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ALTERNATING DIRECTION IMPLICIT METHOD FOR SOLVING EQUATIONS OF 2-D HETEROGENEOUS MODEL OF DEEP-BED GRAIN DRYING

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ABSTRACT

In this paper, numerical studies of two-dimensional mathematical model are presented for deep-bed grain drying based on heat and mass transfer. The two-dimensional dynamic equations are solved numerically by finite difference method with alternating direction implicit algorithm and then applied to simulate humidity and temperature profile of drying gas across dryers together with moisture content and temperature of grain. Line by line algorithm was applied to overcome multidimensional direction. The capabilities of these models were compared with experimental data had been conducted, under variable conditions such as temperature and absolute humidity of drying gas and moisture content of grains. The simulation results show that the two-dimensional dynamic modelling of corn-grain drying can predict the dynamic of drying process.

NOTATION

A	coeficient of algebraic equation		r	radial direction coordinate	m
a	specific surface per volume of bed	m^{-1}	R_C	radius of deep bed	m
Cp_a	specific heat of gas	$J\ kg^{-1}\ K^{-1}$	R_P	radius of grain	m
Cp_p	specific heat of grain	$J\ kg^{-1}\ K^{-1}$	S	source term	
Cp_v	specific heat of water vapor	$J\ kg^{-1}\ K^{-1}$	t	time	s
$D_{z\ eff}$	axial effective difusivity	$m^2\ s^{-1}$	T_b	drying gas temperature	K
$D_{r\ eff}$	radial effective difusivity	$m^2\ s^{-1}$	T_p	grain temperature	K
h	Heat transfer coefficient	$J\ m^{-2}\ s^{-1}\ K^{-1}$	U	linier air velocity	$m\ s^{-1}$
ho	outside heat transfer coefficient of column dryer	$J\ m^{-2}\ s^{-1}\ K^{-1}$	X	grain moisture content	kg moisture / kg dry grain
hi	inside heat transfer coefficient	$J\ m^{-2}\ s^{-1}\ K^{-1}$	Y	absolute air humidity	kg moisture / kg dry air
k_m	Mass transfer coefficient	$kg\ m^{-2}\ s^{-1}$	Y^*	surface moisture	kg moisture /

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L	height of bed (thickness)	m
P_w	vapor pressure of water	mmHg
P_{ws}	vapor pressure of water in the surface of grain	mmHg

Greek Symbols

ΔH_v	vaporization heat of water	$J\ kg^{-1}$
Δx	wall thickness	m
λ_{eff}	Effective thermal conductivity	$J\ m^{-1}\ s^{-1}\ K^{-1}$
δ	thermomigration coefficient	K^{-1}
ε_b	bed porosity	
ε_P	grain porosity	
ρ_a	density of air	$kg\ m^{-3}$
ρ_P	density of grain	$kg\ m^{-3}$
Φ	relative humidity	decimal
ϕ	dependent variable	
ζ	phase conversion factor	

Superscripts

*	equilibrium condition
cr	critical condition

	content of grain	kg dry air
z	axial direction coordinate	m

Subscripts

a	air phase
b	bed or air phase
eff	effective
m	mass type
o	initial
p	grain phase
r	radial
s	surface
v	water vapor
w	water
W	wall
z	axial
∞	environment
i	inlet

1. INTRODUCTION

Heat and mass transfer that occurs during drying with internal vaporization can involve quite complex physical mechanisms that are strongly coupled and highly nonlinear. Consequently, in the past, numerical computation has been the preferred option for solving comprehensive sets of equations. In particular, such models must be supplied with numerous physical properties that are often functions of one or two variables (temperature and moisture content). Even more problematic, some of these parameters are both sensitive and poorly known.

Considerable work has been done in the past decades on the development of theory and mathematical models for grain drying [Thompson *et al*. (1968), Palancz (1985), Abid *et al*. (1990), Courtois (1997), Lopez *et al*. (1998)]. Comprehensive reviews of these models and computer simulation methods are available in the literature [Parry (1985), Franca *et al*. (1994)]. Most of these models are based on systems of coupled nonlinear partial differential equations where temperature and moisture content are the primary variables.

It is clear that the majority of the work - the design of dryer and the improvement of the drying process - is still closely related with empirical knowledge. There is no real evidence of the use of comprehensive models, and typically experiments, curve fitting, and global formulation expressed at scale of a whole dryer are the most popular research tools. Here, mathematical models for grain drying configurations had been developed, based on principles of mass and energy conservation and on empirical relations for heat and mass transfer between phases. The two-dimensional modelling has been developed that known as two-phase model or heterogeneous model [Sitompul *et al.* (2000)].

In view of numerical studies, researchers have been developed numerical method for solving the coupled-partial differential equations. Numerical simulation of deep-bed grain drying has been developed by researchers that is considering drying air phase and grain phase, for malt drying [Lopez *et al.* (1997)], barley drying [Sun *et al.* (1997)], hazelnut drying [Lopez *et al.* (1998)], corn drying [Thompson *et al.* (1968), Courtois (1997)], wheat drying [Giner *et al.* (1996)], and others [Franca *et al.*(1994)]. However, these models have been developed were not a complete model, only one-dimensional, and solve them by finite-difference scheme.

The aim of this research is numerical studies of two-dimensional grain drying models. The second order discretization scheme was developed to predict differential terms by finite-difference method. The two-dimensional mass and heat transfer equations have been developed in the drying gas phase in which consider convective and diffusive mechanism, and the coupled of heat and mass transfers within intraparticle is considered in the grain phase. In order to verify the mathematical models, an experimental prototype of deep-bed grain dryer was built, and the characteristics of the dryers were studied. In order to simulate the drying process using this model, a computer code has been developed using the set of equations proposed in the following section.

2. MATHEMATICAL MODEL

In this paper, we proposed two-phase model for deep-bed dryer by taking into account the conservation of mass and energy within the bed and the spherical grains. The assumptions were made in order to simplify the mathematical modelling and computation, i.e. (a) no shrinkage during drying, (b) uniform grain kernels are in size and internally homogeneous/isotropic spheres, (c) moisture migration path within each particle is in the radial direction only, (d) no conduction between close grains, and (e) moisture transfer within the corn grain is controlled by liquid diffusion only [Sitompul *et al.* (1999)].

The governing equations of moisture and heat balance for describing humidity and temperature of the drying gas are written in a two-dimensional cylindrical coordinate (r, z) system. The moisture balance can be written as follows:

$$\frac{\partial(\rho_a Y)}{\partial t} = \frac{\partial}{\partial z}\left(D_{z\,\mathrm{eff}}\frac{\partial(\rho_a Y)}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\,D_{r\,\mathrm{eff}}\frac{\partial(\rho_a Y)}{\partial r}\right) - \frac{\partial(\rho_a U_z Y)}{\partial z} - \frac{1}{r}\frac{\partial(r\,\rho_a U_r Y)}{\partial r} + k_m\left(Y^* - Y\right)a\left(\frac{1-\varepsilon_b}{\varepsilon_b}\right) \tag{1}$$

Heat balance can be written as follows:

$$\frac{\partial\left(\rho_a\left(Cp_a + YCp_v\right)T_b\right)}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_{az\,\mathrm{eff}}\frac{\partial T_b}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\lambda_{ar\,\mathrm{eff}}\frac{\partial T_b}{\partial r}\right) - \frac{\partial}{\partial z}\left(\rho_a\left(Cp_a + YCp_v\right)U_z T_b\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\rho_a\left(Cp_a + YCp_v\right)U_r T_b\right) - \left(k_m \Delta H_v\left(Y^* - Y\right) + h\left(T_b - T_p\right)\right)a\left(\frac{1-\varepsilon_b}{\varepsilon_b}\right) \tag{2}$$

The governing equations of moisture and heat balance for describing moisture content and temperature of the grains are written in a one-dimensional spherical coordinate (r) system are as follows:

$$\frac{\partial(\rho_p X)}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_{p\,eff}\,\rho_p\frac{\partial X}{\partial r})+\frac{\delta}{r^2}\frac{\partial}{\partial r}(r^2 D_{p\,eff}\,\rho_p\frac{\partial T_p}{\partial r}) \qquad (3)$$

$$\rho_p Cp_p\frac{\partial T_p}{\partial t}=\frac{\lambda_{p\,eff}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_p}{\partial r}\right)-\zeta\rho_p\varepsilon_p\Delta H_v\frac{\partial X}{\partial t} \qquad (4)$$

For this study, model parameters such as $\lambda_{az\ eff}$, $\lambda_{ar\ eff}$, $D_{z\ eff}$, $D_{r\ eff}$ taken as constant parameters along the bed, also for U_z, ρ_a and ρ_p, while $U_r=0$ will be considered. Note that symbol ζ in this equation describes phase conversion factor. However, the effect of momentum balance for grain drying process is not yet considered.

Computation of the equilibrium moisture content of the drying gas having direct contact with the grain surface the following relations can be used [Zahed *et al* (1992)] :

$$p_w=100\exp\left[27.0214-\left(\frac{6887}{T_b}\right)-5.31\ln\left(\frac{T_b}{273.16}\right)\right] \qquad (5)$$

$$p_{w_s}=\Phi\, p_w \qquad (6)$$

$$Y^*=0.622\frac{p_{w_s}}{760-p_{w_s}} \qquad (7)$$

Thompson *et al.* (1968) proposed Φ parameter or equilibrium moisture content relation for corn grain drying:

$$\Phi=1-\exp\left(-8.6541\text{x}10^{-5}\left(100X^*\right)^{1.8634}\left(T_b-273.16+49.81\right)\right) \qquad \text{for } X_s\le X^{cr} \qquad (8)$$

The boundary and initial conditions for all equation developed in above are given elsewhere [Sitompul *et al.* (2000), Sitompul (1994)].

3. NUMERICAL SOLUTION

The heat and mass transfer equations in the fluid and particle phases have been developed in the previous section are coupled, non-linear, unsteady, and involve variable transport properties. The equations are solved numerically by using finite difference method with alternating direction implicit method algorithm applied to two-dimensional cylindrical coordinate together with boundary and initial conditions. The most attractive feature of this method is that the solution converges fast.

3.1. Domain Computation and Boundary Condition

By employing the discretization scheme, the computed domain are divided into a number of control volumes with uniform grid spacing applied on both column dryer and grain section as shown in Fig.1 and Fig.2 respectively.

3.2. Discretization Scheme

The second order discretization was employed to reduce discretization error of differential approximation. The discretization of fluid-phase differential terms is in implicit and explicit at alternate time intervals for the axial and radial direction. The discretization of grain-phase differential terms is in implicit and explicit at alternate time intervals in accordance with fluid phase algorithm. Discretization for time derivative is made use of backward difference method, while spatial derivative are discretized by central difference one. The implicit discretization for all governing equations will give set of linear algebraic equations that can solved using tridiagonal matrix routine (TDMA algorithm) [Davis (1984), Sitompul *et al.* (1999), Hoffmann *et al.* (1993)].

Fig.1. Domain computation and boundary conditions of cylindrical column dryer

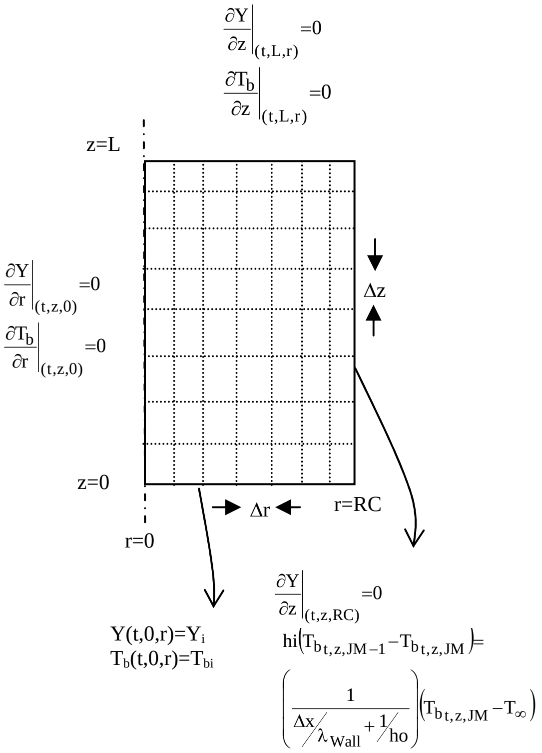
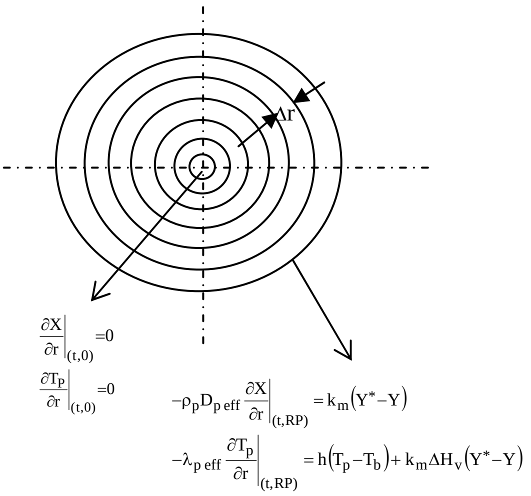


Fig.2. Domain computation and boundary conditions of spherical grain



Generally, the discretization of differential terms are as follows, in which ϕ expresses dependent variables such as drying air temperature (T_b) and absolute humidity (Y), grain temperature (T_p) and moisture content (X):

$$\frac{\partial\phi}{\partial t}=\frac{\phi^n_{i,j}-\phi^{n-1}_{i,j}}{\Delta t}\;;\qquad \frac{\partial\phi}{\partial z}=\frac{\phi^n_{i+1,j}-\phi^n_{i-1,j}}{2\Delta z}\;;\qquad \frac{\partial^2\phi}{\partial z^2}=\frac{\phi^n_{i+1,j}+\phi^n_{i-1,j}-2\phi^n_{i,j}}{(\Delta z)^2}\;;$$

$$\frac{\partial\phi}{\partial r}=\frac{\phi^n_{i,j+1}-\phi^n_{i,j-1}}{2\Delta r}\;;\qquad \frac{\partial^2\phi}{\partial r^2}=\frac{\phi^n_{i,j+1}+\phi^n_{i,j-1}-2\phi^n_{i,j}}{(\Delta r)^2} \tag{9}$$

After rearrangement of the equations above, the discretization equations for two-dimensional model (drying gas phase only) can be written as follows:

$$-A_{i-1,j}\,\phi^n_{i-1,j}+A_{i,j}\,\phi^n_{i,j}-A_{i+1,j}\,\phi^n_{i+1,j}+A_{i,j-1}\,\phi^n_{i,j-1}+A_{i,j+1}\,\phi^n_{i,j+1}=S^{n\text{-}1}_{i,j} \tag{10}$$

However, discretization for one-dimensional model (grain phase only) can be written as follows:

$$-A_{i,j,k-1}\,\phi^n_{i,j,k-1}+A_{i,j,k}\,\phi^n_{i,j,k}-A_{i,j,k+1}\,\phi^n_{i,j,k+1}=S^{n\text{-}1}_{i,j,k} \tag{11}$$

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3.3. Solution of Algebraic Equations

The multidimensional discretization equations applied to two-dimensional partial differential equations results set of algebraic equations that form banded matrix. Direct methods for solving the algebraic equations arising in two-or three-dimensional problems are much more complicated and require rather large amounts of computer storage and time. The use of a direct method is usually not economical.

The simplest of all iterative methods is the Gauss-Seidel method in which the values of the variable are calculated by visiting each grid point in a certain order. In the beginning, these represent the initial guess from the previous iteration. The Gauss-Seidel method does not always converge. A criterion has been satisfied to guarantee the convergence of its method.

A convenient combination of the direct method for one-dimensional situations (tridiagonal matrix algorithm-TDMA) and the Gauss-Seidel method can now be formed. We shall choose a grid line, assume that the neighboring lines are known from their latest values and solve for the chosen line by the TDMA. We shall follow this procedure to sweep all the lines in one direction and repeat it for other direction [Hoffmann *et al.* (1993)].

For the sweeping x-direction:

$$-A_{i-1,j}\,\phi^{n}_{i-1,j} + A_{i,j}\,\phi^{n}_{i,j} - A_{i+1,j}\,\phi^{n}_{i+1,j} = S^{n-1}_{i,j} + A_{i,j-1}\,\phi^{n-1}_{i,j-1} + A_{i,j+1}\,\phi^{n-1}_{i,j+1}$$

while explicit discretization for particle equations.

For the sweeping y-direction:

$$-A_{i,j-1}\,\phi^{n}_{i,j-1} + A_{i,j}\,\phi^{n}_{i,j} - A_{i,j+1}\,\phi^{n}_{i,j+1} = S^{n-1}_{i,j} + A_{i-1,j}\,\phi^{n-1}_{i-1,j} + A_{i+1,j}\,\phi^{n-1}_{i+1,j}$$

while implicit discretization for particle equations.

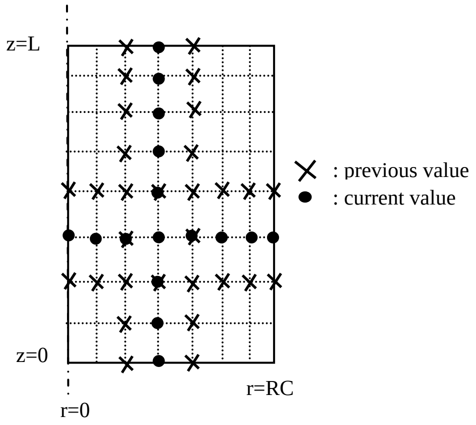

Fig.3. Sweeping representation of the line by line method of drying gas phase

The cross point from Fig.3 presents the latest value of the variables (ϕ), while the dotted point presents the variables value will be find by TDMA algorithm.

4. RESULTS AND DISCUSSION

In this study, a deep bed dryer was designed and built elsewhere [Sitompul *et al.* (2000)]. The numerical model is validated by comparing the computed results with experimental results of the corn-grain drying in the deep-bed dryer apparatus. The relevant experimental parameters used for model validation are listed in Table 1. Moisture profiles along the dryer were calculated for bed thickness of 0.05 m. The simulation results are depicted in Fig.4 to Fig.7.

Table 1. Parameter for the simulation (part of them are taken from experiment)

Parameter	Value	Parameter	Value
Y_i	0.015	$D_{p\ eff}$	$5.98x10^{-11}$
T_{bi}	333	$\lambda_{P\ eff}$	$2.29x10^{-2}$
X_o	0.6	$\lambda_{az\ eff}$	$3.0x10^{-2}$
T_{po}	25	$\lambda_{ar\ eff}$	$2.7x10^{-2}$
U_o	0.09	δ	0.08
a	800	ε_b	0.35
Cp_a	1012	ε_p	0.45
Cp_v	2030	ρ_a	1.057
Cp_p	3505.36	ρ_p	1350
ΔH_v	$2.5x10^6$	k_m	$2.77x10^{-2}$
L	0.05	$D_{z\ eff}$	$1.92x10^{-4}$
h	28	$D_{r\ eff}$	$1.05x10^{-5}$

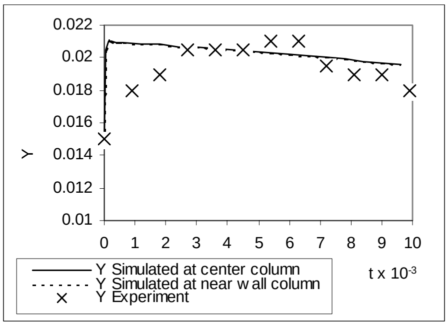

Fig.4. Dynamic evolution of output drying gas humidity at center and near wall dryer column.

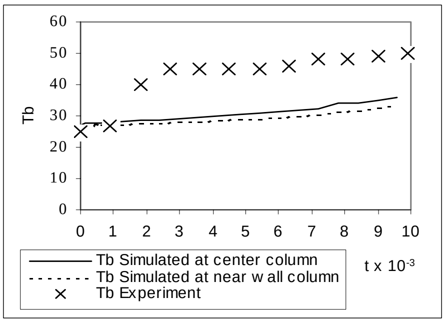

Fig.5. Dynamic evolution of output drying gas temperature at center and near wall dryer column.

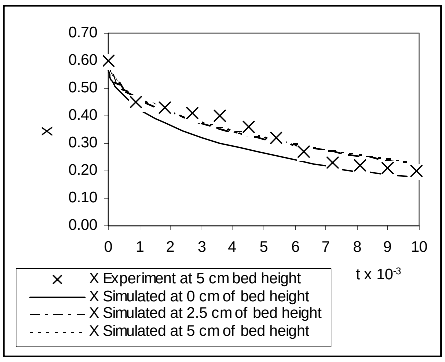

Fig.6. Dynamic evolution of average grain moisture content at various position of bed.

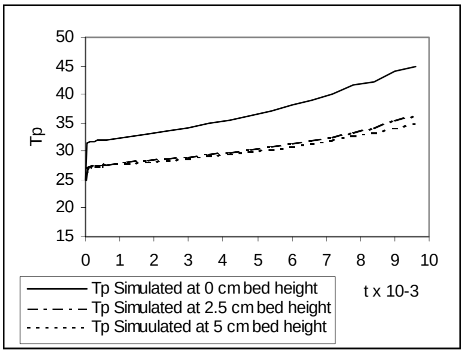

Fig.7. Dynamic evolution of average grain temperature profile at various position of bed.

The determination of step-size combination applied to coupled equations have a difficult one to get the stable solution and realistic physically. The time step (Δt) would have to be small enough, while the spatial step would be reduced to improve the spatial accuracy. In this study, the column dryer was divided into 6x6 uniform grid spacing and 10 uniform grid spacing for grain, while the time step was 0.01 seconds. The

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combination of this spatial step (Δz, Δr_c, and Δr_p) and time step (Δt) results the stable solution and realistic physically.

At initial periods, the bed is predominantly occupied by initial wet grains with large enough moisture must be removed, and that can cause increasing humidity of the outlet drying gas as shown in Fig.4 and the low temperature of the drying air as shown in Fig.5. As can be seen from Fig.4, the humidity of the outlet drying gas decreases at nearly finished drying time due to a little moisture removed from the dried grains, during the course of drying process. Fig.5 shows temperature profile of the outlet drying gas across bed at center and near wall of the column dryer. Temperature profile of the outlet drying gas increases as decreasing the moisture removal.

Fig.6 shows predictions of average moisture content across the grain as the drying proceeds through the bed. The moisture content decrease during removal of the moisture depends on the moisture diffusivity, which depends on moisture content and temperature of grain. As shown on Fig.6, the removal of moisture up to about 0.15 kg moisture/kg dry grain from initial moisture content 0.6 kg moisture/kg dry grain takes place at approximately within range 2.5-3 hours under certain variable conditions. The simulated grain temperature gradually increases throughout the drying proceed as shown in Fig.7, while the temperature of the grain layer on the exhaust drying gas increases near to the wet bulb temperature of the drying air.

5. CONCLUSIONS

Set of partial differential equations can be solved numerically by using finite difference method with alternating direction implicit methods (ADIM) algorithm. Line by line method for sweeping of multidimensional direction can successfully give solution of algebraic equations. As can be seen in the above models, four model parameters are involved as to bring difficulties in determining good ones. Thereafter, the model can be used to simulate the drying process with good accuracy across a wide range of drying conditions.

The two-dimensional mathematical models were developed in this paper to simulate absolute humidity and temperature of drying air, which based on mass and heat transfer in the deep-bed drying of corn grains. The one-dimensional models also were developed to simulate heat and mass transfer process in corn grain drying.

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