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APPLICATION OF REDUCED-ORDER LPV CONTROLLER TO JET ENGINE COMPRESSOR MODEL

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This paper proposes a reduced-order controller of linear parameter varying systems. The parameters are available for measurement while their ranges and rates of variation are assumed to be bounded. A balancing parameter varying systems is first presented. Furthermore, a singular perturbation method of linear time invariant systems is generalized to reduce the order of the balanced systems. Based on the reduced-order model the low-order parameter varying controllers are designed by using parameter dependent H∞ synthesis. Effectiveness of the proposed model reduction method is verified by applying it to jet engine compressor model. Finally, the closed-loop performance of the full-order LPV controllers is compared with thatof reduced-order LPV controllers.

Keywords: bounded parameter variation rates; singular perturbation method; Lyapunov differential inequalities.

1. Introduction

Design techniques for constructing linear parameter varying (LPV) controller with guaranteed H∞ performance usually produce controllers that have the same order as that of the model. Thus, the application of these design techniques to high order models will produce high order controllers. The design and analysis of highorder controllers demand more numerical difficulties while it implementation is very complex. Finding low-order controllers for linear parameter varying system is particularly useful from computation point of view.

A reduced-LPV controller will be found through model reduction. The extension of balanced truncation (BT) to reduce the order of LPV systems has been studied by some authors [El-Zobaidi and Jaimukha (1998); Wood(1996)]. Balanced singular perturbation approximation (BSPA) of linear time invariant (LTI) systems have been published by Liu and Anderson [1989] and Saragih and Yoshida [1998]. Further, Widowati et al. [2004] generalized the BSPA method to reduce the model order of unstable LPV systems with unbounded parameter variation rates.

In this paper we propose generalization of the BSPA method to reduce the LPV systems with bounded parameter variation rates. In comparison with method proposed in Wood [1996], this paper uses BSPA method, whereas in Wood [1996] BT method was used to reduce a high order-model. In BT method, the reducedorder model is obtained by truncating balanced states corresponding to smaller Qe-singular values. In BSPA method, all balanced states are first decomposed into the slow and fast modes by defining the smaller Qe-singular values as the fast mode and the rest as the slow mode. Next, a reduced-order model can be obtained by setting the velocity of the fast mode equal to zero.

From reduced-order model, the reduced-LPV controller is then designed. There are some techniques to design LPV controller [Becker (1994); Lee (1997); Wu et al. (1996)]. In this paper we use design technique developed by Lee [1997] for constructing LPV controller. To verify the validity of the proposed method it is applied to model reduction for the jet engine compressor. The control design objectives of the jet engine compressor are to maintain the state variables inside a neighborhood of the equilibrium (the origin) and to keep derivative of the control input at reasonable level.

2. Induced L_2-norm Analysis

Before defining parameter-dependent stability for LPV systems using parameter dependent Lyapunov functions, we introduce the concept of the set of feasible trajectories.

Given a compact subset $P \subset R^s$, finite non-negative numbers $\{\nu\}_i^s = 1$ with $\nu := [\nu_1 \cdots \nu_s]^T$. Define [Wu et al. (1996)] the parameter ν-variation set as $F_p^\nu := \{\rho(t) \in C^1(R, R^s) : \rho(t) \in P, |\dot{\rho}_i| \leq \nu_i, i = 1, 2, ..., s\}$, where C^1 stands for the class of piecewise continuously differentiable functions.

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Consider the nth-order LPV systems with bounded parameter variation rates, G_p^{v} , is represented by

$$
\begin{aligned}
\dot{x}(t) &= A(\rho(t),\dot{\rho}(t))x(t) + B(\rho(t),\dot{\rho}(t))u(t), \\
y(t) &= C(\rho(t),\dot{\rho}(t))x(t) + D(\rho(t),\dot{\rho}(t))u(t), \qquad \rho(t) \in \mathcal{F}_P^{v}
\end{aligned} \tag{1}
$$

The matrices A : $R^s \rightarrow R^{n\times n}$, B : $R^s \rightarrow R^{n\times m}$, C : $R^s \rightarrow R^{p\times n}$, and D : $R^s \rightarrow R^{p\times m}$ are assumed to be continuous function of the parameter vector $\rho \in R^s$.

The LPV systems G_p^{v} is Qe-stable [Wood (1996); Wu et al. (1996)] if there exits real differentiable positive-definite matrix function P(ρ (t)) = P^T (ρ (t)) > 0 such that

$$
\sum_{i=1}^{s}(\dot{\rho}_i\frac{\partial P(\rho(t))}{\partial \rho_i}) + A^T(\rho(t),\dot{\rho}(t))P(\rho(t)) + P(\rho(t))A(\rho(t),\dot{\rho}(t)) < 0,\ \rho(t) \in \mathcal{F}_P^{v} \tag{2}
$$

The LPV systems G_p^{v} is said to be Qe-stabilizable if there exists a continuous matrix function F(ρ) : $R^s \rightarrow R^{m\times n}$, such that the following systems is Qe-stable for all $\rho \in F_p^{v}$ (for brevity, the dependence ρ on t is omitted)

$$
\dot{x}(t) = (A(\rho,\dot{\rho}) + B(\rho,\dot{\rho})F(\rho))x(t) \tag{3}
$$

The LPV systems G_p^{v} is said to be Q_e-detectable if there exists a continuous matrix function L(ρ) : Rs $R^s \rightarrow R^{n\times p}$ such that the following systems is Qe-stable for all $\rho \in F_p^{v}$

$$
\dot{x}(t) = (A(\rho,\dot{\rho}) + L(\rho)C(\rho,\dot{\rho}))x(t). \tag{4}
$$

The induced L2 norm of a Qe-stable LPV systems, G_p^{v} , with zero initial conditions, is defined as

$$
\|G_P^{v}\|_{i,2} := \sup_{\rho\in\mathcal{F}_P^{v}}\ \sup_{u\neq 0, u\in\mathcal{L}_2} \frac{\|y\|_2}{\|u\|_2}. \tag{5}
$$

Lemma 2.1. [Wood (1996)] Given the LPV system G_p^{v} if there exits W(ρ) = W^T (ρ) > 0 such that the following two conditions hold,

$$
\gamma^2 I - D^T(\rho)D(\rho) > 0.
$$

and

$$
\begin{bmatrix} E(\rho(t),\dot{\rho}) & W(\rho)B(\rho) & C^T(\rho) \\ B^T(\rho)W(\rho) & -\gamma I & D^T(\rho) \\ C(\rho) & D(\rho) & -\gamma I \end{bmatrix} < 0. \tag{6}
$$

fot all $\rho \in F_p^{v}$,

where $E(\rho,\dot{\rho}) = A^T(\rho,\dot{\rho})W(\rho)A(\rho,\dot{\rho}) + \sum_{i=1}^{s}\left(\dot{\rho}_i\frac{\partial w}{\partial_{\rho i}}\right)$ Then the system G_p^{v} is Qe-stable and $\| G_p^{v} \|_{i,2} <$ γ for any allowable trajectories $\rho \in F_p^{v}$

Furthermore, if the two above conditions are satisfied, then it is said that the LPV system satisfies a Q_e γ performance level.

3. Balancing of LPV Systems

Suppose that the LPV system G_p^{v} is Q_e-stable. Let Q(ρ) and P(ρ) be observability and controllability Gramians satisfying differential Lyapunov inequalities:

$$
\sum_{i=1}^{s}(\dot{\rho}_i\frac{\partial Q(\rho)}{\partial \rho_i}) + A^T(\rho,\dot{\rho})Q(\rho) + Q(\rho)A(\rho,\dot{\rho}) + C^T(\rho,\dot{\rho})C(\rho,\dot{\rho}) < 0. \tag{7}
$$

$$-\sum \left(\dot{\rho}_i \frac{\partial P(\rho)}{\partial \rho_i}\right) + A(\rho,\dot{\rho})P(\rho) + P(\rho)A^T(\rho,\dot{\rho}) + B(\rho,\dot{\rho})B^T(\rho,\dot{\rho}) < 0, \forall \rho \in \mathcal{F}_{\mathcal{P}}^{\nu}. \tag{8}$$

where $Q(\rho) = Q^T(\rho) > 0$, $P(\rho) = P^T(\rho) > 0$ are differentiable.

The Q_e-singular values of the LPV system is defined as

$$\sigma_i(\rho) = \sqrt{\lambda_i(Q(\rho)P(\rho))},\ i = 1, 2, 3, \cdots, n. \tag{9}$$

Now let a continuous matrix function $T(\rho)$ [Wood (1996)] where $T^{-1}(\rho)$ exists for all $\rho \in F_p^{\nu}$, and suppose that $T(\rho)$ has continuous partial derivatives with respect to the parameters in the parameter vector. Under these conditions, define the balancing parameter-varying state transformation $x(t) = T(\rho)\tilde{x}(t)$. This gives

$$\begin{aligned} \dot{\tilde{x}}(t) &= (T^{-1}(\rho)A(\rho,\dot{\rho})T(\rho)) - T^{-1}(\rho)\dot{T}(\rho))\tilde{x}(t) + T^{-1}(\rho)B(\rho,\dot{\rho})u(t), \\ y(t) &= C(\rho,\dot{\rho})T(\rho)\tilde{x}(t) + D(\rho,\dot{\rho})u(t), \rho \in \mathcal{F}_{\mathcal{P}}^{\nu}. \end{aligned} \tag{10}$$

where $\dot{T}(\rho) = \sum_{i=1}^{s}\left(\dot{\rho}_i \dfrac{\partial T(\rho)}{\partial_{\rho i}}\right)$ Then the Gramians are transformed to

$$\tilde{Q}(\rho) = \tilde{P}(\rho) = \Sigma(\rho). \tag{11}$$

Where

$$\begin{aligned} &\tilde{Q}(\rho) = T^T(\rho)Q(\rho)T(\rho),\ \tilde{P}(\rho) = T^{-1}(\rho)P(\rho)T^{-T}(\rho), \\ &\Sigma(\rho) = diag(\sigma_1(\rho), \cdots, \sigma_n(\rho)), \\ &\sigma_1(\rho) \geq \sigma_2(\rho) \geq \cdots \geq \sigma_n(\rho) > 0, \forall \rho \in \mathcal{F}_{\mathcal{P}}^{\nu}. \end{aligned}$$

The LPV system which satisfies the equation(11) is called balanced systems.

4. Main Results

In this section, we propose results regarding the generalization of the BSPA method of LTI systems to reduce the order of LPV systems with bounded parameter variation rates. Balanced parameter varying realization (10) can be rewritten as follows.

$$\begin{aligned} G_{\mathcal{P}}^{\nu} &:= \left[\begin{array}{c|c} T^{-1}(\rho)A(\rho,\dot{\rho})T(\rho) - T^{-1}(\rho)\sum_{i=1}^{s}(\dot{\rho}_i\frac{\partial T(\rho)}{\partial \rho_i}) & T^{-1}(\rho)B(\rho,\dot{\rho}) \\ \hline C(\rho,\dot{\rho})T(\rho) & D(\rho,\dot{\rho}) \end{array}\right] \\ &:= \left[\begin{array}{c|c} \bar{A}(\rho,\dot{\rho}) & \bar{B}(\rho,\dot{\rho}) \\ \hline \bar{C}(\rho,\dot{\rho}) & D(\rho,\dot{\rho}) \end{array}\right] \end{aligned} \tag{12}$$

Furthermore, partition the balanced parameter varying conformably with the Gramian diag($\Sigma_1(\rho), \Sigma_2(\rho)$) as

$$G_{\mathcal{P}}^{\nu} := \left[\begin{array}{cc|c} \bar{A}_{11}(\rho,\dot{\rho}) & \bar{A}_{12}(\rho,\dot{\rho}) & \bar{B}_1(\rho) \\ \bar{A}_{21}(\rho,\dot{\rho}) & \bar{A}_{22}(\rho,\dot{\rho}) & \bar{B}_2(\rho) \\ \hline \bar{C}_1(\rho) & \bar{C}_2(\rho) & D(\rho) \end{array}\right]. \tag{13}$$

with $\bar{A}_{11} \in R^{r\times r}$, $\bar{A}_{12} \in R^{r\times(n-r)}$, $\bar{A}_{21} \in R^{(n-r)\times r}$, $\bar{A}_{22} \in R^{(n-r)\times(n-r)}$, $\bar{B}_1 \in R^{r\times m}$, $\bar{B}_2 \in R^{(n-r)\times m}$, $\bar{C}_1 \in R^{p\times r}$, $\bar{C}_2 \in R^{p\times(n-r)}$.

When the system is balanced, states corresponding to smaller Qe-singular values $\Sigma_2(\rho)$ represent the fast dynamics of the systems (i.e. its states have very fast transient dynamics and decay rapidly to certain steady value). Then by ignoring the dynamic of this fast subsystem, we set to zero the derivative of all states corresponding to $\Sigma_2(\rho)$ to approximate the system (13). This gives reduced-order systems with state-space realization

$$\hat{G}_{\mathcal{P}_r}^{\nu} := \left[\begin{array}{c|c} A_s(\rho,\dot{\rho}) & B_s(\rho,\dot{\rho}) \\ \hline C_s(\rho,\dot{\rho}) & D_s(\rho,\dot{\rho}) \end{array}\right]. \tag{14}$$

Where

$$\begin{aligned} A_s(\rho,\dot{\rho}) &= \bar{A}_{11}(\rho,\dot{\rho}) - \bar{A}_{12}(\rho,\dot{\rho})(\bar{A}_{22}(\rho,\dot{\rho}))^{-1}\bar{A}_{21}(\rho,\dot{\rho}), \\ B_s(\rho,\dot{\rho}) &= \bar{B}_1(\rho,\dot{\rho}) - \bar{A}_{12}(\rho,\dot{\rho})(\bar{A}_{22}(\rho,\dot{\rho}))^{-1}\bar{B}_2(\rho,\dot{\rho}), \\ C_s(\rho,\dot{\rho}) &= \bar{C}_1(\rho,\dot{\rho}) - \bar{C}_2(\rho,\dot{\rho})(\bar{A}_{22}(\rho,\dot{\rho}))^{-1}\bar{A}_{21}(\rho,\dot{\rho}), \\ D_s(\rho,\dot{\rho}) &= D(\rho,\dot{\rho}) - \bar{C}_2(\rho,\dot{\rho})(\bar{A}_{22}(\rho,\dot{\rho}))^{-1}\bar{B}_2(\rho,\dot{\rho}). \end{aligned}$$

assuming that $\bar{A}22(\rho,\dot{\rho})$ is invertible $\forall\ \rho \in F_p^{\nu}$.

If the LPV system is not Q_e-stable, then the system can not be directly approximated using the method of previous results. To be able to reduce this unstable LPV systems, we extend of BSPA method by approximating contractive right coprime factorizations (CRCF) with a procedure as follows. Suppose that G_p^{ν} is Q_e-stabilizable and Qe-detectable. Define CRCF of the G_p^{ν} as [Wood (1996)]

$$\mathcal{G}_{\mathcal{P}}^{\nu} := \begin{bmatrix} N_{\mathcal{P}}^{\nu} \\ M_{\mathcal{P}}^{\nu} \end{bmatrix} := \left[\begin{array}{c|c} A(\rho,\dot{\rho}) + B(\rho,\dot{\rho})F(\rho) & B(\rho,\dot{\rho})S^{-1/2}(\rho,\dot{\rho}) \\ \hline C(\rho,\dot{\rho}) + D(\rho,\dot{\rho})F(\rho) & D(\rho,\dot{\rho})S^{-1/2}(\rho,\dot{\rho}) \\ F(\rho,\dot{\rho}) & S^{-1/2}(\rho,\dot{\rho}) \end{array}\right]. \tag{15}$$

where

$$F(\rho,\dot{\rho}) = -S^{-1}(\rho,\dot{\rho})(B^T(\rho,\dot{\rho})X(\rho) + D^T(\rho,\dot{\rho})C(\rho,\dot{\rho})),$$

$$S(\rho) = I + D^T(\rho,\dot{\rho}) \times D(\rho,\dot{\rho}),$$

$$R(\rho,\dot{\rho}) = I + D(\rho,\dot{\rho})D^T(\rho,\dot{\rho})$$

and

$$X(\rho) = X^T(\rho) > 0$$

is a solution of the generalized control Riccati differential inequality (GCRDI)

$$\begin{aligned}&\textstyle\sum_{i=1}^{s}(\dot{\rho}_i \frac{\partial X(\rho)}{\partial \rho_i}) + (A(\rho,\dot{\rho}) - B(\rho,\dot{\rho})S^{-1}(\rho,\dot{\rho})D^T(\rho,\dot{\rho})C(\rho,\dot{\rho}))^T X(\rho) + \\ &X(\rho)(A(\rho,\dot{\rho}) - B(\rho,\dot{\rho})S^{-1}(\rho,\dot{\rho})D^T(\rho,\dot{\rho})C(\rho,\dot{\rho})) - X(\rho)B(\rho,\dot{\rho})S^{-1}(\rho,\dot{\rho}) \times \\ &B^T(\rho,\dot{\rho})X(\rho) + C^T(\rho,\dot{\rho})R^{-1}(\rho,\dot{\rho})C(\rho,\dot{\rho}) < 0,\ \forall \rho \in \mathcal{F}_{\mathcal{P}}^{\nu}.\end{aligned} \tag{16}$$

Let $\bar{Q}(\rho) = X(\rho)$ and $\bar{P}(\rho) = (I + Y(\rho)X(\rho))^{-1}Y(\rho)$ are observability and controllability Gramians of CRCF, where $X(\rho)$ solves GCRDI and $Y(\rho)$ solves generalized filtering Riccati differential inequality (GFRDI)

$$\begin{aligned}&-\textstyle\sum_{i=1}^{s}(\dot{\rho}_i \frac{\partial Y(\rho)}{\partial \rho_i}) + (A(\rho,\dot{\rho}) - B(\rho,\dot{\rho})D^T(\rho,\dot{\rho})R^{-1}(\rho,\dot{\rho})C(\rho,\dot{\rho}))Y(\rho) + \\ &Y(\rho)(A(\rho,\dot{\rho}) - B(\rho,\dot{\rho})D^T(\rho,\dot{\rho})R^{-1}(\rho,\dot{\rho})C(\rho,\dot{\rho}))^T - Y(\rho)C^T(\rho,\dot{\rho})R^{-1}(\rho,\dot{\rho}) \times \\ &C(\rho,\dot{\rho})Y(\rho,\dot{\rho}) + B(\rho,\dot{\rho})S^{-1}(\rho,\dot{\rho})B^T(\rho,\dot{\rho}) < 0,\ \forall \rho \in \mathcal{F}_{\mathcal{P}}^{\nu}.\end{aligned} \tag{17}$$

Further, construct balanced G_p^{ν} by using a state transformation matrix function such that the transformed Gramians $\bar{P}(\rho) = \bar{Q}(\rho) = \Sigma(\rho) = \mathrm{diag}(\sigma_1(\rho), \cdots, \sigma_n(\rho))$ and then apply singular perturbation method to obtain $\hat{G}_{pr}^{v} := \begin{bmatrix} N_{pr}^{v} \\ M_{pr}^{v} \end{bmatrix}$ which has rth-order, r < n. Furthermore, form the reduced-order model $\hat{G}_{pr}^{v} = N_{pr}^{v}\left(M_{pr}^{v}\right)^{-1}$.

5. Simulation Results

This section presents an application of the developed method in previous section to reduce a model of jet engine compressor. The closed-loop performance of the full-order LPV controllers will be compared with that of reduced-order controllers.

5.1. *LPV Modeling of The Jet Engine Compressor*

Consider the Moore-Greitzer compressor model [Bruzelius (2004)],

$$\begin{aligned}\dot{\Phi} &= -\Psi_c - 3\Phi R, \\ \dot{\Psi} &= \tfrac{1}{\beta^2}(\Phi + 1 - v\sqrt{\Psi}), \\ \dot{R} &= \sigma R(1 - \Phi^2 - R),\ R(0) > 0.\end{aligned} \tag{18}$$

where Φ is the annulus averaged mass flow coefficient, Ψ is the plenum pressure rise, R is the squared amplitude of circumferential flow asymmetry, Ψ_c the compressor characteristic relating pressure rise in the plenum to the mass flow, v the control which is proportional to the throttle area and β, σ are system dependent constants. Using a state transformation, the non-stall equilibria can be moved to the origin. For simplicity assume that Φ and Ψ are known such that the transformation

$u=\frac{1}{\beta^2}\left(1-\upsilon\sqrt{\Psi}-\Phi\right)$ makes sense. Further, signal u is used at the control input. For an arbitrary non-stall equilibria with $\Phi > 1$ the model (18) can be described as,

$$
\begin{aligned}
\dot{r} &= \sigma(1-\Phi_0^2)r - \sigma r(2\Phi_0\phi+\phi^2) - \sigma r^2,\\
\dot{\phi} &= -\psi + \tfrac{3}{2}(1-\Phi_0^2)\phi - 3\Phi_0 r - \tfrac{1}{2}\phi(\phi^2+3\Phi_0\phi+6r),\\
\dot{\psi} &= -u.
\end{aligned}
\tag{19}
$$

where $\phi = \Phi - \Phi_0$, $\psi = \Psi - \Psi_c(\phi_0)$, and r = R. Furthermore, assume that R and Φ are measured, one LPV systems that represents model (18) is,

$$
\dot{x} = \begin{bmatrix} f_1(\rho) & 0 & 0 \\ -3\Phi_0 & f_2(\rho) & -1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 11 \end{bmatrix} u.
\tag{20}
$$

Where

$$
\begin{aligned}
f_1(\rho) &= \sigma(\dot{\Phi}_0 - \rho_1 - 2\Phi_0\rho_2 - \rho_3),\\
f_2(\rho) &= \tfrac{3}{2}\dot{\Phi}_0 - 3\rho_1 - \tfrac{3}{2}\Phi_0\rho_2 - \tfrac{1}{2}\rho_3.\\
\rho &= \left[r,\ \phi,\ \phi^2\right]^T,\\
x &= \left[r,\ \phi,\ \psi\right]^T.
\end{aligned}
$$

and

$$\dot{\Phi}_0 = 1 - \Phi_0^2.$$

Taking R $\in$ [1, 2], $\dot{R}$ $\in$ [−0.5, 0.5], $\phi \in$ [−0.1, 5], and $\dot{\phi} \in$ [−1, 1] implies that the parameters are confined in the following set

$$
\begin{aligned}
\rho \in \mathcal{F}_{\mathcal{P}}^{\nu} = \{\rho \mid & 0 \le \rho_1 \le 2,\ -0.1 \le \rho_2 \le 5,\ 0 \le \rho_3 \le 25,\\
& -0.5 \le \dot{\rho}_1 \le 0.5,\ -1 \le \dot{\rho}_2 \le 1,\ -10 \le \dot{\rho}_3 \le 10\}
\end{aligned}
$$

The control design objectives are to maintain the state variables inside a neighborhood of the equilibrium (the origin) and to keep derivative of the control input u at reasonable level. This can be translated in to setting the penalty to the control signal and its derivative as $W_u = k_1 + \frac{k_2 s}{s+\upsilon}$ and to the outputs as WT = diag(c_1, c_2). The augmented plant can be written as,

$$
\begin{aligned}
\dot{x} &= \begin{bmatrix} a_{11}(\rho) & 0 & 0 & 0 \\ -3.9 & a_{22}(\rho) & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -100 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -1 \\ -10 \end{bmatrix} u,\\
z &= \begin{bmatrix} 0.001 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0.11 \end{bmatrix} u,\\
y &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w.
\end{aligned}
\tag{21}
$$

where $a_{11}(\rho) = -2.76-4\rho_1-10.4\rho_2-4\rho_3$, $a_{22}(\rho) = -1.035-3\rho_1-1.95\rho_2-0.5\rho_3$, $\sigma = 4$ and $\Phi_0 =$ 1.3, which corresponds to equilibrium pressure $\Phi_c(1.3)$ = 1.9984 or 94% of the peak value $\Phi_c(1)$ = 2.1496, and the weight constants c_1 = 10−3, c_2 = 0.1, k_1 = 0.001, k_2 = 0.1 and v = 100.

5.2. *Design Results*

The LPV system (21) is not Q_e-stable. Hence, we calculate reduced-order models by using generalized singular perturbation via CRCF suggested in the preceding sections. The constrains given by linear matrix inequalities (LMIs) (16)-(17) are parameter dependent i.e., there is an infinite set of LMIs, one for every value of the parameter. These LMIs can be solved by gridding technique [Lee (1997); Wu et al (1996)]. To be able to solve these infinite set LMIs by gridding, some approximations must be made, by gridding the set P with a finite L points $\{\rho_k\}_{k=1}^{L}$. The infinitedimensional variables (X(ρ), Y (ρ)) in inequalities (16)-(17) are approximated by linear combinations of scalar basis functions. The consequence of this approximation is that the number of LMIs becomes $L^s(2^{s+1}+1)$.

For this problem, we pick a gridding of the parameter space P, consisting of 27 points with 3 points in each dimension uniformly (see Table 1).

Table 1. Grid points of the parameter space

	$\rho_2 = -0.1$			$\rho_2 = 2.5$			$\rho_2 = 5$		
$\rho_1 \setminus \rho_3$	0.5	12.5	25	0.5	12.5	25	0.5	12.5	25
0.2	√	√	√	√	√	√	√	√	√
1	√	√	√	√	√	√	√	√	√
2	√	√	√	√	√	√	√	√	√

Basis functions for X(ρ) and Y (ρ) in LMIs (16)-(17) are chosen as follows
$\{f_i(\rho)\}_{i=1}^{27} = \{g_i(\rho)\}_{i=1}^{27} = \{1,\ \rho_1,\ \rho_2,\ \rho_3,\ \rho_1^2,\ \rho_2^2,\ \rho_3^2,\ \rho_1\rho_2,\ \rho_1\rho_3,\ \rho_2\rho_3,\ \rho_1\rho_2^2,\ \rho_1\rho_3^2,\ \rho_1^2\rho_2,\ \rho_3\rho_2^2,\ \rho_1^2\rho_2^2,\ \rho_3^2\rho_2^2,\ \rho_1^2\rho_3^2,\ \rho_1^3,\ \rho_2^3,\ \rho_3^3,\ \rho_1\rho_2^3,\ \rho_1\rho_3^3,\ \rho_1^3\rho_2,\ \rho_2\rho_3^3,\ \rho_1^2\rho_2^3,\ \rho_2^2\rho_3^3,\ \rho_1^2\rho_3^3\}$.
Hence, the parameter dependent X(ρ) and Y (ρ) are in the form of X(ρ) =
$\sum_{i=1}^{27} f_i(\rho)X_i,\ \ Y(\rho) = \sum_{i=1}^{27} f_i(\rho)Y_i$.
Using the LMI control toolbox for MATLAB [Gahinet et al (1995)] running on pentium(R) 4, 2400 MHz, 18x, 512 MB of RAM we obtain optimum solutions X(ρ) and Y (ρ) after 25 iterations (corresponding to CPU time 5489.1 seconds). Furthermore, generalized controllability and observability Gramians are obtained using the solutions X(ρ) and Y (ρ). Then, reduced-order models can be found by applying generalized BSPA method to balanced CRCF of the LPV systems. Based on the reduced order models the low-order LPV controllers are designed. We use the synthesis procedure developed by Lee [1997] to construct LPV controllers which satisfy $Q_{e,\gamma}$ -performance level. The performance level γ giving the optimum solution of output feedback problems is γ_{opt} = 0.131 for full-order controller. CPU time required to solve output feedback problems is shown in Table 2.

Table 2. CPU time required for finding controller

Order of the plant	4	3	2	1
CPU time(seconds)	8371.7	3232.009	711.344	193.688

Results presented in Table 2 show that the average time required for finding controller increasing with the plant order. The impulse responses of the closed-loop system with the full-order, 3rd-, 2nd-, and 1st-order parameter dependent controllers at grid points of parameter space are depicted in Fig.1. From these figures, it can be seen that the closed-loop systems time responses with full-order and 2nd-, 1st-order controllers are stabilizable within 3 seconds. When the LPV model is reduced to 3th-order, the responses reach steady state after much greater than 3 seconds. It may be caused by the difference of 4th- and 3th-Qe-singular values which is smaller than that of 3th- and 2th-, and that of 2th- and 1th-Qe-singular values.

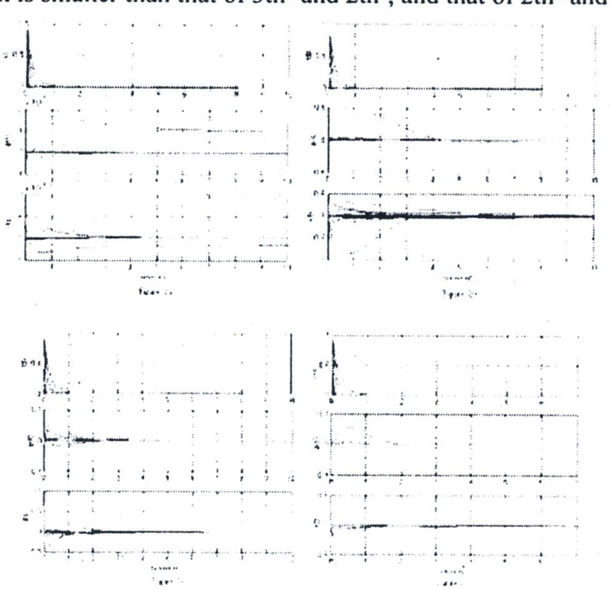

Fig.1. Time responses of closed-loop system with full-order and reduced-order controller at grid points of parameters (2a: full-order, 2b: 3rd-order, 2c: 2nd-order, 2d: 1st-order controller)

6. Conclusions

We have generalized the singular perturbation approach of LTI systems to reduce the order of the LPV systems with bounded parameter variation rates. The reducedorder model was obtained by setting the derivative of all states corresponding to the smaller Qe-singular values of the systems equal to zero. The proposed method was shown to be effective in reducing order of the jet engine compressor model. The reduced-order LPV controller can maintain the performance of the closed-loop system with the full-order LPV controller for all parameter trajectories.

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