec{q})^*\right\} \qquad \text{(B.3)}$$

$$m = \frac{1}{2J_i+1}\sum_{M_i}\sum_{M_f}\left[\sum_{\lambda,\lambda'=\pm1} 2J_\lambda(\vec{q})J_{\lambda'}^*(\vec{q})Q_\lambda^*Q_{\lambda'}\right.$$

$$+\sum_{\lambda=\pm1} 2J_\lambda(\vec{q})\rho^*(\vec{q})\left(\frac{\omega}{|\vec{q}|^2} - Q_0\right)Q_\lambda^*$$

$$\left.+\sum_{\lambda'=\pm1} 2J_{\lambda'}^*(\vec{q})\rho(\vec{q})Q_{\lambda'}\left(\frac{\omega}{|\vec{q}|^2}\vec{q}.\vec{Q} - Q_0\right)^2 + \tfrac{1}{2}q_\mu^2\sum_{\lambda=\pm1} J_\lambda(\vec{q})J_\lambda^*(\vec{q})\right]$$

$$+\left[2\left(\frac{\omega}{|\vec{q}|^2}\vec{q}.\vec{Q} - Q_0\right)^2 - \tfrac{1}{2}q_\mu^2\frac{q_\nu^2}{|\vec{q}|^2}\right]\rho(\vec{q})\rho^*(\vec{q})$$

$$m = \frac{1}{2J_i+1}\sum_{M_i}\sum_{M_f}\Big[\sum_{\lambda,\lambda'=\pm1} 2J_\lambda(\vec{q})J^*_{\lambda'}(\vec{q})Q^*_\lambda Q_{\lambda'}$$

$$+\tfrac{1}{2}q_\mu^2\sum_{\lambda=\pm1} J_\lambda(\vec{q})J_\lambda(\vec{q})\Big]+\left[2(\frac{\omega}{|\vec{q}|^2}\vec{q}.\vec{Q}-Q_0)^2-\tfrac{1}{2}\frac{q_\nu^2}{|\vec{q}|^2}\right]\rho(\vec{q})\rho^*(\vec{q}) \qquad \text{(B.4)}$$

dengan menuliskan :

$$\vec{q}.\vec{Q} = \tfrac{1}{2}(k'-k).(k'+k) = \tfrac{1}{2}(\varepsilon'^2-\varepsilon^2) = \tfrac{1}{2}(\varepsilon'+\varepsilon)(\varepsilon'-\varepsilon) = \tfrac{1}{2}\omega(\varepsilon'+\varepsilon) \qquad \text{(B.5a)}$$

dan

$$Q_0\,\tfrac{1}{2}(\varepsilon'+\varepsilon) \qquad \text{(B.5b)}$$

maka:

$$(\frac{\omega}{|\vec{q}|^2}\vec{q}.\vec{Q}-Q_0)^2 = \left\{2\frac{\omega}{|\vec{q}|^2}(\varepsilon'+\varepsilon)-\tfrac{1}{2}(\varepsilon'+\varepsilon)\right\}^2$$

$$= \tfrac{1}{2}(\varepsilon'+\varepsilon)^2(\frac{\omega^2}{|\vec{q}|^2}-\frac{\vec{q}^2}{|\vec{q}|^2})^2 = \tfrac{1}{2}(\varepsilon'+\varepsilon)^2\left\{\frac{q_\mu^*}{|\vec{q}|^2}\right\}^2 \qquad \text{(B.6)}$$

bentuk :

$$\frac{1}{2J_i+1}\sum_{\lambda,\lambda'=\pm1}\sum_{M_i}\sum_{M_f}2J_\lambda(\vec{q})J^*_{\lambda'}(\vec{q})Q^*_\lambda Q_{\lambda'}$$

$$=\frac{1}{2J_i+1}\sum_{\lambda,\lambda'=\pm1}\left\{\delta_{\lambda\lambda'}(2\pi)\sum_{J\geq1}\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2+\left|\left\langle J_f\left\|\hat{T}^{mag}_J(q)\right\|J_i\right\rangle\right|^2\right\}$$

$$\otimes\left\{\sum_{\lambda=\pm1}Q^*_\lambda Q_\lambda-Q^*_3Q_3\right\}$$

$$=\frac{4\pi}{2J_i+1}\sum_{J\geq1}\left\{\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2+\left|\left\langle J_f\left\|\hat{T}^{mag}_J(q)\right\|J_i\right\rangle\right|^2\right\}\left\{\left|\vec{Q}\right|^2-\left(\frac{\vec{q}.\vec{Q}}{\left|\vec{q}\right|^2}\right)^2\right\}$$

(B.7)

Bentuk :

$$\frac{1}{2J_i+1}\sum_{\lambda=\pm1}\sum_{M_i}\sum_{M_f}2J_\lambda(\vec{q})J^*_{\lambda'}(\vec{q})$$

$$=\frac{4\pi}{2J_i+1}\sum_{J\geq1}\left\{\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2+\left|\left\langle J_f\left\|\hat{T}^{mag}_J(q)\right\|J_i\right\rangle\right|^2\right\} \qquad (B.8)$$

substitusi persamaan (B.6), (B.7) dan (B.8) ke dalam persamaan (B.4), diperoleh:

$$
\begin{aligned}
m &= \frac{1}{2J_i+1}\sum_{M_i}\sum_{M_f}\left\{2J_\lambda(\vec{q})J^*_{\lambda'}(\vec{q})Q^*_\lambda Q_{\lambda'}+\tfrac{1}{2}q^2_\mu J_\nu(\vec{q})J^*_\nu(\vec{q})\right\} \\
&= \frac{4\pi}{2J_i+1}\left\{2\left|\vec{Q}\right|^2-\frac{(\vec{q}.\vec{Q})^2}{\left|\vec{q}\right|^2}+\tfrac{1}{2}q^2_\mu\right\}\sum_{J\geq 1}\left\{\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2+\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2\right\} \\
&\otimes\left\{\tfrac{1}{2}(\varepsilon'+\varepsilon)^2\frac{q^4_\mu}{\left|\vec{q}\right|^4}-\tfrac{1}{2}\frac{q^4_\mu}{\left|\vec{q}\right|^2}\right\}\rho(\vec{q})\rho^*(\vec{q}) \\
&= \frac{4\pi}{2J_i+1}\sum_{M_i}\sum_{M_f}\left[V_T(\theta)\sum_{J\geq 1}\left\{\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2+\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2\right\}\right. \\
&\left.+V_L(\theta)\sum_{J\geq 1}\left|\left\langle J_f\left\|\hat{T}^{el}_J(q)\right\|J_i\right\rangle\right|^2\right]
\end{aligned}
\tag{B.9}
$$

di mana didefinisikan :

$$
V_L(\theta)\equiv\tfrac{1}{2}\frac{q^4_\mu}{\left|\vec{q}\right|^4}\left\{4Q^2_0-\left|\vec{q}\right|^2\right\}, \tag{B.10a}
$$

sebagai faktor kinematika longitudinal, dan :

$$
V_T(\theta)\equiv\left|\vec{Q}\right|^2-\frac{(\vec{q}.\vec{Q})^2}{\left|\vec{q}\right|^2}+\tfrac{1}{2}q^2_\mu \tag{B.10b}
$$

sebagai faktor kinematika transversal.

LAMPIRAN-C

BEBERAPA SIFAT VEKTOR HARMONIK SFERIK

DAN FUNGSI BESSEL SFERIK

1. $\vec{Y}^{M}_{JJl} \equiv \sum_{M\lambda} \left\langle lm1\lambda \middle| l1JM \right\rangle Y_{JM}(\Omega_x)\hat{e}_\lambda$ (C.1)

$$\hat{e}_\lambda = \begin{cases} \hat{e}_\pm = \mp(\frac{1}{2})^{\frac{1}{2}}(\hat{e}_x \pm \hat{e}_y) \\ \hat{e}_0 = \hat{e}_Z = \hat{e}_0^* \end{cases}$$

2. $(J(J+1))^{-\frac{1}{2}} \vec{L} Y_{JM}(\Omega) = (J(J+1))^{-\frac{1}{2}}(-i)(\vec{x} \times \nabla) Y_{JM}(\Omega) = \vec{Y}^{M}_{JJ1}$ (C.2)

3. $$\begin{aligned} \nabla \times (\phi(x)\vec{Y}^{M}_{JJ1}) &= i(\frac{d}{dx} - \frac{J}{x})\phi(x)(\frac{J}{2J+1})^{\frac{1}{2}} \vec{Y}^{M}_{JJ+11} \\ &+ i(\frac{d}{dx} + \frac{J+1}{x})\phi(x)(\frac{J+1}{2J+1})^{\frac{1}{2}} \vec{Y}^{M}_{JJ-11} \end{aligned}$$ (C.3)

4. $\nabla \bullet (\phi(x)\vec{Y}^{M}_{JJ1}) = 0$ (C.4)

5. $$\nabla(\phi(x)\vec{Y}^{M}_{JJ1}) = -(\frac{J+1}{2J+1})^{\frac{1}{2}}(\frac{d}{dx}-\frac{J}{x})\phi(x)\vec{Y}^{M}_{JJ+11} + (\frac{J}{2J+1})^{\frac{1}{2}}(\frac{d}{dx}+\frac{J+1}{x})\phi(x)\vec{Y}^{M}_{JJ-11} \quad (C.5)$$

6. $$(\frac{d}{dqx}-\frac{J}{qx})j_J(qx) = -j_{J+1}(qx) \quad (C.6)$$

7. $$(\frac{d}{dqx}-\frac{J+1}{qx})j_J(qx) = j_{J-1}(qx) \quad (C.7)$$

8. $$j_J(qx) \xrightarrow[qx\to 0]{} = \frac{(qx)^J}{(2J+1)!!} \quad (C.8)$$

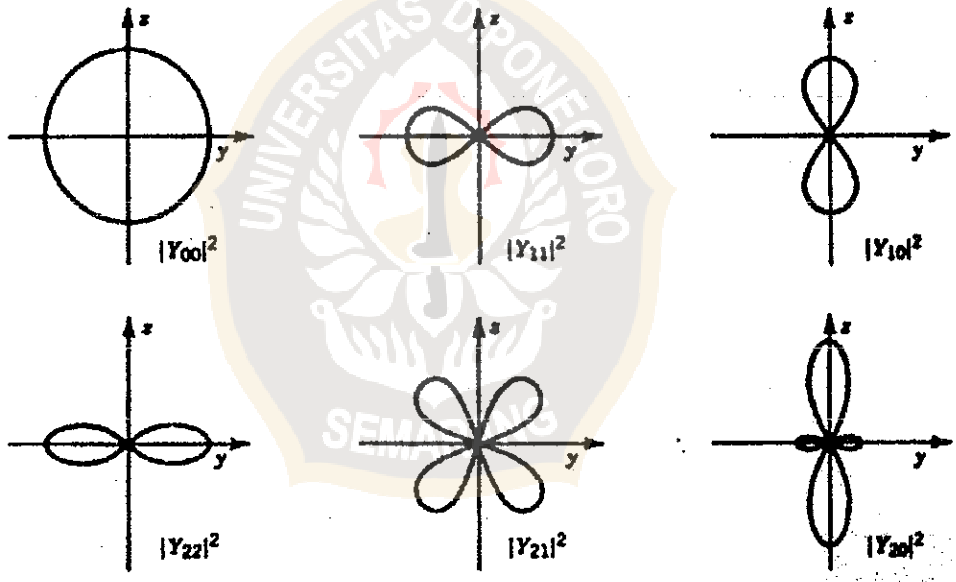
dari persamaan (C.3), (C.6) dan (C.7), diperoleh :

$$\nabla\times(j_J(qx)\vec{Y}^{M}_{JJ1}) = -i(\frac{J}{2J+1})^{\frac{1}{2}} j_J(qx)\vec{Y}^{M}_{JJ+11} + i(\frac{J+1}{2J+1})^{\frac{1}{2}} j_{J-1}(qx)\vec{Y}^{M}_{JJ-11} \quad (C.9)$$

dan dari persamaan (C.5), (C.6) dan (C.7), diperoleh :

$$\nabla(j_J(qx)\vec{Y}_{JM}(\Omega)) = (\frac{J+1}{2J+1})^{\frac{1}{2}} j_{J+1}(qx)\vec{Y}^{M}_{JJ+11} + (\frac{J}{2J+1})^{\frac{1}{2}} j_{J-1}(qx)\vec{Y}^{M}_{JJ-11} \tag{C.10}$$

Distribusi kebolehjadian beberapa orde dari fungsi harmonik.



Gambar C.1Distribusi kebolehjadian spatial dari fungsi harmonik yang menggambarkan struktur monopol, dipol maupun kuadrupol

LAMPIRAN-D

PEMBUKTIAN PERSAMAAN

Ukuran jejari muatan inti

Untuk q kecil atau θ kecil ($q = 2k\sin\frac{1}{2}\theta$), persamaan faktor struktur dalam bentuk eksponensial dapat diuraikan sebagai berikut :

$$\begin{aligned} F(q) &= \int \rho(r)\exp(iq.r)d^3x \\ &= \int \rho(r)\left[1 + iq.r + \tfrac{1}{2!}(iq.r)^2 + \ldots\right]d^3x \\ &= \int \rho(r)d^3x - \tfrac{1}{2!}\int \rho(r)(iq.r)^2 d^3x + \ldots \\ &\approx 0 - \tfrac{1}{6}q^2\int \rho(r)r^2 d^3x = -\tfrac{1}{6}q^2\left\langle r^2\right\rangle \end{aligned} \tag{D.1}$$

sehingga diperoleh :

$$\left\langle r^2\right\rangle = -\frac{6dF}{dq^2}\bigg|_{q^2=0} \tag{D.2}$$

Hasil tersebut dapat juga diperoleh dari persamaan faktor struktur dalam bentuk sinus sebagai fungsi kuadrat alih momentum yaitu :

$$F(q^2) = \frac{4\pi}{Ze}\int_0^\infty r^2\rho(r)\frac{\sin(qr)}{qr}dr \tag{D.3}$$

Turunan pertama terhadap alih momentum, diperoleh :

$$\frac{dF}{dq} = \frac{4\pi}{Ze}\int_0^\infty \rho(r)\frac{d}{dq}\left(\frac{\sin(qr)}{qr}\right)dr$$
$$= \frac{4\pi}{Ze}\int_0^\infty \rho(r)\left[\frac{r\cos(qr)}{qr} - \frac{\sin(qr)}{q^2r}\right]dr \qquad \text{(D.4)}$$

sehingga

$$\frac{dF}{d(q^2)} = \frac{dF}{dq}\frac{dq}{d(q^2)} = \frac{1}{2q}\cdot\frac{4\pi}{Ze}\int_0^\infty \rho(r)\left[\frac{r\cos(qr)}{qr} - \frac{\sin(qr)}{q^2r}\right]dr$$
$$= \frac{2\pi}{Ze}\int_0^\infty \rho(r)\left[\frac{r\cos(qr)}{q^2r} - \frac{\sin(qr)}{q^3r}\right]dr \qquad \text{(D.5)}$$

untuk $q^2 \approx 0$, maka $\left.\frac{dF}{d(q^2)}\right|_{q^2=0}$ dapat dihitung sebagai berikut :

$$\lim_{q\to 0}\left[\frac{r\cos(qr)}{q^2r} - \frac{\sin(qr)}{q^3r}\right] = \lim_{q\to 0}\left[\frac{r - \left[1 - \frac{1}{2}(qr)^2\right]}{q^2r} - \frac{qr - \frac{1}{6}(qr)^3}{q^3r}\right]$$
$$= \lim_{q\to 0}\left[-\frac{1}{3}r^2\right] \qquad \text{(D.6)}$$
$$= -\frac{r^2}{3}$$

diperoleh :

$$\frac{dF}{d(q^2)} = \frac{2\pi}{Ze}\int_0^\infty \rho(r)\left(-\frac{r^2}{3}\right)dr$$

$$= -\frac{1}{6}\cdot\frac{1}{Ze}\int_0^\infty r^2\rho(r)4\pi r^2 dr \qquad \text{(D.7)}$$

$$= -\frac{1}{6}\left\langle r^2\right\rangle$$

persamaan (D.7) diperoleh dari $\left\langle r^2\right\rangle$ yang merupakan nilai harap kuadrat jejari yang dirumuskan :

$$\left\langle r^2\right\rangle = \int |\psi_0|^2 r^2 d^3r \qquad \text{(D.8)}$$

rapat inti yang berbentuk :

$$\rho(r) = Ze|\psi_0|^2 \qquad \text{(D.9)}$$

dan dengan menganggap inti pada *ground state* berbentuk simetri bola, sehingga elemen volume berbentuk :

$$d^3r = 4\pi r^2 dr\ . \qquad \text{(D.10)}$$

Dari persamaan (D.7) jejari muatan inti diperoleh :

$$\langle r^2 \rangle^{\frac{1}{2}} = \left(-6 \frac{dF}{d(q)^2} \right)^{\frac{1}{2}}. \qquad \text{(D.11)}$$

Berdasarkan persamaan (D.11) nilai jejari muatan tersebut dapat diperoleh dari grafik faktor struktur sebagai fungsi kuadrat alih momentum yaitu dengan menghitung gradien grafik pada titik $q^2 \approx 0$.

<u>Contoh menghitung kuadrat jejari muatan</u> :

Menghitung kuadrat jejari muatan proton, dari grafik gambar (5.3) didapatkan bahwa pada $q^2 = 5\ fm^{2}$, nilai faktor struktur listriknya adalah 0,67, sehingga dengan menggunakan persamaan (D.2) ataupun (D.11) diperoleh :

$$\begin{aligned} \langle r^2 \rangle &= \left(-6 \frac{dF}{d(q)^2} \right)_{q^2=0} \\ &= -6 \cdot \frac{F(q_1^2) - F(0)}{q_1^2 - 0} \\ &= -6. \frac{0{,}67 - 1}{5 - 0\ fm^{-2}} \\ &= 0{,}44\ fm^2 \end{aligned} \qquad \text{(D.12)}$$

Pembuktian Persamaan (4.40) menjadi (4.41)

$$\left|\left\langle J_0 J_0 \left|\hat{\bar{\mu}}_{J0}\right| J_0 J_0 \right\rangle\right| = (-1)^{J_0-J_0}\begin{pmatrix} J_0 & 1 & J_0 \\ -J_0 & 0 & J_0 \end{pmatrix}\left|\left\langle J_0 \left\|\hat{\bar{\mu}}_J\right\| J_0 \right\rangle\right|$$

$$= (-1)^{J_0-J_0}(-1)^{J_0-J_0}\frac{J_0}{\sqrt{J_0(2J_0+1)(J_0+1)}}\left|\left\langle J_0 \left\|\hat{\bar{\mu}}_J\right\| J_0 \right\rangle\right| \quad \text{(D.13)}$$

$$= \left(\frac{J_0}{(2J_0+1)(J_0+1)}\right)^{\frac{1}{2}}\left|\left\langle J_0 \left\|\hat{\bar{\mu}}_J\right\| J_0 \right\rangle\right|$$

hasil pengkuadratan kedua ruas diperoleh :

$$\left|\left\langle J_0 J_0 \left|\hat{\bar{\mu}}_{J0}\right| J_0 J_0 \right\rangle\right|^2 = \frac{J_0}{(2J_0+1)(J_0+1)}\left|\left\langle J_0 \left\|\hat{\bar{\mu}}_J\right\| J_0 \right\rangle\right|^2$$

$$\frac{J_0+1}{J_0}\left|\left\langle J_0 J_0 \left|\hat{\bar{\mu}}_{J0}\right| J_0 J_0 \right\rangle\right|^2 = \frac{1}{2J_0+1}\left|\left\langle J_0 \left\|\hat{\bar{\mu}}_J\right\| J_0 \right\rangle\right|^2 \quad \text{(D.14)}$$

Pembuktian persamaan (4.46) :

Persamaan (4.36) yang merupakan operator multipol magnet dapat dituliskan dalam bentuk :

$$-i\hat{T}_{JM}^{mag}(q) = \int\left\{\nabla(J_J(qx)Y_{JM}(\Omega))\right\}\cdot\left\{\frac{L\cdot J}{J(J+1)(J+1)} + \frac{\mu_i S\cdot J}{2MJ(J+1)}\right\}J\, d^3x, \quad \text{(D.15)}$$

elemen matrik :

$$\left\langle \left| \hat{T}_{JM}^{mag}(q) \right| \right\rangle = \int \{\nabla (J_J(qx) Y_{JM}(\Omega))\} \left\{ \frac{\langle |L \cdot J| \rangle}{J(J+1)(J+1)} + \frac{\mu_i \langle |S \cdot J| \rangle}{2MJ(J+1)} \right\} J \, d^3x . \qquad (D.16)$$

Untuk *state triplet*-S $\rightarrow J = 1, \quad S = 1, \quad L = 0$, diperoleh :

$$L \cdot J = \tfrac{1}{2}\big(J(J+1) + L(L+1) - S(S+1)\big) = 0$$

$$S \cdot J = \tfrac{1}{2}\big(J(J+1) - L(L+1) + S(S+1)\big) = 2$$

dengan menerapkan fungsi gelombang *state triplet*-S pada elemen matrik, diperoleh :

$$\langle |L \cdot J| \rangle = 0$$

$$\langle |S \cdot J| \rangle = \frac{2}{4\pi r^2} u^2$$

sehingga didapatkan :

$$\left\{ \frac{\langle |L \cdot J| \rangle}{J(J+1)(J+1)} + \frac{\mu_i \langle |S \cdot J| \rangle}{2MJ(J+1)} \right\} = \frac{1}{4\pi} \left\{ 0 + \frac{\mu_i 2u^2}{2M \cdot 2} \right\} = \frac{(\mu_p + \mu_n) u^2}{2M} \qquad (D.17)$$

Untuk *state tripel*-D $\rightarrow J = 1, \quad S = 1, \quad L = 2$ diperoleh :

$$L \cdot J = \tfrac{1}{2}\big(J(J+1) + L(L+1) - S(S+1)\big) = 3$$

$$S \cdot J = \tfrac{1}{2}\big(J(J+1) - L(L+1) + S(S+1)\big) = -1$$

dengan menerapkan fungsi gelombang *state triplet*-D pada elemen matrik, diperoleh :

$$\langle |L \cdot J| \rangle = \frac{3}{4\pi r^2} w^2$$

$$\langle |S \cdot J| \rangle = -\frac{1}{4\pi r^2} w^2$$

sehingga didapatkan :

$$\left\{ \frac{\langle |L \cdot J| \rangle}{J(J+1)(J+1)} + \frac{\mu_i \langle |S \cdot J| \rangle}{2MJ(J+1)} \right\}$$
$$= \frac{1}{4\pi r^2}\left\{ \frac{3}{4} w^2 - \frac{\mu_i}{2M \cdot 2} w^2 \right\} = \frac{1}{4\pi r^2}\left(\frac{3}{4} - \frac{(\mu_p + \mu_n)}{2} \right) w^2 . \qquad \text{(D.18)}$$

Dengan menggunakan sifat fungsi Bessel dalam persamaan (C.10), didapatkan :

$$\nabla(J_J(qx) Y_{JM}(\Omega)) = \left(\frac{J+1}{2J+1} \right)^{\frac{1}{2}} j_2(qx) Y_{22} + \left(\frac{J}{2J+1} \right)^{\frac{1}{2}} j_0(qx) Y_{00} . \qquad \text{(D.19)}$$

untuk pendekatan q kecil akan diperoleh fungsi Bessel orde-2 yang sangat kecil, sehingga suku pertama persamaan (D.19) dapat diabaikan. Elemen matrik persamaan (D.16) menjadi :

$$\left|\left\langle\left\|\hat{T}^{mag}\right\|\right\rangle\right| = \frac{1}{4\pi r^2}\left(\frac{J}{2J+1}\right)^{\frac{1}{2}}\left[\int\left\{(\mu_p+\mu_n)u^2+\tfrac{1}{2}(\tfrac{3}{2}-(\mu_p+\mu_n))w^2\right]j_0(\tfrac{1}{2}qr)Y_{00}d^3r\right]$$

$$= \frac{1}{\sqrt{4\pi}}\left(\frac{J}{2J+1}\right)^{\frac{1}{2}}\int\left[(\mu_p+\mu_n)u^2+\tfrac{1}{2}(\tfrac{3}{2}-(\mu_p+\mu_n))w^2\right]j_0(\tfrac{1}{2}qr)dr$$

(D.20)

Diperoleh persamaan faktor struktur magnetik deuteron :

$$\left|F_M(q)\right|^2 = \frac{4\pi}{Z^2}\frac{1}{2J_0+1}\left|\left\langle\left\|\hat{T}^{mag}\right\|\right\rangle\right|^2$$

$$= \frac{1}{Z}\frac{1}{(2J+1)}\left[\int\left\{(\mu_p+\mu_n)u^2+\tfrac{1}{2}(\tfrac{3}{2}-(\mu_p+\mu_n))w^2\right]j_0(\tfrac{1}{2}qr)dr\right]$$ (D.21)

LAMPIRAN-E

PERUMUSAN FAKTOR STRUKTUR KUADRUPOL

Menentukan persamaan faktor struktur momen kuadrupol

Dari persamaan :

$$\left|F_Q(q)\right| = \int \ (2uw - \tfrac{2}{\sqrt{8}} w^2) j_2(\tfrac{1}{2} qr) dr. \qquad \text{(E.1)}$$

untuk hasil pendekatan fungsi Bessel sferik berlaku :

$$j_l(qr) \approx \frac{(qr)^l}{1.3.5...(2l+1)!}, \qquad \text{(E.2)}$$

sehingga persamaan (E.1) menjadi :

$$\begin{aligned}
\left|F_Q(q)\right| &= \int \ (2uw - \tfrac{2}{\sqrt{8}} w^2) \frac{(\frac{1}{2} qr)^2}{15} dr. \\
&= \frac{(\frac{1}{2} q)^2}{15} \int \ (2uw - \tfrac{2}{\sqrt{8}} w^2) r^2 \, dr \\
&= \frac{(\frac{1}{2} q)^2}{15} \frac{20}{\sqrt{2}} \frac{\sqrt{2}}{20} \int \ (2uw - \tfrac{2}{\sqrt{8}} w^2) r^2 \, dr \\
&= \frac{(\frac{1}{2} q)^2}{15} \frac{20}{\sqrt{2}} Q \\
&= \frac{\sqrt{8}}{3} (\tfrac{1}{2} q)^2 Q
\end{aligned} , \qquad \text{(E.3)}$$

dengan

$$Q=\frac{\sqrt{2}}{20}\int\ (2uw-\tfrac{2}{\sqrt{8}}w^2)r^2\,dr. \qquad (E.4)$$

Menentukan persamaan fungsi gelombang deuteron

Karena dalam *ground state*-nya deuteron memiliki dua komponen state yaitu $^3S_1$ dan $^3D_1$, maka fungsi gelombang deuteron pada *ground state* berbentuk :

$$\begin{aligned}\psi_D &= \psi(^3S_1)+\psi(^3D_1)\\ &= \frac{1}{\sqrt{4\pi}}\left(\frac{u}{r}+\frac{S_{12}}{\sqrt{8}}\frac{w}{r}\right)\end{aligned} \qquad (E.5)$$

S_{12} merupakan operator tensor yang dirumuskan secara umum (Eisenberg, 1972 : 20) :

$$S_{JLL'}=\frac{6\sqrt{J(J+1)}}{2J+1}\ \text{untuk}\ L'=J-1\ \text{dan}\ L=J+1, \qquad (E.6a)$$

dan

$$S_{JLL'}=\frac{-2(J+2)}{2J+1}\ \text{untuk}\ L'=L=J+1\ . \qquad (E.6b)$$

Karena deuteron memiliki spin total J=1 dan pada komponen state $^3D_1$ L=2, maka operator tensornya adalah S_{12}, yang memiliki nilai :

$$S_{12} = \frac{6\sqrt{1(1+1)}}{2.1+1} = \sqrt{8} \text{ untuk L'=0,} \quad \text{(E.7a)}$$

dan

$$S_{12} = \frac{-2(1+2)}{2.1+1} = -2 \text{ untuk L'=2.} \quad \text{(E.7b)}$$

Oleh karena itu fungsi gelombang *ground state* deuteron untuk masing-masing komponen *state* adalah :

$$\psi(^3S_1) = \frac{1}{\sqrt{4\pi}}\frac{u}{r}, \quad \text{(E.8)}$$

$$\psi(^3D_1) = \frac{1}{\sqrt{4\pi}}\frac{w}{r} \text{ untuk L'=0, dan} \quad \text{(E.9a)}$$

$$\psi(^3D_1) = -\frac{1}{\sqrt{4\pi}}\frac{2}{\sqrt{8}}\frac{w}{r} \text{ untuk L'=2.} \quad \text{(E.9b)}$$

Menetukan persamaan momen kuadrupol Deuteron

Persamaan (E.4) adalah momen kuadrupol deuteron yang diperoleh dengan menghitung nilai harap dari operator kuadrupol Q_{20} yang berbentuk :

$$Q_{20} = (3z^2 - r^2), \quad \text{(E.10)}$$

atau dengan memasukkan bentuk $z = r\cos\theta$, persamaan (E.10) menjadi :

$$Q_{20} = r^2(3\cos^2\theta - 1). \qquad \text{(E.11)}$$

Karena vektor posisi r untuk proton adalah $\frac{1}{2}r$ dan neutron adalah $-\frac{1}{2}r$, maka persamaan (E.11) menjadi :

$$Q_{20} = \tfrac{1}{4}r^2(3\cos^2\theta - 1), \qquad \text{(E.12)}$$

atau

$$Q_{20} = \tfrac{1}{4}r^2\sqrt{\frac{16\pi}{5}}\,Y_{20}(\theta,\phi), \qquad \text{(E.13)}$$

dimana

$$Y_{20}(\theta,\phi) = \sqrt{\frac{5}{16\pi}}\,(3\cos^2\theta - 1). \qquad \text{(E.14)}$$

Nilai harap dari operator momen kuadrupol dirumuskan :

$$Q = \langle Q_{20}\rangle = \int \psi^* Q_{20}\psi\, d^3r. \qquad \text{(E.15)}$$

Adanya dua fungsi gelombang memungkinkan nilai harap operator kuadrupol akan disumbang oleh kedua komponen state dengan bentuk kombinasi :

$$\begin{aligned}\langle Q_{20}\rangle &= \int \psi_D{}^* Q_{20}\psi_D \; d^3r \\ &= \int \psi(^3S_1)^* Q_{20}\psi(^3S_1)\; d^3r + 2\int \psi(^3S_1)Q_{20}\psi(^3D_1). \\ &\quad + \int \psi(^3D_1)Q_{20}\psi(^3D_1)\; d^3r\end{aligned} \qquad \text{(E.16)}$$

Karena diketahui bahwa kuadrupol akan muncul pada L=2, maka suku pertama dari persamaan akan hilang :

$$\begin{aligned}\langle 00|Q_{20}|00\rangle &= (-1)^0\begin{pmatrix}0 & 2 & 0\\ 0 & 0 & 0\end{pmatrix}\langle 0\|Q_{20}\|0\rangle \\ &= 0\end{aligned}$$

akibatnya persamaan (E.16) akan menjadi :

$$\langle Q_{20}\rangle = 2\int \psi(^3S_1)Q_{20}\psi(^3D_1) + \int \psi(^3D_1)Q_{20}\psi(^3D_1)\; d^3r. \qquad \text{(E.17)}$$

Dengan menerapkan fungsi gelombang ground state deuteron pada persamaan (E.8), (E.9) dan operator kuadrupol pada persamaan (E.13) maka nilai harap momen kuadrupol pada persamaan (E.17) menjadi :

$$\langle Q_{20}\rangle = 2\int \frac{1}{\sqrt{4\pi}}\frac{u}{r}\cdot\tfrac{1}{4}r^2\sqrt{\frac{16\pi}{5}}Y_{20}(\theta,\phi)\cdot\frac{1}{\sqrt{4\pi}}\frac{w}{r}d^3r$$

$$+\int \frac{1}{\sqrt{4\pi}}\frac{w}{r}\cdot\tfrac{1}{4}r^2\sqrt{\frac{16\pi}{5}}Y_{20}(\theta,\phi)(-\frac{1}{\sqrt{4\pi}}\frac{2}{\sqrt{8}}\frac{w}{r})d^3r$$

$$=\frac{1}{4\pi r^2}\int (2uw-\tfrac{2}{\sqrt{8}}w^2)\tfrac{1}{4}r^2\sqrt{\frac{16\pi}{5}}Y_{20}(\theta,\phi)\, d^3r \qquad \text{(E.18)}$$

$$=\frac{1}{4\pi r^2}\int (2uw-\tfrac{2}{\sqrt{8}}w^2)\tfrac{1}{4}r^2\sqrt{\frac{16\pi}{5}}\sqrt{\frac{5}{16\pi}}(3\cos^2\theta-1)\frac{4\pi\sqrt{2}\, r^2}{5(3\cos^2\theta-1)}dr$$

$$=\frac{\sqrt{2}}{20}\int (2uw-\tfrac{2}{\sqrt{8}}w^2)r^2dr.$$

LAMPIRAN-F

BEBERAPA BENTUK KOEFISIEN CLEBSCH-GORDAN DAN SIMBOL-3j

$$\begin{pmatrix} j & 0 & j' \\ -m & 0 & m' \end{pmatrix} = \frac{(-1)^{j-m}}{\sqrt{2j+1}} \delta_{jj'} \delta_{mm'} \quad \text{(F.1)}$$

$$\begin{pmatrix} j & 1 & j' \\ -m & 0 & m' \end{pmatrix} = (-1)^{j-m} \frac{m}{\sqrt{j(2j+1)(j+1)}} \quad \text{(F.2)}$$

$$\begin{pmatrix} j & 2 & j' \\ -m & 0 & m' \end{pmatrix} = (-1)^{j-m} \frac{3m^2 - j(j+1)}{\sqrt{(2j-1)j(2j+1)(j+1)(2j+3)}} \quad \text{(F.3)}$$

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ 0 & 0 & 0 \end{pmatrix} = (-1)^g \frac{\sqrt{(2g-2j_1)!(2g-2j_2)!(2g-2j_3)!}}{(2g+1)!} \otimes \frac{g!}{(g-j_1)!(g-j_2)!(g-j_3)!} \quad \text{(F.4)}$$

jika $2g = j_1 + j_2 + j_3 = genap$

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ 0 & 0 & 0 \end{pmatrix} = 0 \quad \text{(F.5)}$$

jika $j_1 + j_2 + j_3 = gasal$

Berlaku pula

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \begin{pmatrix} j_2 & j_3 & j_1 \\ m_2 & m_3 & m_1 \end{pmatrix} = \begin{pmatrix} j_3 & j_1 & j_2 \\ m_3 & m_1 & m_2 \end{pmatrix} \quad \text{(F.6)}$$